Partitioned Matrix-Matrix Multiplication

©2014 R. van de Geijn and M. Myers

$$C = AB$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array}\right) =$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array}\right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array}\right) =$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array}\right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array}\right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array}\right)$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array}\right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array}\right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} =$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array}\right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array}\right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = =$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array}\right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array}\right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} =$$

$$\begin{pmatrix} \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \\ \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} \begin{vmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \\ \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array}\right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array}\right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = =$$

$$\begin{pmatrix} \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} \begin{vmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \\ \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array}\right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array}\right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} =$$

$$\begin{pmatrix} \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} \begin{vmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \\ \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\begin{array}{c|c} C_0 & C_1 \end{array}\right) = A \left(\begin{array}{c|c} B_0 & B_1 \end{array}\right) = \left(\begin{array}{c|c} AB_0 & AB_1 \end{array}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} =$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = =$$

$$\begin{pmatrix} \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} \\ \beta_{1,0} & \beta_{1,1} \\ \beta_{2,0} & \beta_{2,1} \\ \beta_{3,0} & \beta_{3,1} \end{pmatrix} \begin{vmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,2} & \beta_{0,3} \\ \beta_{1,2} & \beta_{1,3} \\ \beta_{2,2} & \beta_{2,3} \\ \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$3\left(\begin{array}{c|c}-1 & 2\end{array}\right) =$$

$$3\left(\begin{array}{c|c}-1 & 2\end{array}\right) = \left(\begin{array}{c|c} (3) \times (-1) & (3) \times (2)\end{array}\right)$$

$$3\left(\begin{array}{cc} -1 & 2 \end{array}\right) = \left(\begin{array}{cc} (3) \times (-1) & (3) \times (2) \end{array}\right)$$

$$A\left(\begin{array}{cc} B_0 & B_1 \end{array}\right) =$$

$$3\left(\begin{array}{ccc} -1 & 2 \end{array}\right) & = & \left(\begin{array}{ccc} (3) \times (-1) & (3) \times (2) \end{array}\right)$$

$$A\left(\begin{array}{cccc} B_0 & B_1 \end{array}\right) & = & \left(\begin{array}{cccc} AB_0 & AB_1 \end{array}\right)$$

$$3\left(\begin{array}{ccc} -1 & 2 \end{array}\right) & = & \left(\begin{array}{ccc} (3) \times (-1) & (3) \times (2) \end{array}\right)$$

$$A\left(\begin{array}{ccc} B_0 & B_1 \end{array}\right) & = & \left(\begin{array}{ccc} AB_0 & AB_1 \end{array}\right)$$

Note:

$$\left(\begin{array}{c|c} (3)\times (-1) & (3)\times (2) \end{array}\right) = \left(\begin{array}{c|c} (-1)\times (3) & (2)\times (3) \end{array}\right)$$

$$3\left(\begin{array}{ccc} -1 & 2 \end{array}\right) & = & \left(\begin{array}{ccc} (3) \times (-1) & (3) \times (2) \end{array}\right)$$

$$A\left(\begin{array}{cccc} B_0 & B_1 \end{array}\right) & = & \left(\begin{array}{cccc} AB_0 & AB_1 \end{array}\right)$$

Note:

$$\left(\begin{array}{ccc} (3) \times (-1) & | & (3) \times (2) \end{array} \right) &= \left(\begin{array}{ccc} (-1) \times (3) & | & (2) \times (3) \end{array} \right)$$

$$\left(\begin{array}{cccc} AB_0 & | & AB_1 \end{array} \right) & \neq \left(\begin{array}{cccc} B_0A & | & B_1A \end{array} \right)$$

$$\left(\frac{C_0}{C_1}\right) =$$

$$\left(\frac{C_0}{C_1}\right) = \left(\frac{A_0}{A_1}\right)B =$$

$$\left(\frac{C_0}{C_1}\right) = \left(\frac{A_0}{A_1}\right)B = \left(\frac{A_0B}{A_1B}\right)$$

