

# Project 4

## Parallel Programming using the Message Passing Interface MPI

### Due date: 23 November 2022, 23:59 (midnight)

This assignment will introduce you to parallel programming using MPI. You will implement a simple MPI message exchange, compute a process topology, parallelize the computation of the Mandelbrot set, and finally either a parallel matrix-vector multiplication within the Power iteration (Option A) or a parallel PageRank Algorithm using the Power iteration (Option B). You may do this project in groups of two or three students. In fact, we prefer that you do so.

### 1. Ring maximum using MPI [10 Points]

This problem demonstrates basic MPI send/receive functionality and identification of neighbours in a 1-dimensional process grid layout. The processes are organized in a circular chain, where each process has 2 neighbours, and the first and last processes are neighbours of one another. In the class, we covered a ring summation algorithm that sends its rank to a neighbour and then every process sends the data it receives from one neighbour to its other neighbour; as a result, all processes obtained the sum of all ranks at the end. The first two iterations of the algorithm are illustrated in Figure 1.

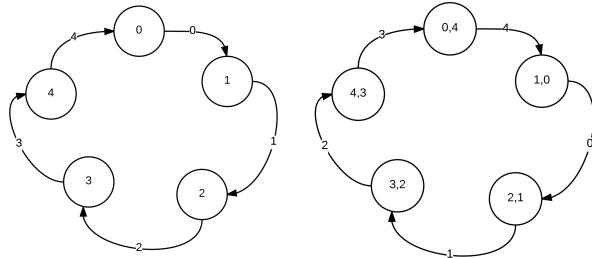


Figure 1: First two iterations of the ring sum algorithm.

**Write a program similar to the ring addition that achieves the following goals:**

1. Determines the left/right neighbours of each process.
2. Every process in the ring should contain the global maximum of the function

$$3 * \text{process\_rank}_i \mod (2 * \text{communicator\_size}), \text{ for } i = 0, 1, \dots, \text{communicator\_size} - 1.$$

Compilation and execution with 4 ranks on the ICS cluster :

```
[user@icslogin01]$ salloc -n 4
[user@icnodeXX]$ module load openmpi
[user@icnodeXX]$ cd ring/
[user@icnodeXX]$ make
[user@icnodeXX]$ mpirun -np 4 ./max_ring
```

The result of the parallel computation should be the maximum of the function of all four MPI ranks (0,1,2,3):

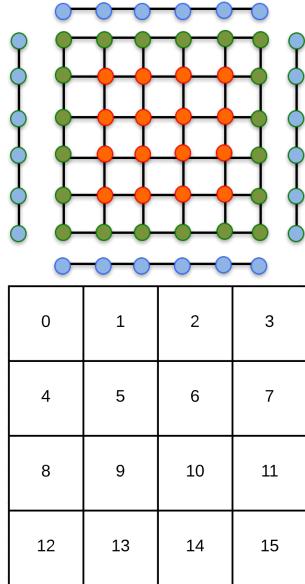


Figure 2: Local domain (left) and processor grid (right).

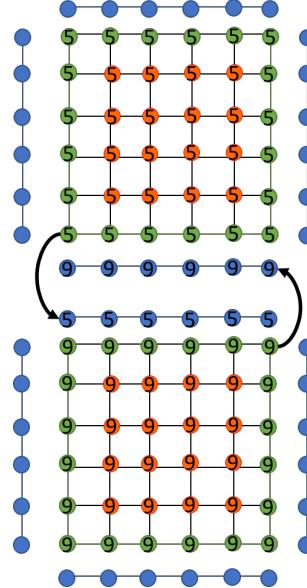


Figure 3: Exchange of the ghost cells.

```

Process 0:      Max = 6
Process 1:      Max = 6
Process 2:      Max = 6
Process 3:      Max = 6

```

## 2. Ghost cells exchange between neighbouring processes [15 Points]

The objective of this problem is to write a parallel program, using MPI, to exchange ghost cells between neighbouring processes. The term “ghost cell” refers to a copy of remote process’ data in the memory space of the current process. The ghost data is often needed for local computations, therefore, before proceeding with the computation, the processes need to exchange their ghost cells first. In this simple example, each process has a  $6 \times 6$  local domain which is extended by 1 row/column from each side to accommodate the copy of its neighbours’ borders (also known as “ghost cells”). The local domain is illustrated in red and green in Figure 2. The green cells are the local data that will be communicated to other processes. On the other hand, the ghost cells, containing the copy of the remote processes’ data, are indicated by the color blue. The local domain extended by ghost cells has size  $(6 + 2) \times (6 + 2)$  (we will ignore the corners). To easily inspect the result of the communication, we initialize the local domain of each process by its `mpi_rank`. Furthermore, we will assume we have an  $n \times n$  grid of processes organized in a Cartesian topology, as shown in the bottom image of Figure 2. We also assume cyclic borders, which means that, for example, process 0 is also a neighbour of process 3 and 12. Each process has 4 neighbours, sometimes referred to as the neighbour north, south, east and west. For testing purposes, always assume 16 processes are being used. The exchange of the borders between process 5 and 9 is illustrated in Figure 3. The compilation and execution on the ICS cluster are shown below:

```

[user@icslogin01]$ salloc -n 16
[user@icsnodeXX]$ module load openmpi
[user@icsnodeXX]$ cd ghost/
[user@icsnodeXX]$ make
[user@icsnodeXX]$ mpirun -np 16 ./ghost

