Partitioned Matrix-Matrix Multiplication

$$C = AB$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array} \right) =$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array}\right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array}\right) =$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array} \right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array} \right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array} \right)$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array} \right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array} \right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array} \right)$$

$$\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) =$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array} \right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array} \right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array} \right)$$

$$\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array} \right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array} \right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array} \right)$$

$$\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =$$

$$\left(\begin{array}{c|c} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \\ \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \end{array} \right)$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array}\right) = A\left(\begin{array}{c|c} B_0 & B_1 \end{array}\right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =$$

$$\left(\begin{array}{c|c} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \\ \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \end{array}\right)$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array} \right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array} \right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array} \right)$$

$$\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =$$

$$\left(\begin{array}{c|c} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \\ \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \end{array} \right)$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array} \right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array} \right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array} \right)$$

$$\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =$$

$$\left(\begin{array}{c|c} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \\ \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \end{array} \right)$$

How to remember

$$3\left(\begin{array}{c|c} -1 & 2 \end{array}\right) =$$

How to remember

$$3\left(\begin{array}{c|c} -1 & 2 \end{array}\right) \quad = \quad \left(\begin{array}{c|c} (3)\times(-1) & (3)\times(2) \end{array}\right)$$

How to remember

$$3\left(\ -1\ \middle|\ 2\ \right) \quad = \quad \left(\ (3)\times(-1)\ \middle|\ (3)\times(2)\ \right)$$

$$A\left(\ B_0\ \middle|\ B_1\ \right) \quad =$$

How to remember

$$3\left(\begin{array}{c|c} -1 & 2 \end{array}\right) = \left(\begin{array}{c|c} (3)\times(-1) & (3)\times(2) \end{array}\right)$$

$$A\left(\begin{array}{c|c} B_0 & B_1 \end{array}\right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array}\right)$$

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Note:

$$\left(\begin{array}{c|c} (3)\times(-1) & (3)\times(2) \end{array} \right) = \left(\begin{array}{c|c} (-1)\times(3) & (2)\times(3) \end{array} \right)$$

How to remember

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Note:

$$\left(\begin{array}{c|c} (3)\times(-1) & (3)\times(2) \end{array}\right) = \left(\begin{array}{c|c} (-1)\times(3) & (2)\times(3) \end{array}\right)$$

$$\left(\begin{array}{c|c} AB_0 & AB_1 \end{array}\right) \neq \left(\begin{array}{c|c} B_0A & B_1A \end{array}\right)$$

$$\left(\frac{C_0}{C_1}\right) =$$

$$\left(\frac{C_0}{C_1}\right) = \left(\frac{A_0}{A_1}\right) B =$$

$$\left(\frac{C_0}{C_1}\right) = \left(\frac{A_0}{A_1}\right) B = \left(\frac{A_0 B}{A_1 B}\right)$$

$$\left(\begin{array}{c} C_0 \\ \hline C_1 \end{array}\right) = \left(\begin{array}{c} A_0 \\ \hline A_1 \end{array}\right) B = \left(\begin{array}{c} A_0 B \\ \hline A_1 B \end{array}\right)$$

$$\left(\begin{array}{cccc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) =$$

$$\left(\begin{array}{c} C_0 \\ \hline C_1 \end{array}\right) = \left(\begin{array}{c} A_0 \\ \hline A_1 \end{array}\right) B = \left(\begin{array}{c} A_0 B \\ \hline A_1 B \end{array}\right)$$

$$\left(\begin{array}{cccc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cccc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} =$$

$$\left(\begin{array}{c} C_0 \\ \hline C_1 \end{array}\right) = \left(\begin{array}{c} A_0 \\ \hline A_1 \end{array}\right) B = \left(\begin{array}{c} A_0 B \\ \hline A_1 B \end{array}\right)$$

$$\left(\begin{array}{cccc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cccc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} =$$

$$\left(\begin{array}{c} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \\ \hline \begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \end{array}\right)$$

$$\left(\begin{array}{c} C_0 \\ \hline C_1 \end{array}\right) = \left(\begin{array}{c} A_0 \\ \hline A_1 \end{array}\right) B = \left(\begin{array}{c} A_0 B \\ \hline A_1 B \end{array}\right)$$

$$\left(\begin{array}{cccc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cccc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} =$$

$$\left(\begin{array}{c} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \\ \hline \begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \end{array}\right)$$