$$\left(\frac{C_0}{C_1}\right) = \left(\frac{A_0}{A_1}\right)B = \left(\frac{A_0B}{A_1B}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \frac{\gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3}}{\gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} =$$

$$\left(\frac{C_0}{C_1}\right) = \left(\frac{A_0}{A_1}\right)B = \left(\frac{A_0B}{A_1B}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix}$$

$$\left(\frac{C_0}{C_1}\right) = \left(\frac{A_0}{A_1}\right)B = \left(\frac{A_0B}{A_1B}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \frac{\gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3}}{\gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \frac{\alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3}}{\alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\frac{C_0}{C_1}\right) = \left(\frac{A_0}{A_1}\right)B = \left(\frac{A_0B}{A_1B}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \frac{\gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3}}{\gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \frac{\alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3}}{\alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\frac{C_0}{C_1}\right) = \left(\frac{A_0}{A_1}\right)B = \left(\frac{A_0B}{A_1B}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \frac{\gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3}}{\gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \frac{\alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3}}{\alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\frac{C_0}{C_1}\right) = \left(\frac{A_0}{A_1}\right)B = \left(\frac{A_0B}{A_1B}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \frac{\gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3}}{\gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \frac{\alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3}}{\alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\frac{-1}{2}\right)3 =$$

$$\left(\frac{-1}{2}\right)3 = \left(\frac{(-1)\times(3)}{(2)\times(3)}\right)$$

$$\left(\frac{-1}{2}\right)3 = \left(\frac{(-1)\times(3)}{(2)\times(3)}\right)$$

$$\left(\frac{A_0}{A_1}\right)B =$$

$$\left(\frac{-1}{2}\right)3 = \left(\frac{(-1)\times(3)}{(2)\times(3)}\right)$$

$$\left(\frac{A_0}{A_1}\right)B = \left(\frac{A_0B}{A_1B}\right)$$

$$\left(\frac{-1}{2}\right)3 = \left(\frac{(-1)\times(3)}{(2)\times(3)}\right)$$

$$\left(\frac{A_0}{A_1}\right)B = \left(\frac{A_0B}{A_1B}\right)$$

Note:

$$\left(\frac{(-1)\times(3)}{(2)\times(3)}\right) = \left(\frac{(3)\times(-1)}{(3)\times(2)}\right)$$

$$\left(\frac{A_0B}{A_1B}\right) \neq \left(\frac{BA_0}{BA_1}\right)$$

C =

$$C = \left(A_0 \mid A_1 \right) \left(\frac{B_0}{B_1} \right) =$$

$$C = \left(A_0 \mid A_1 \right) \left(\frac{B_0}{B_1} \right) = A_0 B_0 + A_1 B_1$$

$$C = \left(\begin{array}{c|c} A_0 & A_1 \end{array}\right) \left(\begin{array}{c|c} B_0 \\ \hline B_1 \end{array}\right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} =$$

$$C = \begin{pmatrix} A_0 & A_1 \end{pmatrix} \begin{pmatrix} B_0 \\ \hline B_1 \end{pmatrix} = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(A_0 \mid A_1 \right) \left(\frac{B_0}{B_1} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,0} & \alpha_{1,1} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(A_0 \mid A_1 \right) \left(\frac{B_0}{B_1} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(A_0 \mid A_1 \right) \left(\frac{B_0}{B_1} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(A_0 \mid A_1 \right) \left(\frac{B_0}{B_1} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} & \alpha_{1,3} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,2} & \alpha_{3,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(A_0 \mid A_1 \right) \left(\frac{B_0}{B_1} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(A_0 \mid A_1 \right) \left(\frac{B_0}{B_1} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} & \alpha_{0,3} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{1,2} & \alpha_{1,3} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{2,2} & \alpha_{2,3} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$C = \left(A_0 \mid A_1 \right) \left(\frac{B_0}{B_1} \right) = A_0 B_0 + A_1 B_1$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\begin{array}{c|c} 3 & 1\end{array}\right) \left(\begin{array}{c} -1 \\ \hline 2\end{array}\right) =$$

