QUARK SOUP: APPLIED SUPERSTRING THEORY

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Many string theorists are excited about recent experiments at the Brookhaven National Laboratory (BNL) on Long Island. Upon visiting, you would find that the BNL site is dominated by a particle accelerator running almost four kilometers in circumference, known as the Relativistic Heavy Ion Collider or RHIC, for short. In experiments at RHIC, gold nuclei are collided together with energies of up to 10^{11} electron-volts per nucleon – recall that each gold nucleus contains 79 protons and 118 neutrons, for a total of 197 nucleons. Each such collision produces a spectacular "explosion" sending literally thousands of subatomic particles out through the detectors enclosing the collision points (see Figure 1). The initial stages of these explosions recreate extreme high temperatures such as may have been found in the very early universe [1].

With the experiments at RHIC, physicists are seeking to better understand quantum chromodynamics (QCD), the theory of the "strong force" determining the physical properties of nuclear matter. At a superficial level, QCD looks like a simple matrix-extension of the more familiar electromagnetism. The force carriers, known as gluons, can be thought of as 3-by-3 matrix-valued photons while the charged matter, known as quarks, can be seen as 3-component vector-valued electrons. However, unlike electromagnetism, quantum fluctuations of the QCD fields play an essential role in determining the force law. In particular, at low energies or large distances (by the standards of subatomic physics), the coupling of QCD is large and

SUMMARY

It is believed that in the first few microseconds after the Big Bang, our universe was dominated by a strongly interacting phase of nuclear matter at extreme temperatures. An impressive experimental program at the Brookhaven National Laboratory on Long Island has been studying the properties of this nuclear plasma with some rather surprising results. We outline how there may be a deep connection between extra-dimensional gravity of String Theory and the fundamental theories of subatomic particles can solve the mystery of the near-ideal fluid properties of the strongly coupled nuclear plasma.

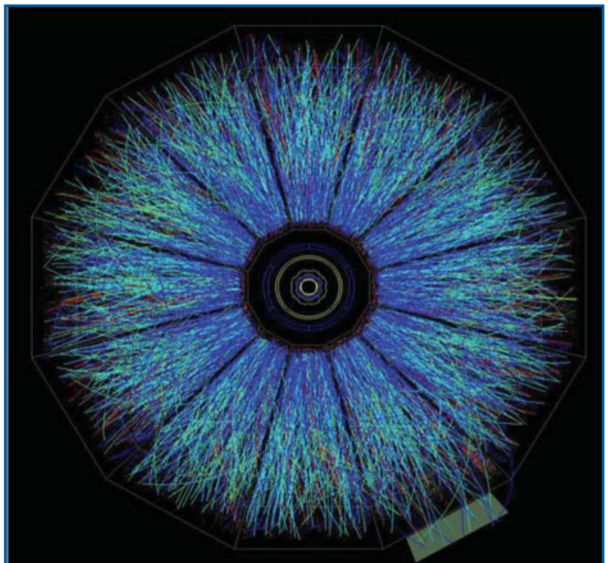

Fig. 1 A high energy collision of gold ions at RHIC as seen by the Solenoidal Tracker at RHIC (STAR) detector. Each radial line indicates the path of a subatomic particle emerging from the collision.

the forces are "strong", as implied by the original name. This is exemplified by the fact that amongst thousands of particle tracks in Figure 1, not a single one corresponds to an individual quark. Rather the long-range force between quarks is so strong that they become "confined" and any experimental detectors only ever see QCD-neutral packages known as hadrons, *e.g.*, protons and neutrons. However, in the opposite regime of very high energies or short distances, the QCD coupling becomes small and correspondingly the forces are weak. This property, known as asymptotic freedom, allows us to detect quarks and gluons, for example, inside the proton with high energy collisions at the Large Hadron Collider.