$$\left(\frac{C_0}{C_1}\right) = \left(\frac{A_0}{A_1}\right) B = \left(\frac{A_0 B}{A_1 B}\right)$$

$$\left(\begin{array}{cccc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cccc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} =$$

$$\left(\begin{array}{c} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \\ \hline \begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \end{array}\right)$$

$$\left(\begin{array}{c} C_0 \\ \hline C_1 \end{array}\right) = \left(\begin{array}{c} A_0 \\ \hline A_1 \end{array}\right) B = \left(\begin{array}{c} A_0 B \\ \hline A_1 B \end{array}\right)$$

$$\left(\begin{array}{cccc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cccc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} =$$

$$\left(\begin{array}{c} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \\ \hline \begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \end{array}\right)$$

How to remember

$$\left(\frac{-1}{2}\right)3 \quad =$$

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$$\left(\frac{-1}{2}\right)3 \quad = \quad \left(\frac{(-1)\times(3)}{(2)\times(3)}\right)$$

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$$\left(\frac{A_0}{A_1}\right)B \quad = \quad \left(\frac{A_0B}{A_1B}\right)$$

Note:

$$\left(\frac{(-1)\times(3)}{(2)\times(3)}\right) \quad = \quad \left(\frac{(3)\times(-1)}{(3)\times(2)}\right)$$

$$\left(\frac{A_0B}{A_1B}\right) \quad \neq \quad \left(\frac{BA_0}{BA_1}\right)$$

$$C =$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array} \right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array} \right) =$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array} \right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array} \right) = A_0 B_0 + A_1 B_1$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array} \right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} =$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array} \right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array} \right) \left(\begin{array}{cccc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array} \right) =$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array} \right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array} \right) \left(\begin{array}{cccc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array} \right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \\ \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array}\right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array}\right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \left(\begin{array}{cccc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \\ \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array} \right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array} \right) \left(\begin{array}{cccc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array} \right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \\ \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array} \right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array} \right) \left(\begin{array}{cccc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array} \right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \\ \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array} \right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array} \right) \left(\begin{array}{cccc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array} \right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \\ \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array} \right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array} \right) \left(\begin{array}{cccc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array} \right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \\ \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array} \right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array} \right) \left(\begin{array}{cccc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array} \right) =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \\ \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

How to remember

$$\left(\begin{array}{c|c} 3 & 1 \end{array}\right)\left(\begin{array}{c} -1 \\ \hline 2 \end{array}\right) \;=$$

How to remember

$$\left(\begin{array}{c|c} 3 & 1 \end{array}\right)\left(\begin{array}{c} -1 \\ \hline 2 \end{array}\right) \quad = \quad (3) \times (-1) + (1) \times (2)$$

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$$\left(\begin{array}{c|c} 3 & 1 \end{array}\right)\left(\begin{array}{c} -1 \\ \hline 2 \end{array}\right) \quad = \quad (3) \times (-1) + (1) \times (2)$$

$$\left(\begin{array}{c|c} A_0 & A_1 \end{array}\right)\left(\begin{array}{c} B_0 \\ \hline B_1 \end{array}\right) \quad =$$

How to remember

$$\left(\begin{array}{c|c} 3 & 1 \end{array} \right) \left(\begin{array}{c} -1 \\ \hline 2 \end{array} \right) = (3) \times (-1) + (1) \times (2)$$

$$\left(\begin{array}{c|c} A_0 & A_1 \end{array} \right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array} \right) = A_0 \times B_0 + A_1 \times B_1$$

How to remember

$$\left(\begin{array}{c|c} 3 & 1 \end{array}\right)\left(\begin{array}{c} -1 \\ \hline 2 \end{array}\right) = (3) \times (-1) + (1) \times (2)$$

$$\left(\begin{array}{c|c} A_0 & A_1 \end{array}\right)\left(\begin{array}{c} B_0 \\ \hline B_1 \end{array}\right) = A_0 \times B_0 + A_1 \times B_1$$

