Swelling and Popping Model

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Biophysics Background

- Water gain by osmosis causes the hydra to 'swell'
- Hydra maintain their water balance by periodically 'popping' removing excess water from the enteron through the mouth by contraction of their body
- A functioning nervous system is necessary for this process - supported by epitheliail Hydra studies (Marcum '78)

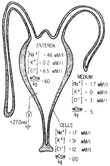


Fig. 8. Schematic diagram summarizing the distribution of Na, K, Cl, and osmolality in the external medium, tissues, and enteron of *Hydra littoralis*. The data are from Benos and Prusch (1972). The transepithelial potential (as measured from the data shown in Fig. 7) is also indicated.

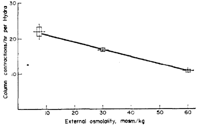


FIG. 1. Rate of column contraction of *H. littoralis* as a function of external osmolality. Horizontal line: mean; vertical line: range; rectangle on either side of mean: two standard errors of the mean. Number of determinations: Versene medium, 7; 30 mOsmole/kg, 3; 60 mOsmole/kg, 4. The osmolality of the Versene medium was increased by the addition of sucrose. The total number of column contractions of thirteen *Hydra* were counted in 1-hr intervals, and the results expressed in terms of contraction rate per individual *Hydra*.

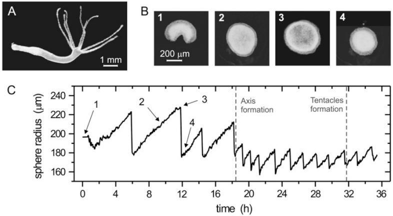
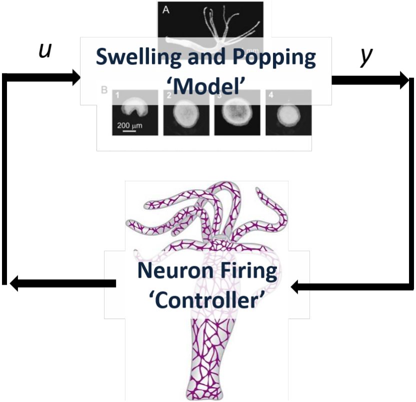


FIGURE 1 (*A*) Adult *Hydra vulgaris*. (*B* and *C*) Example of the swelling-collapse oscillations during regeneration at $T = 20$°C. The snapshots in panel *B* show: 1), the initial fragment of tissue folding to form a sphere; 2), inflation stage; 3), critical inflation and rupture of the *Hydra* shell; and 4), collapse, healing, and beginning of a new inflation cycle.

References: Kucken, Soriano et al. '08; Benos and Prusch '73; Benos, Kirk et al. '77

Volume Model and Neuron Network as Feedback

- Swelling and popping model described by the biophysics is 'controlled' by the neuron network



- Control u: Neuron firing induces contractions and mouth opening which regulates the volume
- Output y: Volume is measured and 'feeds back' to the neuron network

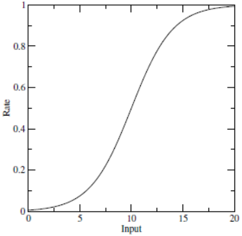
Volume Model

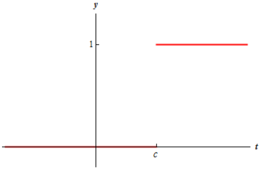
The Hydra swells at the rate of v_0 until sufficient neuron firing $r(t)$ occurs inducing a 'pop'

$$\tau_v \frac{dv}{dt} = v_0 - \Phi_v(u)$$

where

- v is the volume
- v_0 is the swelling rate ("the sphere inflates almost linearly in time" Kucken '08)
- τ_v is the time constant for the volume model
- u is the control signal from the neuron network e.g., an accumulative firing rate
- $\Phi_v(\cdot)$ is the transfer function associated with the 'pop', e.g., sigmoidal or step function





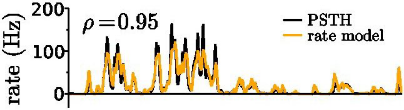
Neuron Firing Rate Model

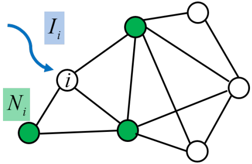
The average firing rate $r_i(t)$ dynamics for neuron i (or subpopulation) is of the form

$$\frac{dr_i}{dt} = f(r_i, \{r_j\}_{N_i}, I_i)$$

where

- $\{r_j\}_{N_i}$ are the firing rates of neurons adjacent to i
- I_i is an external input to i





Rate equation

$$\tau_i \dot{r}_i = -r_i + \Phi_r\left(\sum_{j \in N_i} J_{ij} r_j + I_i\right), \text{ for } i = 1, 2, \ldots, n$$

