1)

$$H(z) = \frac{(1 - \frac{1}{2} z^{-2})}{(1 - \frac{1}{2} z^{-1})(1 - \frac{1}{4} z^{-1})} \; ; \; |z| > \frac{1}{2}$$

a) $$H(z) = Q(z) + \frac{A_1}{1 - \frac{1}{2} z^{-1}} + \frac{A_2}{1 - \frac{1}{4} z^{-1}}$$

$$(1 - \tfrac{1}{2} z^{-1})(1 - \tfrac{1}{4} z^{-1}) = 1 - \tfrac{3}{4} z^{-1} + \tfrac{1}{8} z^{-2}$$

$$\begin{array}{l|l} 1 - \frac{1}{2} z^{-2} & 1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2} \\ \hline -3z^{-1} + 4 + \frac{1}{2} z^{-2} & -4 \\ \hline 5 - 3z^{-1} & \end{array} \qquad ; \; Q(z) = -4$$

$$H(z) = -4 + \underbrace{\frac{5 - 3z^{-1}}{(1 - \frac{1}{2} z^{-1})(1 - \frac{1}{4} z^{-1})}}_{H'(z)}$$

$$A_1 = H'(z)(1 - \tfrac{1}{2} z^{-1})\Big|_{z=\frac{1}{2}} = \left(\frac{5 - 3z^{-1}}{1 - \frac{1}{4} z^{-1}}\right)\Bigg|_{z=\frac{1}{2}} = \frac{-1}{\frac{1}{2}} = -2$$

$$A_2 = H'(z)(1 - \tfrac{1}{4} z^{-1})\Big|_{z=\frac{1}{4}} = \left(\frac{5 - 3z^{-1}}{1 - \frac{1}{2} z^{-1}}\right)\Bigg|_{z=\frac{1}{4}} = \frac{-7}{-1} = 7$$

$$H(z) = -4 - \underbrace{\frac{2}{1 - \frac{1}{2} z^{-1}}}_{|z| > \frac{1}{2}} + \underbrace{\frac{7}{1 - \frac{1}{4} z^{-1}}}_{|z| > \frac{1}{4}} \; , \; |z| > \frac{1}{2}$$

$$h[n] = -4\,\delta[n] - 2\left(\tfrac{1}{2}\right)^n u[n] + 7\left(\tfrac{1}{4}\right)^n u[n]$$

b) $$H(z) = \frac{Y(z)}{X(z)}$$

$$\frac{Y(z)}{X(z)} = \frac{1 - \frac{1}{2} z^{-2}}{(1 - \frac{1}{2} z^{-1})(1 - \frac{1}{4} z^{-1})} \Rightarrow Y(z)\left((1 - \tfrac{1}{2} z^{-1})(1 - \tfrac{1}{4} z^{-1})\right) = X(z)(1 - \tfrac{1}{2} z^{-2})$$

$$\mathcal{Z}^{-1}\left\{Y(z) - \frac{3}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z)\right\} = \mathcal{Z}^{-1}\left\{X(z) - \tfrac{1}{2} z^{-2} X(z)\right\}$$

$$y[n] - 3/4\, y[n-1] + 1/8\, y[n-2] = x[n] - 1/2\, x[n-2]$$

2)

$y[n] = x[-n+3]$

$y'[n] = x[n-3] \Longleftrightarrow Y'(z) = z^{-3}X(z)$

$y[n] = y'[-n] \Longleftrightarrow Y(z) = Y'(z^{-1})$

$Y(z) = z^3 X(z^{-1})$

$$X(z) = \frac{z^{-1}}{(1+\frac{3}{4}z^{-1})(1-z^{-1}+\frac{1}{2}z^{-2})}$$

$$X(z^{-1}) = \frac{z}{(1+\frac{3}{4}z)(1-z+\frac{1}{2}z^{2})}$$

$$Y(z) = \frac{z^4}{(1+\frac{3}{4}z)(1-z+\frac{1}{2}z^2)} = \frac{z^4}{\frac{3}{4}z(\frac{4}{3}z^{-1}+1)\frac{1}{2}z^2(2z^{-2}-2z^{-1}+1)}$$

