Dynamical fixed points in holography

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Based on arXiv:2111.04122

also: arXiv: 1702.01320 (with A.Karapetyan), 1809.08484, 1904.09968, 1912.03566

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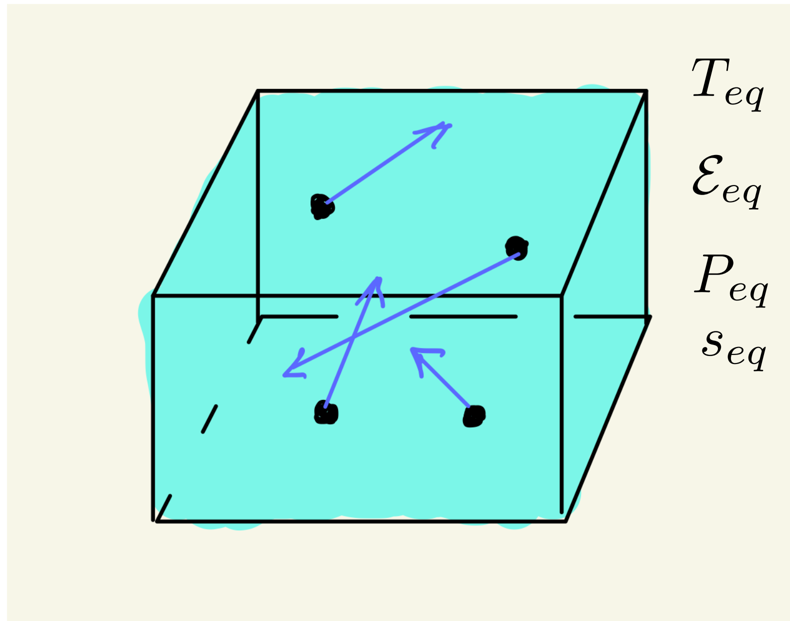
$\Longrightarrow$ Thermodynamic equilibrium



- T_{eq} - the equilibrium temperature
- $\mathcal{E}_{eq}$ - the energy density
- P_{eq} - pressure
- s_{eq} - thermodynamic entropy density

$$\mathcal{F}_{eq} = -P_{eq} = \mathcal{E}_{eq} - s_{eq}\, T_{eq}\,, \qquad d\mathcal{E}_{eq} = T_{eq}\, ds_{eq}$$

$\Longrightarrow$ *Thermodynamic equilibrium* is a late-time attractor of dynamical evolution of isolated interacting quantum system:

$$\lim_{t\to\infty} T_{\mu\nu}(t,\boldsymbol{x}) = \mathrm{diag}\,(\mathcal{E}_{eq}, P_{eq}, \cdots P_{eq})$$

■ $T_{\mu\nu}$ are the component of the stress-energy tensor of the system at time t and the spatial location $\boldsymbol{x}$

$\Longrightarrow$ We also have a theory — **the hydrodynamics** — that describes the approach to that equilibrium (assuming we are not-far from it):

- Given the local energy density $\mathcal{E}$ and the equilibrium equation of state $P_{eq} = P_{eq}(\mathcal{E}_{eq})$ we define the local pressure P

$$\mathcal{E}(t,\boldsymbol{x}) \equiv T_{00}(t,\boldsymbol{x}) \qquad \Longrightarrow \qquad P(t,\boldsymbol{x}) = P_{eq}\left(\mathcal{E}(t,\boldsymbol{x})\right)$$

- and obtain the local entropy density $s(t,\boldsymbol{x})$ and temperature $T(t,\boldsymbol{x})$

$$\mathcal{E} + P = s\,T\,, \qquad d\mathcal{E} = T\,ds$$

- "not-far from equilibrium" is then

$$T \cdot \left| \frac{\partial_\mu \mathcal{E}}{\mathcal{E}} \right| \ll 1 \qquad \underline{\boldsymbol{and}} \qquad T \cdot \left| \nabla_\mu u^\nu \right| \ll 1$$

where $u^\mu = u^\mu(t, \boldsymbol{x})$ is a local fluid 4-velocity, $u^\mu u_\mu = -1$, used to define the hydrodynamic stress-energy tensor

$$T^{\mu\nu} = \underbrace{\mathcal{E}\ u^\mu u^\nu + P\ \Delta^{\mu\nu}}_{\text{"equilibrium'' part}} \qquad + \qquad \underbrace{\mathcal{T}^{\mu\nu}}_{\text{first-order dissipative terms}}$$