- τ_i is a time constant for neuron i
- J_{ij} is the coupling effect of neuron j on neuron i
- $\Phi_r(\cdot)$ is a transfer function e.g., linear, threshold linear, sigmoidal

Combined Dynamics

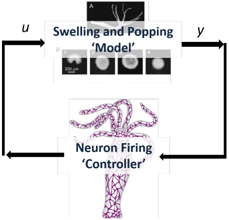
Now to connect the two models...

$$\tau_v \dot{v} = v_0 - \Phi_v(u)$$
$$\tau_i \dot{r}_i = -r_i + \Phi_r(J_i r + I), \text{ for } i = 1, 2, \ldots, n$$

Sensing: $y = I = b(S)$

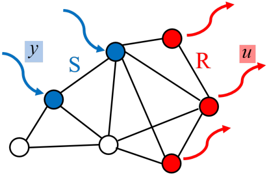
- S is the set of neurons that can 'sense' volume
- $b(S)$ encodes S as a binary vector, i.e.,

$$b_i(S) = \begin{cases} 1 & i \in S \\ 0 & \text{otherwise} \end{cases}$$



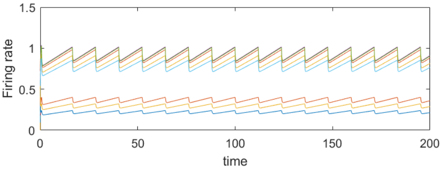
Control: $u = c(R)^T r(t - t_d)$

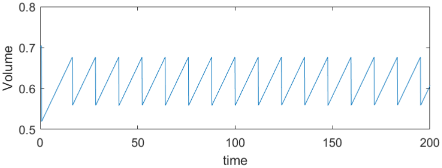
- R is the set of neurons associated with mouth opening ('actuators')
- t_d is the time delay associated with the 'pop'
- $c(R)$ is encodes R as a binary vector

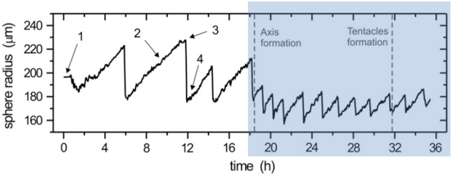


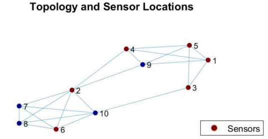
Sample Performance

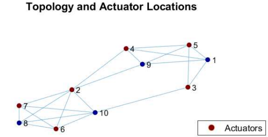
$\Phi_v(\cdot)$ is a step and $\Phi_r(\cdot)$ is linear

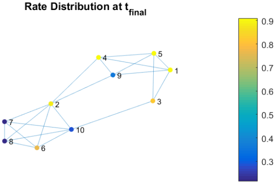




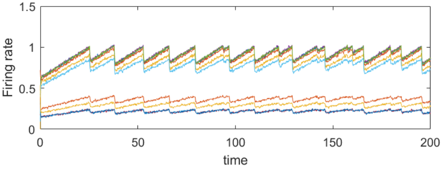


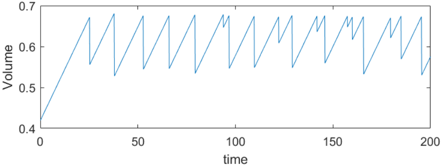


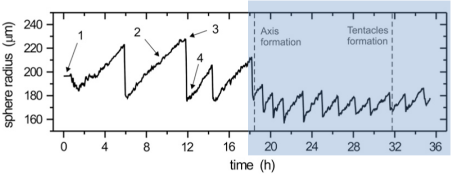


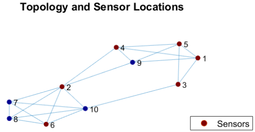


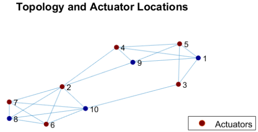
Sample Performance with Noise

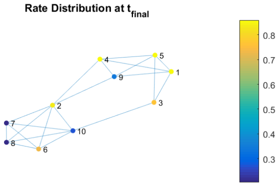












Next steps and directions...

Refining the model

- Is there evidence that supports the proposed connection between the firing rate and mouth opening?
- Apply realistic swelling rates, time constants, scaling factors...
- Apply realistic network topologies, and sensor/actuator locations

Control theory

- What is the role of the network? What characterized favorable network topologies?
- Where are the optimal sensor and actuator locations?
- Are certain configurations more robust to noise in sensors and actuator signals, and model uncertainty?
- What features make the system more amenable for control?
- If we were to perturb/control (or probe/observe) the topologies where are the best attachment (observation) points