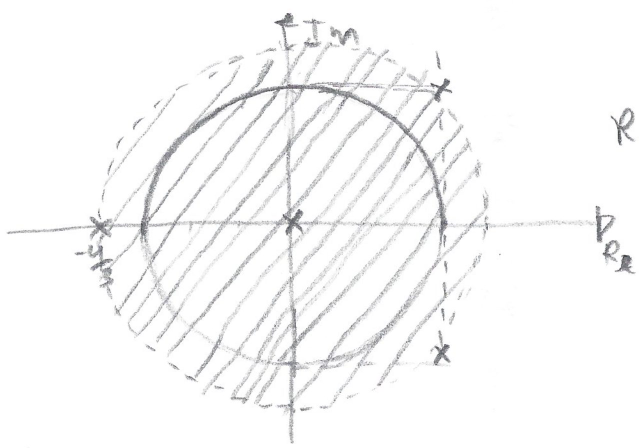
$$Y(z) = \frac{8/3}{z^{-1}(1+\frac{4}{3}z^{-1})(1-2z^{-1}+2z^{-2})}$$

$p_1 = 0, p_2 = -\frac{4}{3}, p_3 = 1+i, p_4 = 1-i$

$x[n]$ é causal, logo,

$x[-n+3]$ é lateral esquerda

RoC: $0 < |z| < 4/3$



3)

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + (2)^n u[-n-1]$$

$$y[n] = 5\left(\frac{1}{3}\right)^n u[n] - 5\left(\frac{2}{3}\right)^n u[n]$$

a) $H(z) = \dfrac{Y(z)}{X(z)}$

$$Y(z) = \frac{5}{1-\frac{1}{3}z^{-1}} - \frac{5}{1-\frac{2}{3}z^{-1}} \quad ; \quad |z| > \frac{2}{3}$$

$$x[n] = \left(\frac{1}{3}\right)^n u[n] - (-(2))^n u[-n-1]$$

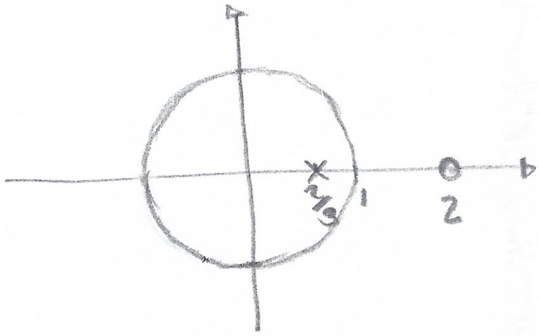
$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-2z^{-1}} \quad ; \quad \frac{1}{3} < |z| < 2$$

$$Y(z) = \frac{5\left(\left(1-\frac{2}{3}z^{-1}\right)-\left(1-\frac{1}{3}z^{-1}\right)\right)}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{2}{3}z^{-1}\right)} = \frac{-\frac{5}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{2}{3}z^{-1}\right)}$$

$$X(z) = \frac{\left(1-2z^{-1}\right)-\left(1-\frac{1}{3}z^{-1}\right)}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-2z^{-1}\right)} = \frac{-\frac{5}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-2z^{-1}\right)}$$

$$H(z) = \frac{\left(\dfrac{-\frac{5}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{2}{3}z^{-1}\right)}\right)}{\left(\dfrac{-\frac{5}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-2z^{-1}\right)}\right)} = \left(\frac{-\frac{5}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{2}{3}z^{-1}\right)}\right)\left(\frac{\left(1-\frac{1}{3}z^{-1}\right)\left(1-2z^{-1}\right)}{-\frac{5}{3}z^{-1}}\right)$$

$$H(z) = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}} \quad ; \quad |z| > \frac{2}{3}$$



b) $H(z) = \frac{1}{1-\frac{2}{3}z^{-1}} - 2z^{-1} \cdot \frac{1}{1-\frac{2}{3}z^{-1}}$, $|z| > \frac{2}{3}$

$h[n] = \left(\frac{2}{3}\right)^n u[n] - 2\left(\frac{2}{n}\right)^{n-1} u[n-1]$

c) O círculo unitário está contido na RoC, portanto o sistema é estável.