- $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$, $\qquad g_{\mu\nu}$ is the background space-time metric

-
$$\mathcal{T}^{\mu\nu} = -\eta\ \sigma^{\mu\nu} - \zeta\ \Delta^{\mu\nu}\ (\nabla \cdot u)$$

where $\sigma^{\mu\nu} \sim \partial^\mu u^\nu$, and $\eta = \eta(\mathcal{E})$, $\zeta = \zeta(\mathcal{E})$ are the shear and the bulk viscosities

- There is no first-principle definition of $\mathcal{S}^\mu$ away from equilibrium; to the first-order in the gradients of the local fluid velocity u^μ,

$$\mathcal{S}^\mu = s\ u^\mu - \frac{1}{T}\ \mathcal{T}^{\mu\nu} u_\nu$$

- from the conservation of the stress-energy tensor,

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \Longrightarrow$$

$$T\ \nabla \cdot \mathcal{S} = \zeta\ \ (\nabla \cdot u)^2 + \frac{\eta}{2} \sigma_{\mu\nu} \sigma^{\mu\nu}\ \ \geq\ 0$$

which is manifestly non-negative, provided the viscosities are positive.

$\Longrightarrow$ As one approaches the equilibrium,

$$\lim_{t\to\infty} u^\mu = u^\mu_{eq} = (1, \mathbf{0}) \qquad \Longrightarrow \qquad \lim_{t\to\infty} T\ \nabla \cdot \mathcal{S} = 0$$

i.e., in the approach to equilibrium the entropy production rate vanishes

We can now provide a formal definition of a dynamical fixed point (DFP):

A *Dynamical Fixed Point* is an internal state of a quantum field theory with spatially homogeneous and time-independent one-point correlation functions of its stress energy tensor $T^{\mu\nu}$, and (possibly additional) set of gauge-invariant local operators $\{\mathcal{O}_i\}$,

and

strictly positive divergence of the entropy current at late-times:

$$\lim_{t\to\infty}\left(\nabla\cdot\mathcal{S}\right) > 0$$

$\Longrightarrow$ Apart from the requirement of the strictly non-zero entropy production rate at late times, characteristics of a DFP coincide with that of the thermodynamic equilibrium.

Why?

$\Longrightarrow$ DFP, *i.e.*, the non-vanishing late-time entropy production in **driven** (open) quantum-mechanical systems/QFT:

- time-dependent coupling constants (quantum quenches)
- time-dependent masses
- time-dependent external EM fields, etc

<u>and</u>

- QFTs in cosmological backgrounds, asymptotically de Sitter space-times in particular

$\Longrightarrow$ To study DFPs means to classify the end-of-time dynamics of <u>*massive*</u> QFTs, in cosmologies with dark energy

The rest of the talk

- A trivial DFP: thermal states of $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) in de Sitter from holography
- Nontrivial DFP

$\Longrightarrow$ Holographic picture for $\mathcal{N} = 4$ SYM in de Sitter

$$S_{\mathcal{N}=4} = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} d^5\xi \sqrt{-g} \left[R + \frac{12}{L^2} \right]$$

$$L^4 = \ell_s^4 \; N g_{YM}^2 \,, \qquad G_5 = \frac{\pi L^3}{2N^2} \,, \qquad 4\pi g_s = g_{YM}^2$$

$\Longrightarrow$ Consider general spatially homogeneous, time-dependent states:

$$ds_5^2 = 2dt \; (dr - A dt) + \Sigma^2 \; d\boldsymbol{x}^2$$

$$A = A(t, r) \,, \qquad \Sigma = \Sigma(t, r)$$

$\Longrightarrow$ We are interested in spatially homogeneous and isotropic states of $\mathcal{N}=4$ SYM in FLRW, so the bulk metric warp approach the AdS boundary $r\to\infty$ as

$$\Sigma = \frac{a(t)r}{L} + \mathcal{O}(r^0)\,, \qquad A = \frac{r^2}{2L^2} + \mathcal{O}(r^1)$$

Indeed, as $r\to\infty$,

$$ds_5^2 = \frac{r^2}{L^2}\underbrace{\left(-dt^2 + a(t)^2 d\boldsymbol{x}^2\right)}_{\text{boundary FLRW}} + \cdots$$