Note:

$$(3) \times (-1) + (1) \times (2) = (-1) \times (3) + (2) \times (1)$$

$$A_0 \times B_0 + A_1 \times B_1 \neq B_0 \times A_0 + B_1 \times A_1$$

$$\left(\begin{array}{c|c} C_{00} & C_{01} \\ \hline C_{10} & C_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right)$$

$$=$$

$$
\left(\begin{array}{c|c} C_{00} & C_{01} \\ \hline C_{10} & C_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right)
$$

$$
= \left(\begin{array}{c|c} A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\ \hline A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11} \end{array}\right)
$$

$$
\left(\begin{array}{c|c} C_{00} & C_{01} \\ \hline C_{10} & C_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right)
$$

$$
= \left(\begin{array}{c|c} A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\ \hline A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11} \end{array}\right)
$$

$$
\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) =
$$

$$\left(\begin{array}{c|c} C_{00} & C_{01} \\ \hline C_{10} & C_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right)$$

$$= \left(\begin{array}{c|c} A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\ \hline A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11} \end{array}\right)$$

$$\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =$$

$$\left(\begin{array}{c|c} C_{00} & C_{01} \\ \hline C_{10} & C_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right)$$

$$= \left(\begin{array}{c|c} A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\ \hline A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11} \end{array}\right)$$

$$\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =$$

$$\left(\begin{array}{c|c} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \\ \hline \begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \end{array}\right)$$

$$
\left(\begin{array}{c|c} C_{00} & C_{01} \\ \hline C_{10} & C_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right)
$$

$$
= \left(\begin{array}{c|c} A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\ \hline A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11} \end{array}\right)
$$

$$
\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =
$$

$$
\left(\begin{array}{c|c}
\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \\
\hline
\begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix}
\end{array}\right)
$$

$$
\left(\begin{array}{c|c} C_{00} & C_{01} \\ \hline C_{10} & C_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right)
$$

$$
= \left(\begin{array}{c|c} A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\ \hline A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11} \end{array}\right)
$$

$$
\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =
$$

$$
\left(\begin{array}{c|c}
\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} &
\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \\ \hline
\begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} &
\begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix}
\end{array}\right)
$$

$$\left(\begin{array}{c|c} C_{00} & C_{01} \\ \hline C_{10} & C_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right)$$

$$= \left(\begin{array}{c|c} A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\ \hline A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11} \end{array}\right)$$

$$\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =$$

$$\left(\begin{array}{c|c} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \\ \hline \begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \end{array}\right)$$

$$\left(\begin{array}{c|c} C_{00} & C_{01} \\ \hline C_{10} & C_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right)$$

$$= \left(\begin{array}{c|c} A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\ \hline A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11} \end{array}\right)$$

$$\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =$$

$$\left(\begin{array}{c|c} \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \\ \hline \begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} & \begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \end{array}\right)$$

$$
\left(\begin{array}{c|c} C_{00} & C_{01} \\ \hline C_{10} & C_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right)
$$

$$
= \left(\begin{array}{c|c} A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\ \hline A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11} \end{array}\right)
$$

$$
\left(\begin{array}{cc|cc} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \hline \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{array}\right) = \left(\begin{array}{cc|cc} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \hline \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{array}\right) \left(\begin{array}{cc|cc} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \hline \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{array}\right) =
$$

$$
\left(\begin{array}{c|c}
\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} &
\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} \\ \alpha_{1,0} & \alpha_{1,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix} \\ \hline
\begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} &
\begin{pmatrix} \alpha_{2,0} & \alpha_{2,1} \\ \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix}
\end{array}\right)
$$

How to remember

$$\left(\begin{array}{c|c} 1 & -1 \\ \hline 2 & 3 \end{array}\right)\left(\begin{array}{c|c} -2 & 0 \\ \hline 1 & 2 \end{array}\right) =$$

How to remember

$$\left(\begin{array}{c|c} 1 & -1 \\ \hline 2 & 3 \end{array}\right)\left(\begin{array}{c|c} -2 & 0 \\ \hline 1 & 2 \end{array}\right) = \left(\begin{array}{c|c} (1)\times(-2)+(-1)\times(1) & (1)\times(0)+(-1)\times(2) \\ \hline (2)\times(-2)+(3)\times(1) & (2)\times(0)+(3)\times(2) \end{array}\right)$$

How to remember

$$\left(\begin{array}{c|c} 1 & -1 \\ \hline 2 & 3 \end{array}\right)\left(\begin{array}{c|c} -2 & 0 \\ \hline 1 & 2 \end{array}\right) = \left(\begin{array}{c|c} (1)\times(-2)+(-1)\times(1) & (1)\times(0)+(-1)\times(2) \\ \hline (2)\times(-2)+(3)\times(1) & (2)\times(0)+(3)\times(2) \end{array}\right)$$

$$\left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right)\left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right) =$$