```

**Note:** If you are running more processes than the available number of CPUs, you will need to oversubscribe your CPU, see here:

```
[user@icsnodeXX]$ mpirun -np 16 --oversubscribe ./ghost
```

The result of the boundary exchange on rank 9 should be

```
9.0 5.0 5.0 5.0 5.0 5.0 5.0 5.0 9.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
8.0 9.0 9.0 9.0 9.0 9.0 9.0 10.0
9.0 13.0 13.0 13.0 13.0 13.0 13.0 9.0
```

**Note:** When running on the ICS cluster , mpirun automatically uses all cores allocated to the job by slurm . Therefore, it is not necessary to provide this number again following the mpirun command as an argument.

Solve the following tasks:

1. Create a Cartesian 2-dimensional communicator ( $4 \times 4$ ) with periodic boundaries and use it to find your neighbouring ranks in all dimensions in a cyclic manner.
2. Create a derived data type for sending a column border (east and west neighbours).
3. Exchange ghost cells with the neighbouring cells in all directions and verify that correct values are in the ghost cells after the communication phase.

### 3. Parallelizing the Mandelbrot set using MPI [20 Points]

In this subproject, we will parallelize the Mandelbrot set computation from Project 2 using MPI. The computation of the Mandelbrot set will be partitioned into a set of parallel MPI processes, where each process will compute only its local portion of the Mandelbrot set. Examples of possible partitioning are illustrated in Figure 4. After each process completes its computation, the local domain is sent to the master process that will handle the I/O and create the output image containing the whole Mandelbrot set.

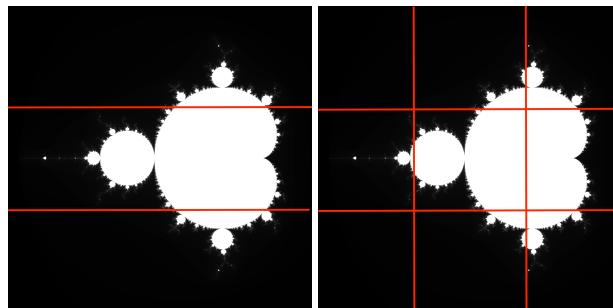


Figure 4: Possible partitionings of the Mandelbrot set.

We introduce two structures, which are defined in `consts.h`. The structure `Partition` represents the layout of the grid of processes and contains information such as the number of processes in  $x$  and  $y$  directions and the coordinates of the current MPI process.

```
typedef struct
{
    int x; // Coordinates of the current MPI process within the processor grid
    int y;
    int nx; // Dimensions of the processor grid
    int ny;
```

```
    MPI_Comm comm;  
} Partition;
```

The second structure `Domain` holds the information about the local domain of the current MPI process, such as its size (number of pixels in each dimension) and its global indices (index of the first and the last pixel in the full image of the Mandelbrot set that will be computed by the current process).

```
typedef struct  
{  
    long nx; // Size of the local domain  
    long ny;  
    long startx; // Begining of the local domain (global index)  
    long starty;  
    long endx; // End of the local domain (global index)  
    long endy;  
} Domain;
```

The code, which you will find on iCorsi or the Git repository, is initialized in a way that each process computes the whole Mandelbrot set. Your task will be to partition the domain (so that each process only computes an appropriate part), compute the local part of the image, and send the local data to the master process that will create the final complete image. The compilation and execution on the ICS cluster are shown below:

```
[user@icslogin01]$ salloc -n 16  
[user@icsnodeXX]$ module load openmpi  
[user@icsnodeXX]$ cd mandel/  
[user@icsnodeXX]$ make  
[user@icsnodeXX]$ mpirun -np 16 ./mandel_mpi
```

Running the benchmark and creating the performance plot (please compile and run on the compute node):

```
[user@icslogin01]$ salloc --exclusive  
[user@icsnodeXX]$ module load openmpi gnuplot  
[user@icsnodeXX]$ cd mandel/  
[user@icsnodeXX]$ make  
[user@icsnodeXX]$ ./run_perf.sh  
[user@icsnodeXX]$ ./plot_perf.sh
```

Solve the following tasks:

1. Create the partitioning of the image by implementing a body of functions `Partition createPartition(int mpi_rank, int mpi_size)` and `Partition updatePartition(Partition p, int mpi_rank)`. You can find a dummy implementation of these functions in `consts.h`.
2. Determine the dimensions and the start/end of the local domain based on the computed partitioning by implementing a function `Domain createDomain(Partition p)`. The function is defined in the `consts.h`.
3. Send the local domain to the master process if `mpi_rank > 0` and receive it at the master process where `mpi_rank == 0`. **Compare the output of the parallelized program to that of the sequential program in a graphic and verify that it is correct.**
4. Analyze the performance in the graph `perf.ps`. It shows the computational time of each process for multiple runs with a varying number of processes. **Comment on the benefits of the MPI parallelization and load balancing you have observed. How does the performance change when running on 1 compute node vs. multiple nodes?**

## 4. Option A: Parallel matrix-vector multiplication and the power method [40 Points]

In this subproject<sup>1</sup>, you will write a parallel program to multiply a matrix  $A$  by a vector  $x$ , and use this routine in an implementation of the power method to find the absolute value of the largest eigenvalue of the matrix. Your code will call routines that we supply to generate matrices, record timings, and validate the answer.