$$\left(\begin{array}{c|c}3 & 1\end{array}\right) \left(\frac{-1}{2}\right) = (3) \times (-1) + (1) \times (2)$$

$$\left(\begin{array}{c|c} 3 & 1 \end{array}\right) \left(\begin{array}{c} -1 \\ \hline 2 \end{array}\right) & = & (3) \times (-1) + (1) \times (2)$$

$$\left(\begin{array}{c|c} A_0 & A_1 \end{array}\right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array}\right) & = & \end{array}$$

$$\left(\begin{array}{c|c} 3 & 1 \end{array}\right) \left(\begin{array}{c} -1 \\ \hline 2 \end{array}\right) = (3) \times (-1) + (1) \times (2)$$

$$\left(\begin{array}{c|c} A_0 & A_1 \end{array}\right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array}\right) = A_0 \times B_0 + A_1 \times B_1$$

$$\left(\begin{array}{c|c} 3 & 1 \end{array}\right) \left(\begin{array}{c} -1 \\ \hline 2 \end{array}\right) = (3) \times (-1) + (1) \times (2)$$

$$\left(\begin{array}{c|c} A_0 & A_1 \end{array}\right) \left(\begin{array}{c} B_0 \\ \hline B_1 \end{array}\right) = A_0 \times B_0 + A_1 \times B_1$$

Note:

$$(3) \times (-1) + (1) \times (2) = (-1) \times (3) + (2) \times (1)$$

 $A_0 \times B_0 + A_1 \times B_1 \neq B_0 \times A_0 + B_1 \times A_1$

$$\left(\begin{array}{c|c|c} C_{00} & C_{01} \\ \hline C_{10} & C_{11} \end{array}\right) = \left(\begin{array}{c|c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right)$$

$$\left(\begin{array}{c|c|c}
C_{00} & C_{01} \\
\hline
C_{10} & C_{11}
\end{array}\right) = \left(\begin{array}{c|c|c}
A_{00} & A_{01} \\
A_{10} & A_{11}
\end{array}\right) \left(\begin{array}{c|c|c}
B_{00} & B_{01} \\
B_{10} & B_{11}
\end{array}\right) \\
= \left(\begin{array}{c|c|c}
A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\
A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11}
\end{array}\right)$$

$$\left(\begin{array}{c|c|c}
C_{00} & C_{01} \\
\hline
C_{10} & C_{11}
\end{array}\right) = \left(\begin{array}{c|c|c}
A_{00} & A_{01} \\
\hline
A_{10} & A_{11}
\end{array}\right) \left(\begin{array}{c|c|c}
B_{00} & B_{01} \\
\hline
B_{10} & B_{11}
\end{array}\right) \\
= \left(\begin{array}{c|c|c}
A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\
\hline
A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11}
\end{array}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} =$$