$\lim_{z \to \infty} H(z) = \lim_{z \to \infty} \frac{1-z^{-1}}{1-\frac{2}{3}z^{-1}} = 1$

Como o limite de $H(z)$ é definido para $z \to \infty$, o sistema é causal e pode ser aplicado o teorema do valor inicial.

4)

$$h[n] = \begin{cases} a^n, & n \geq 0 \\ 0, & n < 0 \end{cases} = a^n u[n]$$

$$x[n] = \begin{cases} 1, & 0 \leq n \leq (N-1) \\ 0, & c.c. \end{cases}$$

$$H(z) = \frac{1}{1 - az^{-1}} \; ; \; |z| > a$$

$$X(z) = \sum_{n=0}^{N-1} z^{-n} \; ; \; p/ \; z \neq 0$$

$$Y(z) = H(z)X(z) = \frac{1}{1 - az^{-1}} \cdot \sum_{i=0}^{N-1} z^{-i} \; ; \; |z| > a$$

$$= \sum_{i=0}^{N-1} H(z) z^{-i}$$

$$y[n] = \sum_{i=0}^{N-1} h[n-i]$$

5) $y[n] + 3y[n-1] = x[n]$

$x[n] = \left(\frac{1}{2}\right)^n u[n] \, , \; y[-1] = 1$

$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \, , |z| > \frac{1}{2}$

$\mathcal{Z}_u\{y[n] + 3y[n-1]\} = \mathcal{Z}_u\{x[n]\}$

$Y(z) + 3\left(y[-1] + z^{-1}Y(z)\right) = X(z)$

$$Y(z) = \frac{X(z) - 3}{1 + 3z^{-1}} = \frac{X(z)}{1+3z^{-1}} - \frac{3}{1+3z^{-1}}$$

$$= \underbrace{\frac{1}{1-\frac{1}{2}z^{-1}} \cdot \frac{1}{1+3z^{-1}}}_{Y'(z)} - \frac{3}{1+3z^{-1}}$$

$$Y(z) = \frac{A_1}{1-\frac{1}{2}z^{-1}} + \frac{A_2}{1+3z^{-1}} - \frac{3}{1+3z^{-1}}$$

$$A_1 = Y'(z) \cdot \left(1 - \frac{1}{2}z^{-1}\right)\Big|_{z=\frac{1}{2}} = \frac{1}{1+6} = \frac{1}{7}$$

$$A_2 = Y'(z) \cdot \left(1 + 3z^{-1}\right)\Big|_{z=-3} = \frac{1}{1+\frac{1}{6}} = \frac{6}{7}$$

$$Y(z) = \frac{1}{7} \cdot \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{6}{7} \cdot \frac{1}{1+3z^{-1}} - \frac{3}{1+3z^{-1}}$$

$$= \frac{1}{7} \cdot \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{15}{7} \cdot \frac{1}{1+3z^{-1}} \; ; |z| > 3$$

$$y[n] = \frac{1}{7}\left(\frac{1}{2}\right)^n u[n] - \frac{15}{7}(-3)^n u[n] \, , \; p/ \; n \geq 0$$

6) a) Por $y[n]$ ser "estável", $Y(z)$ deve conter a circunferência unitária e a única RoC possível que inclui é: $\frac{1}{2} < |z| < 2$.

b) Pelo formato da região de convergência de $Y(z)$, $y[n]$ é, portanto, bilateral.

c) Novamente, a estabilidade implica que $|z| = 1 \in$ RoC.
Portanto das regiões possíveis:

$|z| < \frac{1}{2}$; $\frac{1}{2} < |z| < \frac{3}{4}$; $|z| > \frac{3}{4}$

somente $|z| > \frac{3}{4}$ contém a circunferência unitária

d) Devido a RoC, sabe-se que $x[n]$ é lateral direita sendo necessário mostrar $x[n] = 0$ p/ $n < 0$, isso pode ser demostrado pelo teorema do valor inicial ou pelo formato $\mathcal{Z}^{-1}\{X(z)\}$.