### 4.1. Mathematical background

A square matrix  $A$  is an  $n$ -by- $n$  array of numbers. The entry in row  $i$ , column  $j$  of  $A$  is written either  $a_{ij}$  or  $A(i,j)$ . The rows and columns are numbered from 1 to  $n$ . A vector is a one-dimensional array  $x$  whose  $i$ -th entry is  $x_i$  or  $x(i)$ . Recall the definition of matrix-vector multiplication: The product  $y = Ax$  is a vector  $y$  whose elements are

$$y_i = \sum_{j=1}^n a_{ij}x_j. \quad (1)$$

In words, each element of  $y$  is obtained from one row of  $A$  and all of  $x$ , by computing an inner product (that is, by adding up the pointwise products). Every element of  $x$  contributes to every element of  $y$ ; each element of  $A$  is used exactly once. The *power method* uses matrix-vector multiplication to estimate the size of the largest absolute eigenvalue of the matrix  $A$ , also known as the spectral radius of  $A$ . It works as follows: Start with an arbitrary vector  $x$ . Then repeat the following two steps: divide each element of  $x$  by the norm of  $x$ ; second, replace  $x$  by the matrix-vector product  $Ax$ . The norm of the vector eventually converges to the spectral radius of  $A$ . In your code, you will repeat the matrix-vector product 1000 times. There is a sequential Matlab code `powermethod` for the Power method on the iCorsi course web page<sup>2</sup> and on the git repository<sup>3</sup>.

```
function lambda = powermethod(A)
% POWERMETHOD : Power method to estimate norm of dominant eigenvalue
%
% lambda = powermethod(A);
%
% This routine generates a random vector, then repeatedly
% multiplies matrix A by the vector and normalizes the result
% to have length 1. The norm converges to the magnitude of
% the dominant eigenvalue, which is returned as lambda.
%
% This is a sequential Matlab template -- your task is
% to implement this in parallel.
%
[n,n] = size(A);           % number of rows and columns in A
x = rand(n,1);             % creates a starting n-vector
for i = 1:1000
    x = x / norm(x);      % norm(x) is sqrt(x(1)^2 + x(2)^2 + ... + x(n)^2)
    x = A * x;              % use your parallel matrix * vector routine here
end;
lambda = norm(x);
```

Your task is to write a parallel C/MPI code that does the same computation.

<sup>1</sup>Only one of the two subprojects *Option A* or *Option B* is needed to obtain all points. You can either select *Option A* or *Option B*. If you decide to submit a solution for both subprojects then we will only count the best of these two submissions towards your entire grade for this project.

<sup>2</sup><https://www.icorsi.ch/course/view.php?id=14652>

<sup>3</sup><https://github.com/oschenk/hpc2022>

## 4.2. Test harnesses

We supply three routines that you can call from your code for testing, they are a matrix generator `hpc_generateMatrix`, a validator to check the accuracy of the results `hpc_verify` (for the particular matrix we generate), and a timer `hpc_timer`. We will use these routines in grading your program.

## 4.3. What to implement

You need to write the following MPI parallel C routines:

- `generateMatrix`: Although we will supply a matrix generator for grading, you should also generate various matrices for testing your code. This routine should generate a matrix of specified size, with the data distributed across the processors as specified in the next section.
- `powerMethod`: Implements the power method on a given matrix (which is already distributed across the processors). This routine calls `norm` and `matVec`.
  - `norm`: Computes the norm of a given vector.
  - `matVec`: Multiplies a given matrix (which is already distributed across the processors) by a given vector.
- `main`: The main routine that calls `generateMatrix` and e.g.  $k = 1000$  times the `powerMethod` routine.

## 4.4. Where's the data?

To simplify the parallel code you may assume that  $n$ , the number of rows and columns of the matrix, is divisible by  $p$ , the number of processors. Distribute the matrix across the processors by rows, with the same number of rows on each processor; thus, processor 0 gets rows 1 through  $n/p$  of  $A$ , processor 1 gets rows  $n/p + 1$  through  $2n/p$ , and so forth. Your `generateMatrix` routine should not do any communication except for the value of  $n$ ; each processor should generate its own rows of the matrix, independently of the others, in parallel. Put the vector on processor 0. For our purposes, the “arbitrary vector” you start with can be a vector of random doubles. When you write your `matVec` routine, you should use collective communication like `MPI_Bcast`, `MPI_Gather` and `MPI_Allgather`; you will find the code to be much simpler this way than using point-to-point communication, like `MPI_Send` and `MPI_Recv`.