Because of the strong coupling, our understanding of many aspects of QCD remains incomplete. For example, while it has now been more than thirty-five years since the discovery of asymptotic freedom (and five years since a Nobel prize was awarded to Gross, Politzer and Wilczek for its discovery), a complete theoretical understanding of confinement remains elusive. Of course, great progress has been made on various theoretical fronts. One idea was to study QCD away from its ground state. In particular, asymptotic freedom indicates interactions are weaker at short distances and high energies and so one might expect

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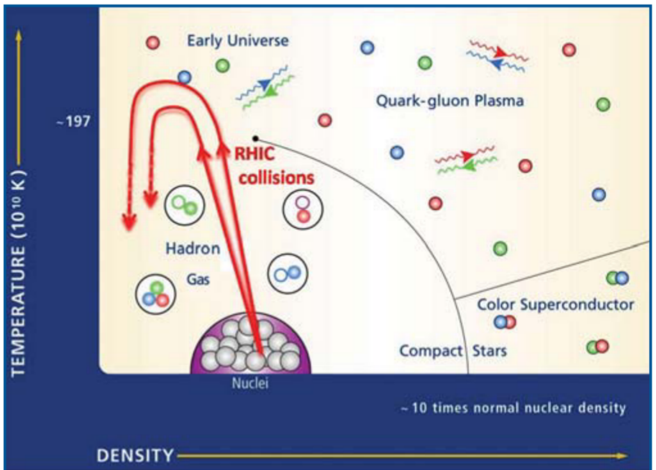

Fig. 2 Phase diagram of QCD according to theorists. The red arrows show caricatures of the evolution of matter in typical RHIC collisions.

to find new behaviour in QCD at high densities and/or high temperatures. With this in mind, theorists mapped out the phase diagram of QCD illustrated in Figure 2.

As illustrated by the red arrows in the Figure 2, the RHIC experiments are probing new regions of this phase diagram. As indicated, the collisions raise the temperature to roughly 2×10^{12} degrees Kelvin. In fact, the experiments have discovered that a surprising new phase of nuclear matter, called the strongly coupled Quark-Gluon Plasma (sQGP), emerges in this regime. In this phase, quarks (and gluons) are neither confined nor free but instead form a strongly interacting "soup", which seems to behave like a *near perfect* fluid. Of course, the plasma expands (*i.e.*, it "explodes") and cools down again to a temperature where the quarks are again confined into hadrons which then escape out into the detectors. A precise interpretation of the RHIC experiments calls for a quantitative understanding of both strong coupling and dynamical properties of QCD. This presents a substantial challenge as few (if any) techniques exist to calculate in this regime.

At this point, the reader may well be wondering what any of this has to do with superstring theory, which appears in our title. Quite surprisingly, it turns out a great deal! In a parallel set of developments, string theorists have been uncovering deep connections between gravity and strongly coupled non-abelian gauge theories (*i.e.*, a broad class of the field theories with "matrix" structure similar to QCD). Broadly these connections are known as gauge/gravity dualities and the best understood example of such a duality is the AdS/CFT correspondence [2]. These dualities realize a holographic description of quantum gravity in which the theory has an equivalent formulation in terms of a non-gravitational theory in a spacetime with fewer dimensions. In this framework, one can study the gauge theory at strong coupling with simple calculations in classical gravity, as we will explain below.

THE ADS/CFT CORRESPONDENCE

In the 1990's, it was realized that string theory is much more than just a theory of strings. In particular, it also contains heavy extended objects known as D-branes. In general, Dp-branes can be visualized as membrane-like objects, extended in p (spatial) dimensions, on which open strings can end (see Figure 3). Two important parameters which characterize both the strings and the D-branes are the string coupling constant g_s and the string length scale ℓ_s. The string coupling controls the strength of interactions of the (open and closed) strings amongst themselves and with D-branes. The string scale fixes the (rest) energy of typical excited string states as $E_0 \sim \hbar c / \ell_s$.