$$\begin{pmatrix} \frac{C_{00}}{C_{10}} & \frac{C_{01}}{C_{11}} \end{pmatrix} = \begin{pmatrix} \frac{A_{00}}{A_{10}} & \frac{A_{01}}{A_{11}} \end{pmatrix} \begin{pmatrix} \frac{B_{00}}{B_{10}} & \frac{B_{01}}{B_{11}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{A_{00}B_{00} + A_{01}B_{10}}{A_{10}B_{00} + A_{11}B_{10}} & \frac{A_{00}B_{01} + A_{01}B_{11}}{A_{10}B_{01} + A_{11}B_{11}} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\gamma_{0,0}}{\gamma_{0,1}} & \frac{\gamma_{0,2}}{\gamma_{0,3}} & \frac{\gamma_{0,3}}{\gamma_{1,0}} & \frac{\gamma_{1,1}}{\gamma_{1,2}} & \frac{\gamma_{1,3}}{\gamma_{1,3}} & \frac{\alpha_{1,0}}{\alpha_{1,0}} & \frac{\alpha_{1,1}}{\alpha_{1,1}} & \frac{\alpha_{1,2}}{\alpha_{1,2}} & \frac{\alpha_{1,3}}{\alpha_{2,0}} \end{pmatrix} \begin{pmatrix} \frac{\beta_{0,0}}{\beta_{0,1}} & \frac{\beta_{0,2}}{\beta_{0,0}} & \frac{\beta_{0,3}}{\beta_{0,2}} & \frac{\beta_{0,3}}{\beta_{1,0}} & \frac{\beta_{1,0}}{\beta_{1,1}} & \frac{\beta_{1,2}}{\beta_{1,2}} & \frac{\beta_{1,3}}{\beta_{2,2}} & \frac{\beta_{2,3}}{\beta_{3,3}} \end{pmatrix} = \begin{pmatrix} \frac{\alpha_{0,0}}{\alpha_{1,1}} & \frac{\alpha_{0,1}}{\alpha_{2,2}} & \frac{\alpha_{2,1}}{\alpha_{2,2}} & \frac{\alpha_{2,2}}{\alpha_{2,3}} & \frac{\alpha_{2,3}}{\beta_{3,0}} & \frac{\beta_{2,1}}{\beta_{2,1}} & \frac{\beta_{2,2}}{\beta_{2,2}} & \frac{\beta_{2,3}}{\beta_{3,3}} \end{pmatrix} = \begin{pmatrix} \frac{\alpha_{0,0}}{\alpha_{1,1}} & \frac{\alpha_{0,1}}{\alpha_{1,1}} & \frac{\alpha_{1,2}}{\alpha_{1,2}} & \frac{\alpha_{1,3}}{\alpha_{2,2}} & \frac{\alpha_{2,3}}{\alpha_{3,1}} & \frac{\beta_{2,0}}{\beta_{2,1}} & \frac{\beta_{2,2}}{\beta_{2,2}} & \frac{\beta_{2,3}}{\beta_{3,3}} \end{pmatrix} = \begin{pmatrix} \frac{\alpha_{0,0}}{\alpha_{1,1}} & \frac{\alpha_{0,1}}{\alpha_{2,2}} & \frac{\alpha_{2,3}}{\alpha_{3,1}} & \frac{\alpha_{2,2}}{\alpha_{2,2}} & \frac{\alpha_{2,3}}{\alpha_{3,0}} & \frac{\beta_{2,1}}{\beta_{2,2}} & \frac{\beta_{2,2}}{\beta_{2,3}} & \frac{\beta_{2,2}}{\beta_{2,3}} & \frac{\beta_{2,3}}{\beta_{3,0}} & \frac{\beta_{3,1}}{\beta_{3,2}} & \frac{\beta_{3,2}}{\beta_{3,3}} & \frac{\beta_{3,2}}{\beta_{3,2}} & \frac{\beta_{3,3}}{\beta_{3,2}} & \frac{\beta$$

$$\begin{pmatrix}
C_{00} & C_{01} \\
C_{10} & C_{11}
\end{pmatrix} = \begin{pmatrix}
A_{00} & A_{01} \\
A_{10} & A_{11}
\end{pmatrix} \begin{pmatrix}
B_{00} & B_{01} \\
B_{10} & B_{11}
\end{pmatrix}$$

$$= \begin{pmatrix}
A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\
A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11}
\end{pmatrix}$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} \beta_{2,1} \\ \beta_{3,0} \beta_{3,1} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,2} \beta_{0,3} \\ \beta_{1,2} \beta_{1,3} & \beta_{1,2} \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} \\ \alpha_{1,2} \alpha_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} \beta_{2,3} \\ \beta_{3,2} \beta_{3,3} & \beta_{3,1} & \alpha_{3,2} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} \\ \alpha_{1,2} \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} \beta_{2,1} \\ \beta_{3,0} \beta_{3,1} & \beta_{3,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} \\ \alpha_{1,2} \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} \beta_{2,1} \\ \beta_{3,0} \beta_{3,1} & \beta_{3,1} & \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} \beta_{0,3} \\ \beta_{0,2} \beta_{0,3} \\ \beta_{1,2} \beta_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} \beta_{2,3} \\ \alpha_{3,2} \alpha_{3,3} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\begin{pmatrix}
C_{00} & C_{01} \\
C_{10} & C_{11}
\end{pmatrix} = \begin{pmatrix}
A_{00} & A_{01} \\
A_{10} & A_{11}
\end{pmatrix} \begin{pmatrix}
B_{00} & B_{01} \\
B_{10} & B_{11}
\end{pmatrix}
= \begin{pmatrix}
A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\
A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11}
\end{pmatrix}$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,1} \\ \beta_{3,0} \beta_{3,1} & \alpha_{3,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} \\ \beta_{1,2} \beta_{1,3} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,2} \alpha_{2,3} \\ \alpha_{3,2} \alpha_{3,3} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \beta_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,2} \alpha_{2,3} \\ \beta_{3,2} \beta_{3,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{2,0} \alpha_{2,1} \\ \alpha_{3,0} \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{$$