## 4.5. What experiments to do

First, debug your code on very small matrices, using one MPI process, then two processes, then several MPI processes. Two matrices you can use for debugging are the  $n$ -by- $n$  matrix of all ones (whose spectral radius is  $n$ ) and the identity matrix (with ones on the main diagonal and zeros elsewhere), whose spectral radius is 1. For debugging, you should probably only do a few iterations of the power method instead of 1000. Your code should only time the call to `powerMethod`, not the matrix generation! You should call our harness routines to start and stop the “official” timer; you can also use `MPI_Wtime` for your timings. Here are some experiments to do:

- **Strong scaling analysis.** Choose a value of  $n$  for which your code runs on one process in a reasonable amount of time, say 30 seconds to a minute, with 1,000 iterations of the main loop. (On my laptop,  $n = 10,000$  takes about 50 seconds.) Run your code for  $p = 1, 4, 8, 12, 16, 32$ , and (if possible) 64. Also, run your code using a various number of ICS cluster nodes. For each run, report the execution time and the parallel efficiency. Plot the execution time versus  $p$  and the parallel efficiency versus  $p$ .
- **Weak scaling analysis.** Modify your program to do only 100 iterations of the main loop, to make the experiments in this part run faster. Now, choose a starting value of  $n$  to use for  $p = 1$ , possibly the same  $n$  as above. Run your code for  $p = 1, 4, 8, 12, 16, 32$ , and 64, but this time use a different  $n$  for each  $p$ , chosen so that  $n$  is

(nearly) proportional to  $\sqrt{p}$ . Since both total memory and total work scale as  $n^2$ , this implies that the memory required per processor and the work done per processor will remain constant as you increase  $p$ . This is called weak scaling. Again, report the execution time and the parallel efficiency for each run, and make plots similar to the previous experiment.

## 5. Option B: Parallel PageRank Algorithm and the Power method [40 Points]

The purpose of this subproject<sup>4</sup> is to learn the importance of parallel numerical linear algebra algorithms to solve fundamental linear algebra problems that occur in search engines.

### 5.1. The initial Page-Rank Algorithm – Parallel Eigenvalue Graph Computation

One of the first reasons why Google is such an effective search engine is the Page-Rank algorithm developed by Google's founders, Larry Page and Sergey Brin, when they were graduate students at Stanford University. PageRank is determined entirely by the link structure of the World Wide Web. In 2002, it was recomputed about once a month and did not involve the actual content of any Web pages or individual queries. Then, for any particular query, Google finds the pages on the Web that match that query and lists those pages in the order of their PageRank. Imagine surfing the Web, going from page to page by randomly choosing an outgoing link from one page to get to the next. This can lead to dead ends at pages with no outgoing links, or cycles around cliques of interconnected pages. So, a certain fraction of the time, simply choose a random page from the Web. This theoretical random walk is known as a *Markov chain* or *Markov process*. The limiting probability that an infinitely dedicated random surfer visits any particular page is its PageRank.

A page has a high rank if other pages with high ranks link to it. Let  $W$  be the set of Web pages that can be reached by following a chain of hyperlinks starting at some root page and let  $n$  be the number of pages in  $W$ . Let  $G$  be the  $n$ -by- $n$  connectivity matrix of a portion of the Web, that is  $g_{ij} = 1$  if there is a hyperlink to page  $i$  from page  $j$  and zero otherwise. The matrix  $G$  can be huge, but it is very sparse. Its  $j$ -th column shows the links on the  $j$ -th page. The number of nonzeros in  $G$  is the total number of hyperlinks in  $W$ . Let  $r_i$  and  $c_j$  be the row and column sums of  $G$ :

$$r_i = \sum_j g_{ij}, \quad c_j = \sum_i g_{ij}, \quad (2)$$

The quantities  $r_j$  are the *in-degree* and  $c_j$  are the *out-degree* of the  $j$ -th page. Let  $p$  be the probability that the random walk follows a link. A typical value is  $p = 0.85$ . Then  $1 - p$  is the probability that some arbitrary page is chosen and  $\delta = (1 - p)/n$  is the probability that a particular random page is chosen.

Let  $A$  be the  $n$ -by- $n$  matrix whose elements are

$$a_{ij} = \begin{cases} p g_{ij}/c_j + \delta & c_j \neq 0 \\ 1/n & c_j = 0. \end{cases} \quad (3)$$

Notice that  $A$  comes from scaling the connectivity matrix by its column sums. The  $j$ -th column is the probability of jumping from the  $j$ -th page to the other pages on the Web. If the  $j$ -th page is a dead-end, that is, it has no out-links, then we assign a uniform probability of  $1/n$  to all the elements in its column. Most of the elements of  $A$  are equal to  $\delta$ , the probability of jumping from one page to another without following a link. If  $n = 4 \cdot 10^9$  (current estimation of webpages) and  $p = 0.85$ , then  $\delta = 3.75 \cdot 10^{-11}$ . Matrix  $A$  is the transition probability matrix of the Markov chain. Its elements are all strictly between zero and one and its column sums are all equal to one. An important result in matrix

<sup>4</sup>This description is a revised version of a SIAM book chapter from *Numerical Computing with Matlab* from Cleve B. Moler that has been published in 2002.

theory known as the *Perron-Frobenius theorem* applies to such matrices. It concludes that a nonzero solution of the equation

$$x = Ax \quad (4)$$

exists and is unique to within a scaling factor. If this scaling factor is chosen so that

$$\sum_i x_i = 1, \quad (5)$$

then  $x$  is the *state vector* of the Markov chain and is *Google's PageRank*. The elements of  $x$  are all positive and less than one. For modest  $n$ , an easy way to compute  $x$  using Matlab notations is to start with some approximate solution, such as the PageRanks from the previous month, or

```
x = ones(n, 1) / n
```