$\lim_{z \to \infty} X(z)$; $X(z) = \frac{1 - \frac{1}{4}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{3}{4}z^{-1})}$

$\lim_{z \to \infty} X(z) = \frac{1}{1 \cdot 1} = 1$,

$x[n]$ é causal por o teorema de valor inicial ser definido.

e) Pelo teorema de valor inicial:

$x[0] = \lim_{z \to \infty} X(z) = 1$

6) f)

$$Y(z) = \frac{z^{-1}(1-\frac{1}{4}z^{-1})}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})} \quad ; \quad X(z) = \frac{1-\frac{1}{4}z^{-1}}{(1+\frac{3}{4}z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}(1-\frac{1}{4}z^{-1})}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})} \cdot \frac{(1+\frac{3}{4}z^{-1})(1-\frac{1}{2}z^{-1})}{1-\frac{1}{4}z^{-1}}$$

$$H(z) = \frac{z^{-1}(1+\frac{3}{4}z^{-1})}{(1-2z^{-1})} \quad ; \quad |z| < 2$$

e) $H(z) = z^{-1} \cdot \frac{1}{1-2z^{-1}} + \frac{3}{4}z^{-2}\frac{1}{1-2z^{-1}}$

$h[n] = (-2)^{n-1} u[-n-1-1] + \frac{3}{4}(-2)^{n-2} u[-n-3]$

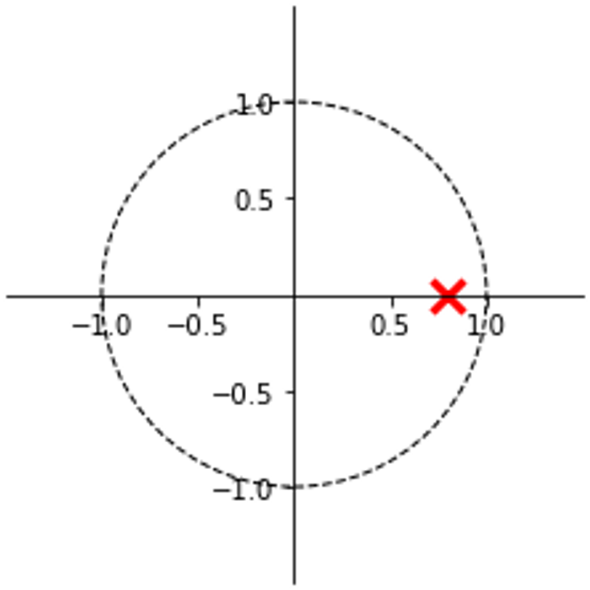
$h[n] = 0$ p/ $n > -2$, portanto $h[n]$ é anticausal.

7) a) $y[n] - 0.8\,y[n-1] = x[n]$

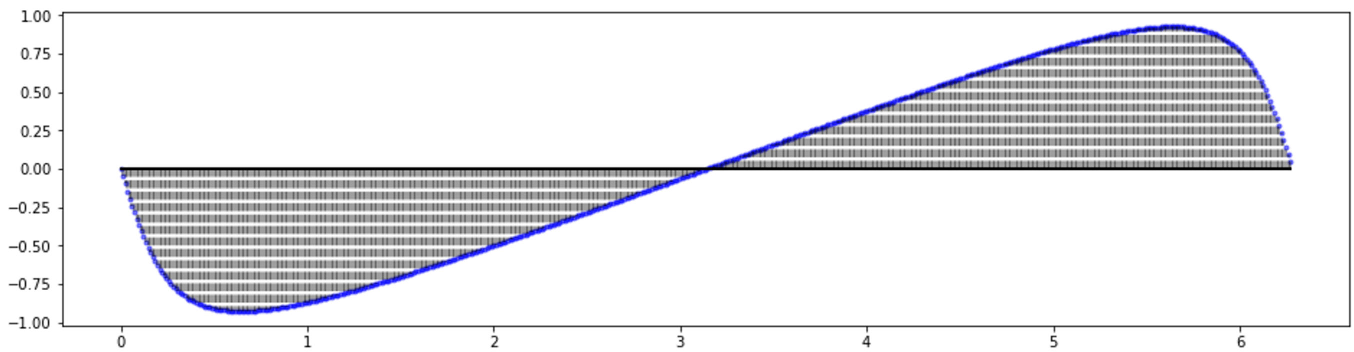
$$\mathcal{Z}\{y[n] - 0.8\,y[n-1]\} = \mathcal{Z}\{x[n]\}$$