$$\left(\begin{array}{c|c|c}
C_{00} & C_{01} \\
\hline
C_{10} & C_{11}
\end{array}\right) = \left(\begin{array}{c|c|c}
A_{00} & A_{01} \\
A_{10} & A_{11}
\end{array}\right) \left(\begin{array}{c|c|c}
B_{00} & B_{01} \\
B_{10} & B_{11}
\end{array}\right) \\
= \left(\begin{array}{c|c|c}
A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\
A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11}
\end{array}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} \beta_{2,1} \\ \beta_{3,0} \beta_{3,1} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,2} \beta_{0,3} \\ \beta_{1,2} \beta_{1,3} & \beta_{1,2} \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} \\ \alpha_{1,2} \alpha_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} \beta_{2,3} \\ \beta_{3,2} \beta_{3,3} & \beta_{3,1} & \alpha_{3,2} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} \\ \alpha_{1,2} \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} \beta_{2,1} \\ \beta_{3,0} \beta_{3,1} & \beta_{3,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} \\ \alpha_{1,2} \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} \beta_{2,1} \\ \beta_{3,0} \beta_{3,1} & \beta_{3,1} & \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} \beta_{0,3} \\ \beta_{0,2} \beta_{0,3} \\ \beta_{1,2} \beta_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} \beta_{2,3} \\ \alpha_{3,2} \alpha_{3,3} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\begin{array}{c|c|c}
C_{00} & C_{01} \\
\hline
C_{10} & C_{11}
\end{array}\right) = \left(\begin{array}{c|c|c}
A_{00} & A_{01} \\
A_{10} & A_{11}
\end{array}\right) \left(\begin{array}{c|c|c}
B_{00} & B_{01} \\
B_{10} & B_{11}
\end{array}\right) \\
= \left(\begin{array}{c|c|c}
A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\
A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11}
\end{array}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} \beta_{2,1} \\ \beta_{3,0} \beta_{3,1} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,2} \beta_{0,3} \\ \beta_{1,2} \beta_{1,3} & \beta_{1,2} \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} \\ \alpha_{1,2} \alpha_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} \beta_{2,3} \\ \beta_{3,2} \beta_{3,3} & \beta_{3,1} & \alpha_{3,2} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} \\ \alpha_{1,2} \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} \beta_{2,1} \\ \beta_{3,0} \beta_{3,1} & \beta_{3,1} & \beta_{1,2} & \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} \\ \alpha_{1,2} \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} \beta_{2,1} \\ \beta_{3,0} \beta_{3,1} & \beta_{3,1} & \alpha_{3,0} & \alpha_{3,1} \end{pmatrix} \begin{pmatrix} \beta_{0,2} \beta_{0,3} \\ \beta_{0,2} \beta_{0,3} \\ \beta_{1,2} \beta_{1,3} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} \beta_{2,3} \\ \alpha_{3,2} \alpha_{3,3} & \beta_{3,2} & \beta_{3,3} \end{pmatrix}$$

$$\left(\begin{array}{c|c|c}
C_{00} & C_{01} \\
\hline
C_{10} & C_{11}
\end{array}\right) = \left(\begin{array}{c|c|c}
A_{00} & A_{01} \\
A_{10} & A_{11}
\end{array}\right) \left(\begin{array}{c|c|c}
B_{00} & B_{01} \\
B_{10} & B_{11}
\end{array}\right) \\
= \left(\begin{array}{c|c|c}
A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\
A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11}
\end{array}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,2} \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{0,2} & \alpha_{0,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{0,2} & \alpha_{0,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{0,2} & \alpha_{0,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{0,2} & \alpha_{0,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{0,2} & \alpha_{0,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{0,2} & \alpha_{0,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha$$