Then simply repeat the assignment statement

```
x = x / norm(x); % norm(x) is sqrt(x(1)^2 + x(2)^2 + ... + x(n)^2)
x = A*x;
```

until successive vectors agree to within a specified tolerance. This is known as the Power method and is about the only possible approach for very large  $n$ . In practice, the matrices  $G$  and  $A$  are never actually formed. One step of the power method would be done by one pass over a database of Web pages, updating weighted reference counts generated by the hyperlinks between pages. The best way to compute PageRank in Matlab is to take advantage of the particular structure of the Markov matrix. Here is an approach that preserves the sparsity of  $G$ . The transition matrix can be written

$$A = pGD + ez^T, \quad (6)$$

where  $D$  is the diagonal matrix formed from the reciprocals of the out-degrees,

$$d_{jj} = \begin{cases} 1/c_j & c_j \neq 0 \\ 0 & c_j = 0. \end{cases} \quad (7)$$

$e$  is the  $n$ -vector of all ones, and  $z$  is the vector with components

$$z_j = \begin{cases} \delta = (1-p)/n & c_j \neq 0 \\ 1/n & c_j = 0. \end{cases} \quad (8)$$

The rank-one matrix  $ez^T$  accounts for the random choices of Web pages that do not follow links. The **Power method** can be implemented in a way that does not actually form the Markov matrix and so preserves sparsity. Compute

```
G = p*D;
z = ((1-p)*(c~=0) + (c==0))/n; // this implements in Matlab the equation (8)
```

Start with

```
x = e/n
```

and then repeat the statement

```
x = G*x + e*(z*x)
```

until  $x$  settles down to several decimal places. The full Page Rank method in Matlab is shown below.

```
function x = pagerank(G,U)
% PAGERANK Google's PageRank
% x = pagerank(U, G, p) uses the power method to compute the page
% rank for a connectivity matrix G with a damping factory p,
% (default is p = 0.85).
p = .85;
```

```
% Eliminate any self-referential links
G = G - diag(diag(G));
% c = out-degree
[n,n] = size(G);
c = sum(G,1);
% Form the components of the Markov transition matrix.
k = find(c~=0);
D = sparse(k,k,1./c(k),n,n);
G = p*D;
e = ones(n,1);
z = ((1-p)*(c~=0) + (c==0))/n;
% Power method to find Markov vector, A*x = x.
x = e/n;
it = 0;
xs = zeros(n,1);
while norm(x-xs,inf)> 1.e-6*norm(x,inf)
    xs = x;
    x = G*x + e*(z*x);
    it = it + 1;
    disp(['Iteration #' , int2str(it), '; Error:' , num2str(norm(x-xs,inf))]);
end
```

## 5.2. Example – A tiny webgraph

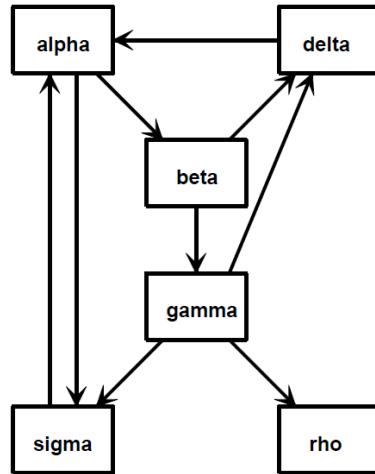


Figure 5: A tiny web graph.

Figure 5 is the graph from a tiny web example, with  $n = 6$ . This small example involves 6 vertices (websites):

```
U = {'www.alpha.com'
      'www.beta.com'
      'www.gamma.com'
      'www.delta.com'
      'www.rho.com'
      'www.sigma.com'}
```

We can generate the connectivity graph matrix by specifying the pairs of indices  $(i, j)$  of the nonzero elements. Because there is a link to `www.beta.com` ( $i = 2$ ) from `www.alpha.com` ( $j = 1$ ), the  $(2, 1)$  element of  $G$  is nonzero. The nine connections  $(i, j)$  of  $G$  are described by

```
i = [ 2 6 3 4 4 5 6 1 1]
j = [ 1 1 2 2 3 3 3 4 6]
```

A sparse matrix is stored in a data structure that requires memory only for the nonzero elements and their indices. This is hardly necessary for a 6-by-6 matrix with only 27 zero entries, but it becomes crucially important for larger problems. The Matlab statements

```
n = 6
G = sparse(i, j, 1, n, n);
full(G)
```

generate the sparse representation of an  $n$ -by- $n$  matrix with ones in the positions specified by the vectors  $i$  and  $j$  and display its full representation of  $G$

```
0 0 0 1 0 1
1 0 0 0 0 0
0 1 0 0 0 0
0 1 1 0 0 0
0 0 1 0 0 0
1 0 1 0 0 0
```