How to remember

$$\left(\begin{array}{c|c} 1 & -1 \\ \hline 2 & 3 \end{array}\right)\left(\begin{array}{c|c} -2 & 0 \\ \hline 1 & 2 \end{array}\right) = \left(\begin{array}{c|c} (1)\times(-2)+(-1)\times(1) & (1)\times(0)+(-1)\times(2) \\ \hline (2)\times(-2)+(3)\times(1) & (2)\times(0)+(3)\times(2) \end{array}\right)$$

$$\left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right)\left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right) = \left(\begin{array}{c|c} A_{00}B_{00}+A_{01}B_{10} & A_{00}B_{01}+A_{01}B_{11} \\ \hline A_{10}B_{00}+A_{11}B_{10} & A_{10}B_{01}+A_{11}B_{11} \end{array}\right)$$

Let $C \in \mathbb{R}^{m \times n}$, $A \in \mathbb{R}^{m \times k}$, and $B \in \mathbb{R}^{k \times n}$, and $C = AB$.

Partition, conformally,

$$
\left(\begin{array}{c|c|c|c}
C_{0,0} & C_{0,1} & \cdots & C_{0,N-1} \\ \hline
C_{1,0} & C_{1,1} & \cdots & C_{1,N-1} \\ \hline
\vdots & \vdots & \ddots & \vdots \\ \hline
C_{M-1,0} & C_{M-1,1} & \cdots & C_{M-1,N-1}
\end{array}\right) =
$$

$$
\left(\begin{array}{c|c|c|c}
A_{0,0} & A_{0,1} & \cdots & A_{0,K-1} \\ \hline
A_{1,0} & A_{1,1} & \cdots & A_{1,K-1} \\ \hline
\vdots & \vdots & \ddots & \vdots \\ \hline
A_{M-1,0} & A_{M-1,1} & \cdots & A_{M-1,K-1}
\end{array}\right)
\left(\begin{array}{c|c|c|c}
B_{0,0} & B_{0,1} & \cdots & B_{0,N-1} \\ \hline
B_{1,0} & B_{1,1} & \cdots & B_{1,N-1} \\ \hline
\vdots & \vdots & \ddots & \vdots \\ \hline
B_{K-1,0} & B_{K-1,1} & \cdots & B_{K-1,N-1}
\end{array}\right)
$$

Let $C \in \mathbb{R}^{m \times n}$, $A \in \mathbb{R}^{m \times k}$, and $B \in \mathbb{R}^{k \times n}$, and $C = AB$.

Partition, conformally,

$$\left(\begin{array}{c|c|c|c} \gamma_{0,0} & \gamma_{0,1} & \cdots & \gamma_{0,N-1} \\ \hline \gamma_{1,0} & \gamma_{1,1} & \cdots & \gamma_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \gamma_{M-1,0} & \gamma_{M-1,1} & \cdots & \gamma_{M-1,N-1} \end{array}\right) =$$

$$\left(\begin{array}{c|c|c|c} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,K-1} \\ \hline \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,K-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \alpha_{M-1,0} & \alpha_{M-1,1} & \cdots & \alpha_{M-1,K-1} \end{array}\right) \left(\begin{array}{c|c|c|c} \beta_{0,0} & \beta_{0,1} & \cdots & \beta_{0,N-1} \\ \hline \beta_{1,0} & \beta_{1,1} & \cdots & \beta_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \beta_{K-1,0} & \beta_{K-1,1} & \cdots & \beta_{K-1,N-1} \end{array}\right),$$

Let $C \in \mathbb{R}^{m \times n}$, $A \in \mathbb{R}^{m \times k}$, and $B \in \mathbb{R}^{k \times n}$, and $C = AB$.

Partition, conformally,

$$C = \left(\begin{array}{c|c|c|c} C_{0,0} & C_{0,1} & \cdots & C_{0,N-1} \\ \hline C_{1,0} & C_{1,1} & \cdots & C_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline C_{M-1,0} & C_{M-1,1} & \cdots & C_{M-1,N-1} \end{array}\right) A = \left(\begin{array}{c|c|c|c} A_{0,0} & A_{0,1} & \cdots & A_{0,K-1} \\ \hline A_{1,0} & A_{1,1} & \cdots & A_{1,K-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline A_{M-1,0} & A_{M-1,1} & \cdots & A_{M-1,K-1} \end{array}\right)$$

$$\text{and} \quad B = \left(\begin{array}{c|c|c|c} B_{0,0} & B_{0,1} & \cdots & B_{0,N-1} \\ \hline B_{1,0} & B_{1,1} & \cdots & B_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline B_{K-1,0} & B_{K-1,1} & \cdots & B_{K-1,N-1} \end{array}\right),$$

Let $C \in \mathbb{R}^{m \times n}$, $A \in \mathbb{R}^{m \times k}$, and $B \in \mathbb{R}^{k \times n}$, and $C = AB$.