$$\left(\begin{array}{c|c|c}
C_{00} & C_{01} \\
\hline
C_{10} & C_{11}
\end{array}\right) = \left(\begin{array}{c|c|c}
A_{00} & A_{01} \\
\hline
A_{10} & A_{11}
\end{array}\right) \left(\begin{array}{c|c|c}
B_{00} & B_{01} \\
\hline
B_{10} & B_{11}
\end{array}\right) \\
= \left(\begin{array}{c|c|c}
A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\
\hline
A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11}
\end{array}\right)$$

$$\begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,0} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} & \beta_{0,1} & \beta_{0,2} & \beta_{0,3} \\ \beta_{1,0} & \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,0} & \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,0} & \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} = \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,2} & \alpha_{0,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{0,0} \beta_{0,1} & \beta_{0,2} \beta_{0,3} \\ \beta_{3,0} \beta_{3,1} & \beta_{3,2} \beta_{3,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} & \alpha_{2,3} \\ \alpha_{1,2} \alpha_{1,3} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \beta_{2,0} \beta_{2,1} \\ \beta_{3,0} \beta_{3,1} & \beta_{3,2} \beta_{3,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} & \alpha_{0,1} \\ \alpha_{1,0} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} \end{pmatrix} \begin{pmatrix} \alpha_{0,0} \alpha_{0,1} \\ \beta_{1,2} \beta_{1,3} & \beta_{1,2} \beta_{1,3} \end{pmatrix} + \begin{pmatrix} \alpha_{0,2} \alpha_{0,3} \\ \alpha_{1,2} \alpha_{2,3} & \alpha_{3,3} \end{pmatrix} \begin{pmatrix} \beta_{2,2} \beta_{2,3} \\ \beta_{3,2} \beta_{3,3} & \beta_{3,2} \beta_{3,3} \end{pmatrix}$$

$$\left(\begin{array}{c|c|c} 1 & -1 \\ \hline 2 & 3 \end{array}\right) \left(\begin{array}{c|c} -2 & 0 \\ \hline 1 & 2 \end{array}\right) =$$

$$\left(\begin{array}{c|c|c} 1 & -1 \\ \hline 2 & 3 \end{array}\right) \left(\begin{array}{c|c|c} -2 & 0 \\ \hline 1 & 2 \end{array}\right) = \left(\begin{array}{c|c|c} (1) \times (-2) + (-1) \times (1) & (1) \times (0) + (-1) \times (2) \\ \hline (2) \times (-2) + (3) \times (1) & (2) \times (0) + (3) \times (2) \end{array}\right)$$

$$\left(\begin{array}{c|c|c} 1 & -1 \\ \hline 2 & 3 \end{array}\right) \left(\begin{array}{c|c} -2 & 0 \\ \hline 1 & 2 \end{array}\right) = \\ \left(\begin{array}{c|c|c} (1) \times (-2) + (-1) \times (1) & (1) \times (0) + (-1) \times (2) \\ \hline (2) \times (-2) + (3) \times (1) & (2) \times (0) + (3) \times (2) \end{array}\right)$$

$$\left(\begin{array}{c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right) =$$

$$\left(\begin{array}{c|c|c} 1 & -1 \\ \hline 2 & 3 \end{array}\right) \left(\begin{array}{c|c} -2 & 0 \\ \hline 1 & 2 \end{array}\right) = \\ \left(\begin{array}{c|c|c} (1) \times (-2) + (-1) \times (1) & (1) \times (0) + (-1) \times (2) \\ \hline (2) \times (-2) + (3) \times (1) & (2) \times (0) + (3) \times (2) \end{array}\right)$$

$$\left(\begin{array}{c|c|c} A_{00} & A_{01} \\ \hline A_{10} & A_{11} \end{array}\right) \left(\begin{array}{c|c|c} B_{00} & B_{01} \\ \hline B_{10} & B_{11} \end{array}\right) = \left(\begin{array}{c|c|c} A_{00}B_{00} + A_{01}B_{10} & A_{00}B_{01} + A_{01}B_{11} \\ \hline A_{10}B_{00} + A_{11}B_{10} & A_{10}B_{01} + A_{11}B_{11} \end{array}\right)$$