The statement

```
c = full(sum(G))
```

computes the column sums (or out-degree of these webpages)

```
2 2 3 1 0 1
```

Notice that  $c(5) = 0$  because the 5th page, labeled  $\rho$ , has no out-links. For this tiny example with  $p = .85$ , the smallest element of the Markov transition matrix  $A$  is  $\delta = .15/6 = .0250$ .

```
A =
0.0250 0.0250 0.0250 0.8750 0.1667 0.8750
0.4500 0.0250 0.0250 0.0250 0.1667 0.0250
0.0250 0.4500 0.0250 0.0250 0.1667 0.0250
0.0250 0.4500 0.3083 0.0250 0.1667 0.0250
0.0250 0.0250 0.3083 0.0250 0.1667 0.0250
0.4500 0.0250 0.3083 0.0250 0.1667 0.0250
```

Notice that the column sums of  $A$  are all equal to one. The Power method applied to

```
x = G*x + e*(z*x)
```

and implemented in pagerank.m (Matlab) using

```
i = [2 6 3 4 4 5 6 1 1];
j = [1 1 2 2 3 3 3 4 6];
n = 6;
G = sparse(i, j, 1, n, n);
U = {'http://www.alpha.com';
      'http://www.beta.com' ;
      'http://www.gamma.com';
      'http://www.delta.com';
      'http://www.rho.com' ;
      'http://www.sigma.com'};
x = pagerank(G,U)
```

or in pagerank.c

```
./pagerank tinygraph.mat
```

solves for the PageRank vector  $x$  until it settles down to several decimal places to produce

```
x =  
0.3210  
0.1705  
0.1066  
0.1368  
0.0643  
0.2007
```

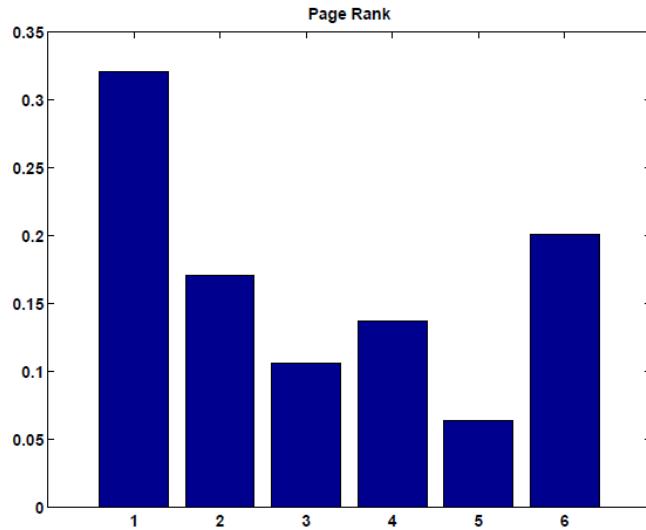


Figure 6: Page Rank for the tiny web graph.

The bar graph of  $x$  is shown in Figure 6. If the URLs are sorted in PageRank order and listed along with their in- and out-degrees, and

```
x = pagerank(G,U)
```

results in

|   | page-rank | in | out | url           |
|---|-----------|----|-----|---------------|
| 1 | 0.3210    | 2  | 2   | www.alpha.com |
| 6 | 0.2007    | 2  | 1   | www.sigma.com |
| 2 | 0.1705    | 1  | 2   | www.beta.com  |
| 4 | 0.1368    | 2  | 1   | www.delta.com |
| 3 | 0.1066    | 1  | 3   | www.gamma.com |
| 5 | 0.0643    | 1  | 0   | www.rho.com   |

We see that alpha has a higher PageRank than delta or sigma, even though they all have the same number of in-links. A random surfer will visit alpha over 32% of the time and rho only about 6% of the time.

### 5.3. PageRank for graphs from the Stanford Network Dataset Collection (SNAP)

Since a vertex will be connected to only a few other vertices, most of the entries in the weighted adjacency matrix will be zero. The resulting matrix will be very sparse. Also, a sizeable webgraph from the Stanford Large Network Dataset Collection (SNAP) will have tens of thousands of nodes. Hence we will need a technique that can efficiently store these large sparse matrices without running out of memory. A common way for storing large sparse matrices is either (i) the compressed column storage (CCS) or (ii) the compressed row storage scheme (CRS). We will provide both versions of the project to you.

The CCS storage technique can be used in our HPC implementation and it consists of the creation of three arrays:

- G – Holds all non zero entries of the connectivity graph  $G$ .
- indices - Contains the corresponding row indices of the elements in G.
- colptr - Contains the indices of indices array of those elements which start a new column in G.

The CRS storage technique can be used as well in our HPC implementation and it consists of the creation of three arrays:

- G – Holds all non zero entries of the connectivity graph  $G$ .
- indices - Contains the corresponding column indices of the elements in G.
- rowptr - Contains the indices of indices array of those elements which start a new row in G.