$\Longrightarrow$ Given the metric ansatz, we can derive derive EOMs (without loss of generality we set $L = 2$):

$$
\begin{aligned}
0 &= (d_+\Sigma)' + 2\Sigma'\, d_+ \ln\Sigma - \frac{\Sigma}{2} \\
0 &= A'' - 6(\ln\Sigma)'\, d_+ \ln\Sigma + \frac{1}{2} \\
0 &= \Sigma'' \\
0 &= d_+^2\Sigma - 2A\Sigma' - (4A\Sigma' + A'\Sigma)\, d_+ \ln\Sigma + \Sigma A
\end{aligned}
$$

where

$$
{}' = \frac{\partial}{\partial r}, \qquad \dot{} = \frac{\partial}{\partial t}, \qquad d_+ = \frac{\partial}{\partial t} + A\frac{\partial}{\partial r}
$$

$\Longrightarrow$ These equations can be solve in all generality for arbitrary $a(t)$:

$$A = \frac{(r+\lambda)^2}{8} - (r+\lambda)\,\frac{\dot{a}}{a} - \dot{\lambda} - \frac{r_0^4}{8a^4(r+\lambda)^2}\,,$$

$$\Sigma = \frac{(r+\lambda)a}{2}$$

where

- r_0 is a single constant parameter
- $\lambda(t)$ is an arbitrary function - the leftover diffeomorphism of the 5d gravitational metric reparametrization $r \to \bar{r} = r - \lambda(t)$:

$$A(t,r) \to \bar{A}(t,\bar{r}) = A(t, r+\lambda(r)) - \dot{\lambda}(t)$$

$$\Sigma(t,r) \to \bar{\Sigma}(t,\bar{r}) = \Sigma(t, r+\lambda(t))$$

$\Longrightarrow$

$$ds_5^2 \;\Longrightarrow\; d\bar{s}_5^2 \qquad = 2dt\,(d\bar{r} - \bar{A}dt) + \bar{\Sigma}^2\, d\boldsymbol{x}^2$$

$\Longrightarrow$ Identifying

$$\frac{r_0}{2} \equiv T_0$$

$\Longrightarrow$ from holographic computation of the boundary stress energy tensor,

$$\mathcal{E}(t) = \frac{3}{8}\pi^2 N^2 T(t)^4 + \frac{3N^2}{32\pi^2}\frac{(\dot{a})^4}{a^4}\,, \qquad P(t) = \frac{1}{3}\mathcal{E}(t) - \frac{N^2}{8\pi^2}\frac{(\dot{a})^2\ddot{a}}{a^3}$$
$$T(t) = \frac{T_0}{a(t)}$$

Precisely as expected from the Weyl transformation of the thermal state from Minkowski to FLRW!

$\Longrightarrow$ Holography buys us more:

- Chesler-Yaffe pioneered numerical studies of EF metrics:

$$ds_5^2 = 2dt\ (dr - Adt) + \Sigma^2\ d\boldsymbol{x}^2$$

- such metrics has an **apparent horizon** (AH) at r_{AH}

$$d_+\Sigma\Big|_{r=r_{AH}} = 0 \qquad \Longrightarrow \qquad r_{AH} = \frac{r_0}{a(t)} - \lambda(t)$$

- causal dependence **must** include

$$r \in [r_{AH}, +\infty)$$

- region

$$r < r_{AH}$$

  is causally disconnected from the holographic dynamics and **must be** excised

- AH is a dynamical horizon

- $$\underbrace{\left.\frac{\Sigma^3}{4G_5}\right|_{r=r_{AH}}}_{\text{comoving Bekenstein entropy of the AH}} = \frac{N^2 r_0^3}{128\pi}$$

$$= \underbrace{s_{comoving}}_{\text{SYM comoving entropy density in FLRW}} = a(t)^3 s(t) = \frac{\pi^2}{2} N^2 T_0^3$$

- Note that:

$$\frac{d}{dt} s_{comoving} = 0$$

as expected, since a thermal state of a CFT in Minkowski is an adiabatic state in de Sitter

$\Longrightarrow$ In general:

- We identify the non-equilibrium entropy density $s(\tau)$ with the Bekenstein entropy density of the AH in a holographic dual. Such a definition
  - leads to, $\mathcal{S}^\mu = s(t)u^\mu$, u^μ is a local rest frame with respect to slicing involved in definition of the AH,

$$T\ \nabla\cdot\mathcal{S} = \boxed{\zeta}\ (\nabla\cdot u)^2 + \frac{\boxed{\eta}}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} + \mathcal{O}((\partial u)^4)$$

where the viscosities $\boxed{\zeta}$ and $\boxed{\eta}$ can be computed independently from the 2-point equilibrium correlation functions