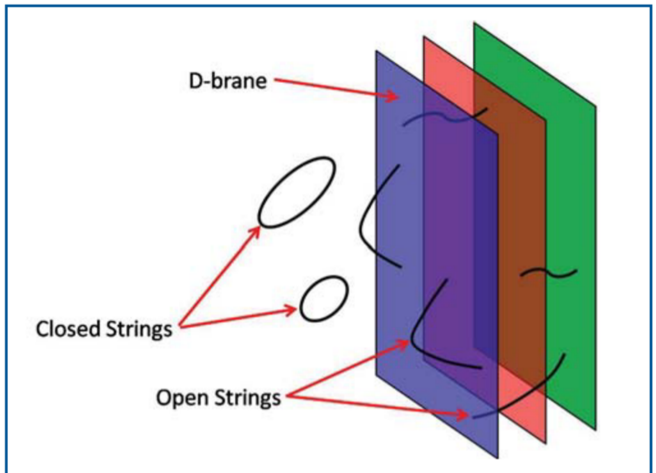

Fig. 3 D-branes are "solitonic" objects of string theory.

D-branes have different descriptions, depending on where in "parameter space" we are calculating. For example, consider the low energy limit of N D3-branes sitting on top of each other, *i.e.*, consider only processes at energies $E \ll \hbar c / \ell_s$. This physics is dominated by the massless open string excitations on the D3-branes, which it turns out can be described by a non-abelian gauge theory. This four-dimensional theory is known as $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory with gauge group $SU(N)$. The matrix structure of the gauge fields, *e.g.*, $(A_\mu)_i^{\ j}$ with $i, j = 1, \ldots, N$, can be visualized as labeling on which D-brane the corresponding open string begins and ends. Further, the string coupling is simply related to the coupling constant of the gauge theory with $g^2_{YM} = 4\pi\, g_s$.

Another interesting regime to consider is the strong coupling limit. One finds that the gravitational field of N coincident D3-branes is proportional to $g_s N$. Hence if we consider a limit where $g_s N$ becomes large, we cannot ignore how branes deform the spacetime geometry. One finds that the geometry near the D3-branes takes the form of $AdS_5 \times S^5$. The first part here is five-dimensional anti-de Sitter (AdS) space, a homogenous spacetime with fixed negative curvature. The second factor is a five-dimensional sphere. The radius of curvature for both the AdS and the sphere is given by

$$\frac{R^4}{\ell_s^4} = 4\pi g_s N = g_{YM}^2 N. \qquad (1)$$

In 1997, Maldacena realized that if the low energy and strong coupling limits were applied one after the other, two radically different pictures emerged depending on the order of limits, as shown in Figure 4. However, his bold conjecture was that these two pictures should still describe the same physics. Hence the AdS/CFT correspondence proposes that the four-dimensional $\mathcal{N} = 4$ SYM theory at strong coupling should be equivalent to ten-dimensional superstring theory on the $AdS_5 \times S^5$ background. While this equivalence remains a conjecture, it has survived the scrutiny of hundreds of tests since it was discovered.

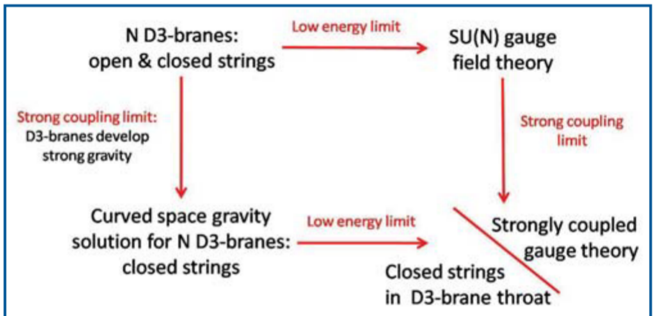

Fig. 4 "Derivation" of the AdS/CFT correspondence.