Partition, conformally,

$$C = \left(\begin{array}{c|c|c|c} C_{0,0} & C_{0,1} & \cdots & C_{0,N-1} \\ \hline C_{1,0} & C_{1,1} & \cdots & C_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline C_{M-1,0} & C_{M-1,1} & \cdots & C_{M-1,N-1} \end{array} \right) A = \left(\begin{array}{c|c|c|c} A_{0,0} & A_{0,1} & \cdots & A_{0,K-1} \\ \hline A_{1,0} & A_{1,1} & \cdots & A_{1,K-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline A_{M-1,0} & A_{M-1,1} & \cdots & A_{M-1,K-1} \end{array} \right)$$

$$\text{and} \quad B = \left(\begin{array}{c|c|c|c} B_{0,0} & B_{0,1} & \cdots & B_{0,N-1} \\ \hline B_{1,0} & B_{1,1} & \cdots & B_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline B_{K-1,0} & B_{K-1,1} & \cdots & B_{K-1,N-1} \end{array} \right),$$

Then

$$C_{i,j} = \sum_{p=0}^{K-1} A_{i,p} B_{p,j}.$$

Let $C \in \mathbb{R}^{m \times n}$, $A \in \mathbb{R}^{m \times k}$, and $B \in \mathbb{R}^{k \times n}$, and $C = AB$.

Partition, conformally,

$$C = \left(\begin{array}{c|c|c|c} C_{0,0} & C_{0,1} & \cdots & C_{0,N-1} \\ \hline C_{1,0} & C_{1,1} & \cdots & C_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline C_{M-1,0} & C_{M-1,1} & \cdots & C_{M-1,N-1} \end{array}\right) A = \left(\begin{array}{c|c|c|c} A_{0,0} & A_{0,1} & \cdots & A_{0,K-1} \\ \hline A_{1,0} & A_{1,1} & \cdots & A_{1,K-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline A_{M-1,0} & A_{M-1,1} & \cdots & A_{M-1,K-1} \end{array}\right)$$

$$\text{and} \quad B = \left(\begin{array}{c|c|c|c} B_{0,0} & B_{0,1} & \cdots & B_{0,N-1} \\ \hline B_{1,0} & B_{1,1} & \cdots & B_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline B_{K-1,0} & B_{K-1,1} & \cdots & B_{K-1,N-1} \end{array}\right),$$

Then

$$C_{i,j} = \sum_{p=0}^{K-1} A_{i,p} B_{p,j}.$$

Let $C \in \mathbb{R}^{m\times n}$, $A \in \mathbb{R}^{m\times k}$, and $B \in \mathbb{R}^{k\times n}$, and $C = AB$.

$$C = \left(\begin{array}{c|c|c|c} \gamma_{0,0} & \gamma_{0,1} & \cdots & \gamma_{0,N-1} \\ \hline \gamma_{1,0} & \gamma_{1,1} & \cdots & \gamma_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \gamma_{M-1,0} & \gamma_{M-1,1} & \cdots & \gamma_{M-1,N-1} \end{array}\right) A = \left(\begin{array}{c|c|c|c} \alpha_{0,0} & \alpha_{0,1} & \cdots & \alpha_{0,K-1} \\ \hline \alpha_{1,0} & \alpha_{1,1} & \cdots & \alpha_{1,K-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \alpha_{M-1,0} & \alpha_{M-1,1} & \cdots & \alpha_{M-1,K-1} \end{array}\right),$$

$$\text{and} \quad B = \left(\begin{array}{c|c|c|c} \beta_{0,0} & \beta_{0,1} & \cdots & \beta_{0,N-1} \\ \hline \beta_{1,0} & \beta_{1,1} & \cdots & \beta_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \beta_{K-1,0} & \beta_{K-1,1} & \cdots & \beta_{K-1,N-1} \end{array}\right),$$

Then

$$\gamma_{i,j} = \sum_{p=0}^{K-1} \alpha_{i,p}\beta_{p,j}.$$

Summary

Blocked matrix-matrix multiplication:

- Exactly like matrix-matrix multiplication
- **except** matrix-matrix multiplication usually does not commute.