  - Using bulk Einstein equations of the holographic dual, a theorem:

$$\nabla\cdot\mathcal{S} \geq 0$$

  - If the system equilibrates,

$$\lim_{t\to\infty} \nabla\cdot\mathcal{S} = 0\,, \qquad \lim_{t\to\infty} s(t) = s_{eq}$$

- Restricting to de Sitter background,

$$a(t) = e^{Ht}$$

$\Longrightarrow$ the entropy production rate $\mathcal{R}(t)$

$$\mathcal{R}(t) \equiv \nabla \cdot S = \frac{1}{a(t)^3}\ \frac{d}{dt}\left(a(t)^3 s(t)\right) = \frac{1}{a(t)^3}\ \frac{d}{dt} s_{comoving}(t)$$

- If the system evolves to a DFP,

$$\lim_{t\to\infty} \mathcal{R}(t) = \text{finite} = 3H \cdot \lim_{t\to\infty} s(t) = 3H \cdot s_{ent}$$

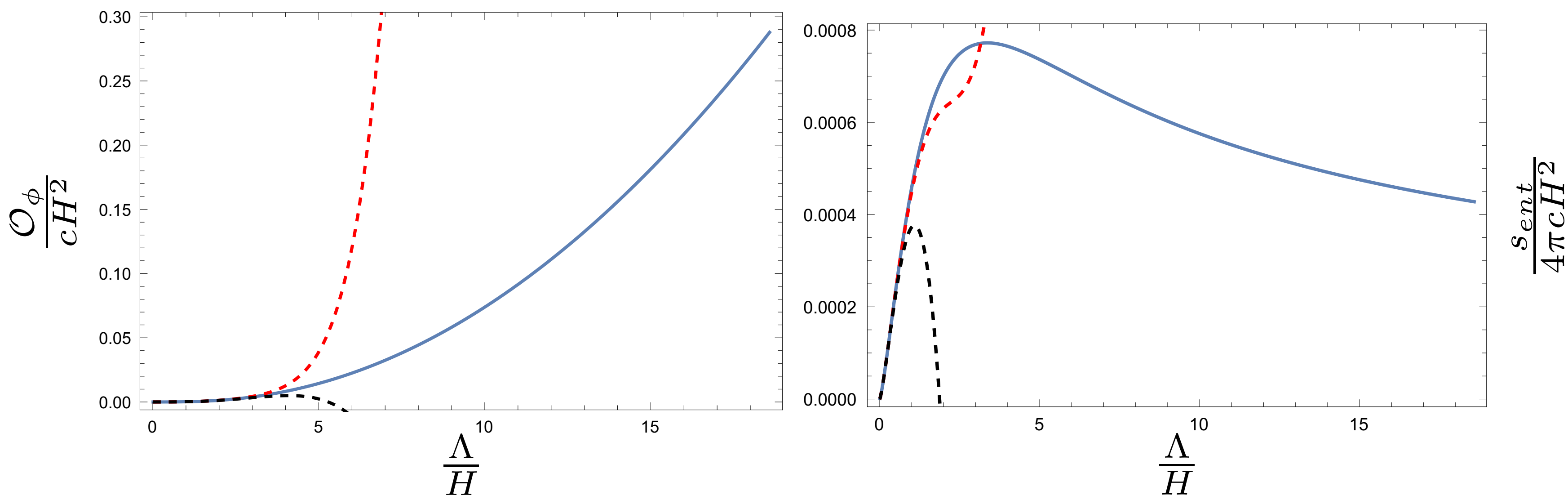
- In the previous example, and for *arbitrary* CFT dynamics

$$s_{ent}\Big|_{\mathrm{CFT}} = 0$$

$\Longrightarrow$ To have a DFP, we need a where the boundary theory is non-conformal, *i.e.*, has a mass scale Λ:

$$S_{\text{non}-\text{conformal}} = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} d^5\xi \sqrt{-g} \left[R + \text{scalars} + \text{scalar potential} \right]$$

$\Longrightarrow$ Here is an example:



$\langle \mathcal{O}_\phi \rangle$ and s_{ent} of the DFP

- c is the central charge of the theory
- Note that $s_{ent} \to 0$ as $\Lambda \to 0$ — recovering the conformal limit of trivial DFP
- Dashed lines are near-conformal perturbation theory (analytics)

In progress:

- Study DFP in 'realistic' QCD-like model:

  - top-down string theory holographic example (not a toy)
  - Λ is a strong coupling scale, as in QCD
  - Like QCD, the theory confined
  - Like in QCD, there is chiral symmetry