The correspondence becomes simpler upon refining the limits further. First, if we keep the curvature scale larger than the string scale, *i.e.*, $R / \ell_s \gg 1$, then stringy corrections to the geometric side of the duality are minimized. In this regime, we may work with low energy gravity alone (rather than the full string theory). Similarly if the string coupling is kept small, i.e., $g_s \ll 1$, the quantum corrections in the gravitational physics are also minimized. Hence this part of the correspondence reduces to classical gravity. Given the relation of the SYM and string couplings, one might worry that the latter limit yields a trivial gauge theory. However, comparing with eq. (1), we see that maintaining the first inequality above requires

$$N \to \infty, \qquad \lambda \equiv g_{YM}^2 N \gg 1. \qquad (2)$$

This limit (2) is well-known in gauge theories as the 't Hooft limit, where $\mathcal{N}$ is taken to infinity while keeping λ fixed. Therefore, in the limit of classical gravity, the dual gauge theory has become strongly coupled in λ.

CONNECTING SYM AND THE SQGP

So what do we have here so far? Well, we have uncovered a remarkable new method to study the $\mathcal{N} = 4$ SYM theory in the 't Hooft limit (2). However, the goal given in the introduction was to study QCD. Unfortunately, SYM is very different from QCD. In QCD, the gauge group is $SU(3)$, *i.e.*, $N = 3$, while we wish to work in the limit of large N in the $SU(N)$ SYM theory. Further the quarks of QCD are fermions transforming in the fundamental representation, *i.e.*, they carry a single $SU(3)$ index. In contrast, the matter sector of SYM contains fermions and scalars, both of which transform in the adjoint representation, *i.e.*, they carry a pair of indices like the gluons. With these extra fields, the gauge theory becomes supersymmetric (the "S" in SYM) with a precise match of the bosonic and fermionic degrees of freedom. We should also note that four-dimensional gauge theories are Conformal Field Theories (CFT) classically, in that they do not have any intrinsic length or energy scale. However, this property is typically lost in the quantum theory. For example, QCD produces a dynamical scale which is related to the confining process. In contrast, the supersymmetry of SYM protects the conformal symmetry so that it remains a CFT even as a quantum theory.

Well, so far the comparison seems hopeless then. However, we are really interested in QCD at finite temperature. The sQGP phase appears just beyond the critical temperature below which the theory becomes confining. For the SYM theory, a finite temperature breaks supersymmetry and introduces an energy scale in the theory (the temperature). Hence in this regime, both systems contain strongly coupled plasmas of gluons and various matter fields. Further we emphasize that we would like to model these plasmas with *fluid dynamics* and so we only care about long wavelength phenomena and not the microscopic dynamics. Therefore, it is very useful to have any model of a strongly coupled gauge theory plasma for which we can do *analytic* calculations.

How do we calculate properties of the SYM plasma that we can then compare with the sQGP? First, we need to introduce a finite temperature into the AdS/CFT correspondence. It turns out that this corresponds to putting a black hole in AdS_5 spacetime. The gauge theory temperature T then corresponds to that of the Hawking radiation emitted by the horizon. We might then use this new gravity background to calculate any of a number of properties of the dual plasma but we focus here on one in particular, the shear viscosity. In any field theory, the shear viscosity can be determined using the so-called Kubo formula

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x\, e^{i\omega t} \left\langle \left[T_{xy}\left(t, \vec{x}\right), T_{xy}\left(0, 0\right) \right] \right\rangle. \qquad (3)$$

The last factor in this expression is a certain correlation function of the gauge theory's stress-energy tensor. While it is next to impossible to calculate this correlator in the gauge theory at strong coupling and finite temperature, the AdS/CFT correspondence translates this factor to a Green's function for gravity waves in the AdS black hole. The dissipative nature of the viscosity then comes from the absorption of the gravity waves by the black hole's event horizon. From this gravity calculation, the value of the shear viscosity of the $\mathcal{N} = 4$ SYM plasma is found to be

$$\eta = \frac{\pi}{8} N^2 T^3. \qquad (4)$$

Given the factor of N^2, the viscosity seems to be very large but one must ask: "Large compared to what?" It turns out that the natural quantity with which to make a comparison is the entropy density of the plasma. Again, the gravity theory yields