The iCorsi and git repository contains the C codes pagerank.c (matrix is stored in CCS format) or another version of pagerank\_csr.c (matrix is stored in CRS format), where both are serial implementations of pagerank computation for datasets from SNAP. Here is an example of how to run it using the SNAP matrix soc-LiveJournal1 that is stored in MatrixMarket format and comes from an online social network application:

```
[user@icslogin01]$ salloc -n 1
[user@icsnodeXX]$ module load gcc
[user@icsnodeXX]$ cd pagerank
[user@icsnodeXX]$ wget https://sparse.tamu.edu/MM/SNAP/soc-LiveJournal1.tar.gz
[user@icsnodeXX]$ gunzip soc-LiveJournal1.tar.gz
[user@icsnodeXX]$ tar -xvf soc-LiveJournal1.tar
[user@icsnodeXX]$ make
[user@icsnodeXX]$ mpirun ./pagerank soc-LiveJournal1/soc-LiveJournal1mtx
```

This results in

```
[USI HPC] Number of nodes 4847571 and edges 68993773 in webgraph soc-LiveJournal1mtx
[USI HPC] (rank=0) nedges_local=68993773 nedges_local_a=68993773
[USI HPC] 21 PageRank iterations with norm of 9.635308e-07 computed in 7.337764 sec.
//*****
// Sol. Norm Squared : 7.992768e-07
//*****
```

and the pagerank vector is available in the file PageRank.dat.

## 5.4. What experiments to do

- To familiarize yourself with the Power method you may read the chapter 8 from the book "A First Course on Numerical Methods" by C. Greif and U. Ascher.
- Study the Power method, the Matlab template pagerank.m

```
G = p*G*D
z = ((1-p)*(c~=0) + (c==0))/n;
while termination_test
x = G*x + e*(z*x)
end
```

and the C code pagerank.c or the pagerank\_csr.c which implement the Power method in C using an appropriate test for terminating.

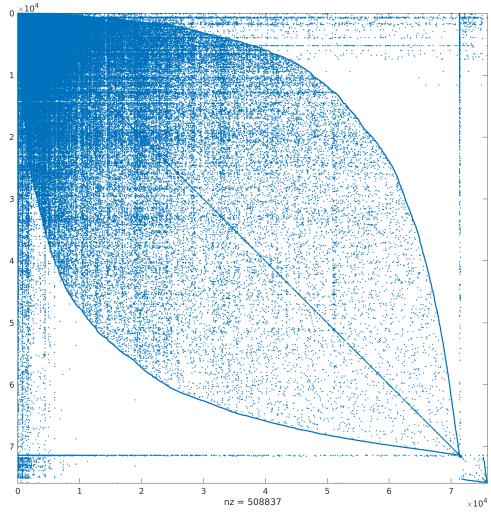


Figure 7: The soc-Epinions1 graph.

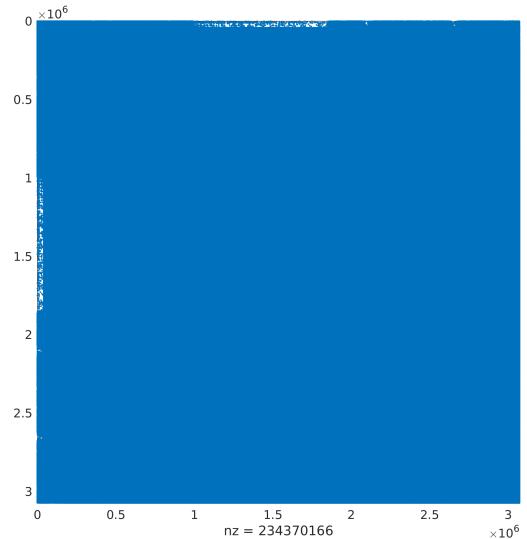


Figure 8: The com-Orkut graph.

- We have provided this serial implementation. It will be your task to make the necessary MPI changes to obtain a parallel implementation to achieve favourable performance across a range of ICS cluster nodes. We need to be able to build and execute your implementations for you to receive credit. Spell out in your report the Makefile targets that we should build for the different parts of your report. Here are some items you need to add to your report for the parallel experiments on three SNAP graphs:
  - A description of the communication you used in the distributed memory implementation.
  - A description of the design choices that you tried and how did they affect the performance.
  - Speedup plots that show how closely your MPI code approaches the idealized  $p$ -times speedup and a discussion on whether it is possible to do better.
  - Where does the time go? Consider breaking down the runtime into computation time, synchronization time and/or communication time. How do they scale with  $p$ ? A discussion on using MPI will be useful as well.
- Benchmark your code on the three SNAP examples below. Please make sure that you download the SNAP examples either directly to your `$SCRATCH` directory (and not to your own local `$HOME` directory). As an alternative, all these three examples are also available in `/cluster/scratch/oschenk/`.
  - `soc-LiveJournal1 mtx` will result using one compute core in

```
[user@icsnodeXX] $ mpirun ./pagerank download_directory/soc-LiveJournal1 mtx
[USI HPC] Number of nodes 4847571 and edges 68993773 in soc-LiveJournal1 mtx
[USI HPC] 21 PageRank iterations with norm of 9.6e-07 computed in 6.4 sec.
```
  - `com-Friendster mtx` will result using one compute core in

```
[user@icsnodeXX] $ mpirun ./pagerank download_directory/com-Friendster mtx
[USI HPC] Number of nodes 65608366 and edges 1806067135 in com-Friendster mtx
[USI HPC] 21 PageRank iterations with norm of 7.8e-07 computed in 704.5 sec.
```
- and
  - `com-Orkut mtx` will result using one compute core in