Partition, conformally,

$$\frac{\vdots}{C_{M-1,0}} \frac{\vdots}{C_{M-1,1}} \frac{\vdots}{\cdots} \frac{\vdots}{C_{M-1,N-1}} = \frac{\vdots}{C_{M-1,0}} \frac{\vdots}{C_{M-1,1}} \frac{\vdots}{\cdots} \frac{\vdots}{C_{M-1,N-1}} \frac{\vdots}{C_{M-1,N-1}} \frac{A_{0,K-1}}{A_{0,K-1}} \frac{A_{0,K-1}}{A_{1,K-1}} \frac{B_{0,0}}{B_{0,1}} \frac{B_{0,1}}{B_{1,0}} \frac{B_{0,1}}{B_{1,1}} \frac{B_{1,0}}{\vdots} \frac{B_{1,1}}{\vdots} \frac{B_{M-1,0}}{B_{M-1,1}} \frac{B_{M-1,1}}{B_{M-1,1}} \frac{B_{M-1,1}}{B_{M-1$$

$$\begin{pmatrix}
B_{0,0} & B_{0,1} & \cdots & B_{0,N-1} \\
B_{1,0} & B_{1,1} & \cdots & B_{1,N-1} \\
\vdots & \vdots & \ddots & \vdots \\
B_{K-1,0} & B_{K-1,1} & \cdots & B_{K-1,N-1}
\end{pmatrix}$$

Partition, conformally,

$$\begin{pmatrix} \frac{\gamma_{0,0} & \gamma_{0,1} & \cdots & \gamma_{0,N-1}}{\gamma_{1,0} & \gamma_{1,1} & \cdots & \gamma_{1,N-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{M-1,0} & \gamma_{M-1,1} & \cdots & \gamma_{M-1,N-1} \end{pmatrix} = \\ \begin{pmatrix} \frac{\alpha_{0,0}}{\alpha_{1,0}} & \alpha_{0,1} & \cdots & \alpha_{0,K-1} \\ \frac{\alpha_{1,0}}{\alpha_{1,0}} & \alpha_{1,1} & \cdots & \alpha_{1,K-1} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{M-1,0} & \alpha_{M-1,1} & \cdots & \alpha_{M-1,K-1} \end{pmatrix} \begin{pmatrix} \frac{\beta_{0,0}}{\beta_{0,1}} & \frac{\beta_{0,1}}{\beta_{1,0}} & \cdots & \frac{\beta_{0,N-1}}{\beta_{1,N-1}} \\ \frac{\beta_{1,0}}{\beta_{1,1}} & \cdots & \frac{\beta_{1,N-1}}{\beta_{1,N-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\beta_{K-1,0}}{\beta_{K-1,1}} & \frac{\beta_{K-1,1}}{\beta_{K-1,N-1}} \end{pmatrix}$$

Partition, conformally,

$$C = \begin{pmatrix} C_{0,0} & C_{0,1} & \cdots & C_{0,N-1} \\ \hline C_{1,0} & C_{1,1} & \cdots & C_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline C_{M-1,0} & C_{M-1,1} & \cdots & C_{M-1,N-1} \end{pmatrix} A = \begin{pmatrix} A_{0,0} & A_{0,1} & \cdots & A_{0,K-1} \\ \hline A_{1,0} & A_{1,1} & \cdots & A_{1,K-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline A_{M-1,0} & A_{M-1,1} & \cdots & A_{M-1,K-1} \end{pmatrix}$$

and
$$B = \begin{pmatrix} B_{0,0} & B_{0,1} & \cdots & B_{0,N-1} \\ \hline B_{1,0} & B_{1,1} & \cdots & B_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline B_{K-1,0} & B_{K-1,1} & \cdots & B_{K-1,N-1} \end{pmatrix},$$