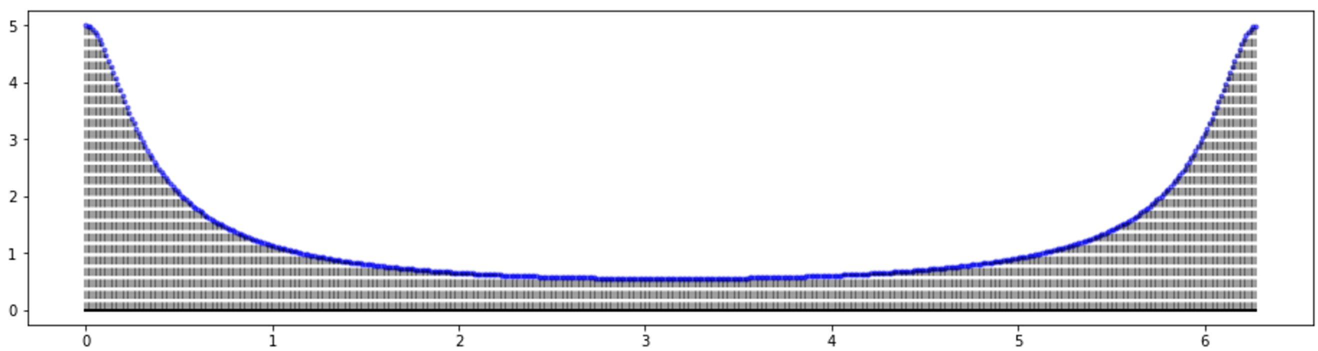
$$Y(z) - 0.8\,z^{-1}Y(z) = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{1-0.8z^{-1}} = H(z)$$



b)





9) a)

$$Y(z) = H_2(z)\left(X(z)\left(1 + H_1(z)\right)\right)$$

$$\frac{Y(z)}{X(z)} = H_2(z)(1 + H_1(z)) = H_T(z)$$

$$H_1(z) = \frac{1}{4}z^{-2} - z^{-1}$$

$$H_2(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$H_T(z) = H_2(z) + H_1(z)H_2(z)$$

$$= H_2(z) + \frac{1}{4}z^{-2}H_2(z) - z^{-1}H_2(z)$$

$$h_T[n] = h_2[n] + \left(\frac{1}{2}\right)^2 h_2[n-2] - h_2[n-1]$$

$$= \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{\cancel{2}}\left(\frac{1}{2}\right)^{n-\cancel{2}} u[n-2] - \left(\frac{1}{2}\right)^{n-1} u[n-1]$$

$$= \left(\frac{1}{2}\right)^n \left(u[n] + u[n-2] - 2u[n-1]\right)$$

$$= \left(\frac{1}{2}\right)^n \left(\delta[n] - \delta[n-1]\right)$$

$$h_T[n] = \delta[n] - \frac{1}{2}\delta[n-1]$$

b) $H_T(z) = H_2(z) + H_1(z)H_2(z)$

$$\frac{Y(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{4}z^{-2}}{1-\frac{1}{2}z^{-1}} \frac{-z^{-1}}{1-\frac{1}{2}z^{-1}} = \frac{1+\frac{1}{4}z^{-2}-z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$Y(z)(1-\tfrac{1}{2}z^{-1}) = X(z)(1+\tfrac{1}{4}z^{-2}-z^{-1})$$

$$\mathcal{Z}^{-1}\{Y(z) - \tfrac{1}{2}z^{-1}Y(z)\} = \mathcal{Z}^{-1}\{X(z) + \tfrac{1}{4}z^{-2}X(z) - z^{-1}X(z)\}$$

$$y[n] - \tfrac{1}{2}y[n-1] = x[n] - x[n-1] + \tfrac{1}{4}x[n-2]$$

c) O sistema é causal já que $h[n] = 0$ para $n < 0$ e é estável pois $h[n]$ é absolutamente somável.

d) $H_T(z) = H_2(z) + H_1(z)H_2(z)$

$$H_T(e^{j\omega}) = H_2(e^{j\omega})(1 + H_1(e^{j\omega}))$$

$$= \frac{1}{1-\frac{1}{2}e^{-j\omega}}\left(1 + \frac{1}{4}e^{-j\omega 2} - e^{-j\omega}\right)$$

e)