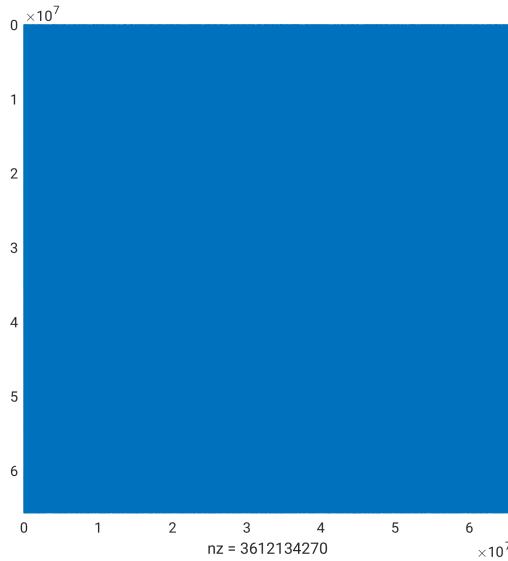


Figure 9: The com-Friendster graph.

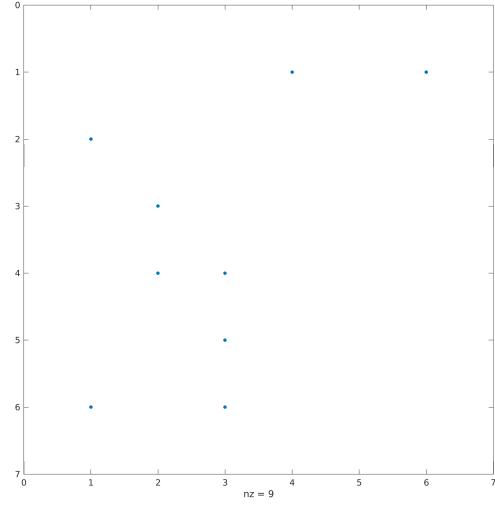


Figure 10: The tiny webgraph matrix.

```
[user@icsnodeXX] $ mpirun ./pagerank download_directory/com-Orkut.mtx
[USI HPC] Number of nodes 3072441 and edges 117185083 in webgraph com-Orkut.mtx
[USI HPC] 16 PageRank iterations with norm of 6.6e-07 computed in 18.6 sec.
```

and report the node number, pagerank, in-degree and out-degree of the five node with the highest pagerank value similar to the table below from the tiny graph from our example above.

| node | page-rank | in | out |
|------|-----------|----|-----|
| 1    | 0.3210    | 2  | 2   |
| 6    | 0.2007    | 2  | 1   |
| 2    | 0.1705    | 1  | 2   |
| 4    | 0.1368    | 2  | 1   |
| 3    | 0.1066    | 1  | 3   |
| 5    | 0.0643    | 1  | 0   |

Since some of the simulations require a substantial amount of memory, be careful to request enough memory for your jobs to run. This is done above with the option

- --mem=MB ,

which requests XX MB of memory . Keep the latter fact in mind when running in parallel. For example when the total amount of memory needed is 40 GB and you run on 8 processes, then the memory request per process should be 5 GB:

```
[user@icsnodeXX] $ mpirun ./pagerank download_directory/com-Friendster.mtx
```

Please also note that the actual wall-clock run time of the jobs may be slightly longer due to the sizable I/O.

## 5.5. MATLAB Visualization of the SNAP Graph Networks

Graphs model the connections in a network and are widely applicable to a variety of physical, biological, and information systems. You can use graphs to model the neurons in a brain, the flight patterns of an airline, and much more. The structure of a graph is comprised of “nodes” and “edges”. Each node represents an entity, and each edge represents a connection between two nodes. In our case above we studied the SNAP graphs. You can visualize the connectivity of these SNAP networks using the following commands in MATLAB and the file mmread.m:

```
wget https://math.nist.gov/MatrixMarket/mmio/matlab/mmread.m
matlab -nodesktop
>> [A,rows,cols,entries,rep,field,symm] = mmread('soc-LiveJournal1 mtx');
>> spy(A);
>> [A,rows,cols,entries,rep,field,symm] = mmread('com-Orkut.mtx');
>> spy(A);
>> [A,rows,cols,entries,rep,field,symm] = mmread('com-Friendster.mtx');
>> spy(A);
```

The visualization is shown in the Figures 7 to 10. You might take into consideration the irregular structure of these SNAP graphs when parallelizing the pagerank computation using MPI.

## 6. Task: Quality of the Report [15 Points]

Each project will have 100 points (out of which 15 points will be given to the general quality of the written report).

### Additional notes and submission details

We only accept submissions using our Latex template and C/C++ code. Please submit the source code files (together with your used `Makefile`) in an archive file (tar, zip, etc.), and summarize your results and observations for all exercises by writing an extended Latex report. Use the Latex template provided on the webpage and upload the Latex summary as a PDF to iCorsi .

- Your submission should be a gzipped tar archive, formatted like `project_number_lastname_firstname.zip` or `project_number_lastname_firstname.tgz`. It should contain:
  - all the source codes of your MPI solutions;
  - your write-up with your name `project_number_lastname_firstname.pdf`.
- Submit your `.zip/.tgz` through iCorsi .