Partition, conformally,

$$C = \begin{pmatrix} C_{0,0} & C_{0,1} & \cdots & C_{0,N-1} \\ \hline C_{1,0} & C_{1,1} & \cdots & C_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline C_{M-1,0} & C_{M-1,1} & \cdots & C_{M-1,N-1} \end{pmatrix} A = \begin{pmatrix} A_{0,0} & A_{0,1} & \cdots & A_{0,K-1} \\ \hline A_{1,0} & A_{1,1} & \cdots & A_{1,K-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline A_{M-1,0} & A_{M-1,1} & \cdots & A_{M-1,K-1} \end{pmatrix}$$

and $B = \begin{pmatrix} B_{0,0} & B_{0,1} & \cdots & B_{0,N-1} \\ \hline B_{1,0} & B_{1,1} & \cdots & B_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline B_{K-1,0} & B_{K-1,1} & \cdots & B_{K-1,N-1} \end{pmatrix},$

Then

nen
$$C_{i,j} = \sum_{p=0}^{K-1} A_{i,p} B_{p,j}. \label{eq:constraint}$$

Partition, conformally,

$$C = \begin{pmatrix} C_{0,0} & C_{0,1} & \cdots & C_{0,N-1} \\ \hline C_{1,0} & C_{1,1} & \cdots & C_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline C_{M-1,0} & C_{M-1,1} & \cdots & C_{M-1,N-1} \end{pmatrix} A = \begin{pmatrix} A_{0,0} & A_{0,1} & \cdots & A_{0,K-1} \\ \hline A_{1,0} & A_{1,1} & \cdots & A_{1,K-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline A_{M-1,0} & A_{M-1,1} & \cdots & A_{M-1,K-1} \end{pmatrix}$$

and $B = \begin{pmatrix} B_{0,0} & B_{0,1} & \cdots & B_{0,N-1} \\ \hline B_{1,0} & B_{1,1} & \cdots & B_{1,N-1} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline B_{K-1,0} & B_{K-1,1} & \cdots & B_{K-1,N-1} \end{pmatrix},$

Then

nen
$$C_{i,j} = \sum_{p=0}^{K-1} A_{i,p} B_{p,j}. \label{eq:constraint}$$

$$C = \begin{pmatrix} \frac{\gamma_{0,0}}{\gamma_{1,0}} & \gamma_{0,1} & \cdots & \gamma_{0,N-1} \\ \frac{\gamma_{1,0}}{\vdots} & \vdots & \ddots & \vdots \\ \frac{\gamma_{M-1,0}}{\gamma_{M-1,1}} & \gamma_{M-1,1} & \cdots & \gamma_{M-1,N-1} \end{pmatrix} A = \begin{pmatrix} \frac{\alpha_{0,0}}{\alpha_{1,0}} & \alpha_{0,1} & \cdots & \alpha_{0,K-1} \\ \frac{\alpha_{1,0}}{\alpha_{1,1}} & \cdots & \alpha_{1,K-1} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\alpha_{M-1,0}}{\alpha_{M-1,1}} & \cdots & \alpha_{M-1,K-1} \end{pmatrix},$$
 and
$$B = \begin{pmatrix} \frac{\beta_{0,0}}{\beta_{1,0}} & \beta_{0,1} & \cdots & \beta_{0,N-1} \\ \frac{\beta_{1,0}}{\beta_{1,1}} & \cdots & \beta_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\beta_{K-1,0}}{\beta_{K-1,1}} & \beta_{K-1,1} & \cdots & \beta_{K-1,N-1} \end{pmatrix},$$

Then

hen
$$\gamma_{i,j} = \sum_{p=0}^{K-1} \alpha_{i,p} \beta_{p,j}.$$

Summary

Blocked matrix-matrix multiplication:

Exactly like matrix-matrix multiplication

except matrix-matrix multiplication usually does not commute.