

# Formule za elektrodinamiku

$$\rho(\mathbf{r})=\sum_{\alpha}q_a\delta^{(3)}(\mathbf{r}-\mathbf{r}_{\alpha})\quad (1)$$

$$\boldsymbol{\nabla}\cdot\mathbf{j}+\frac{\partial\rho}{\partial t}=0\quad (2)$$

$$\begin{aligned}\mathbf{F}&=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})\\ \mathbf{F}&=\int\rho\,\mathrm{d}^3r\,(\mathbf{E}+\mathbf{v}\times\mathbf{B})\\ &=\int(\rho\mathbf{E}+\mathbf{j}\times\mathbf{B})\,\mathrm{d}^3r\end{aligned}\quad (3)$$

$$\mathbf{p}=\int_V\rho(\mathbf{r})\mathbf{r}\mathrm{d}V\quad (4)$$

$$\phi=\frac{1}{4\pi\epsilon_0}\int_V\frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}V'\quad (5)$$

$$\phi=\frac{1}{4\pi\epsilon_0}\left[\frac{Q}{r}+\frac{\mathbf{r}\cdot\mathbf{p}}{r^3}+\frac{x_iD_{ij}x_j}{2r^5}\right]\quad (6)$$

$$\mathbf{E}=\frac{1}{4\pi\epsilon_0}\left(\frac{3(\mathbf{p}\cdot\mathbf{r})\mathbf{r}-r^2\mathbf{p}}{r^5}-\frac{4\pi}{3}\mathbf{p}\delta^{(3)}(\mathbf{r})\right)\quad (7)$$

$$\mathbf{E}(\mathbf{r})=\frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r}')\,\mathrm{d}^3r'}{|\mathbf{r}-\mathbf{r}'|^3}(\mathbf{r}-\mathbf{r}')\quad (8)$$

$$\begin{aligned}D_{ij}&=\int\rho(\mathbf{r})(3x_ix_j-\delta_{ij}r^2)\mathrm{d}V\\ D_{ij}&=\sum_{\alpha}q_{\alpha}(3x_i^{(\alpha)}x_j^{(\alpha)}-\delta_{ij}(r^{(\alpha)})^2);\quad\mathrm{Tr}\,\hat{D}=0\end{aligned}\quad (9)$$

$$\mathbf{E}=-\boldsymbol{\nabla}\phi\quad (10)$$

$$\mathbf{j}=\rho\mathbf{v}\quad (11)$$

$$\mathbf{A}(\mathbf{r})=\frac{\mu_0}{4\pi}\int\frac{\mathbf{j}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}V'\quad (12)$$

$$\mathbf{A}(\mathbf{r})=\frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{r}}{r^3}\quad (13)$$

$$\mathbf{B}=\frac{\mu_0}{4\pi}\left(\frac{3(\mathbf{m}\cdot\mathbf{r})\mathbf{r}-r^2\mathbf{m}}{r^5}+\frac{8\pi}{3}\mathbf{m}\delta^{(3)}(\mathbf{r})\right)\quad (14)$$

$$\mathbf{m}(\mathbf{r})=\frac{1}{2}\int\mathbf{r}\times\mathbf{j}(\mathbf{r})\mathrm{d}V\quad (15)$$

$$\mathbf{B}(\mathbf{r})=\boldsymbol{\nabla}\times\mathbf{A}=\frac{\mu_0}{4\pi}\int\frac{\mathbf{j}(\mathbf{r}')\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}V'\quad (16)$$

$$\mathbf{E}=-\frac{\partial\mathbf{A}}{\partial t}-\boldsymbol{\nabla}\phi\quad (17)$$

$$\begin{aligned}\boldsymbol{\nabla}\cdot\mathbf{A}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}&=0\\ \nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}&=-\frac{\rho}{\epsilon_0}\end{aligned}\quad (18)$$

$$\nabla^2\mathbf{A}-\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}=-\mu_0\mathbf{j}\quad (19)$$

$$\langle\rho\rangle=\frac{1}{T}\int_0^T\rho dt\quad (19)$$

$$\begin{aligned}D_{2n}-D_{1n}&=\sigma\\ B_{2n}-B_{1n}&=0\\ P_{2n}-P_{1n}&=-\sigma_{vez}\\ E_{2n}-E_{1n}&=\frac{1}{\epsilon_0}\sigma\\ E_{2t}-E_{1t}&=0\\ \mathbf{n}\times(\mathbf{H}_2-\mathbf{H}_1)&=\mathbf{i}\\ M_{2t}-M_{1t}&=\mathbf{i}_{vez}\times\mathbf{n}\end{aligned}\quad (20)$$

$$\begin{aligned}\boldsymbol{\nabla}\cdot\mathbf{E}&=\frac{\rho}{\epsilon_0}\\ \boldsymbol{\nabla}\cdot\mathbf{B}&=0\\ \boldsymbol{\nabla}\times\mathbf{E}&=-\frac{\partial\mathbf{B}}{\partial t}\end{aligned}\quad (21)$$

$$\boldsymbol{\nabla}\times\mathbf{B}=\mu_0\left(\mathbf{j}+\epsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$$

$$\begin{aligned}\boldsymbol{\nabla}\cdot\mathbf{D}&=\rho+\rho_{ext}\\ \boldsymbol{\nabla}\cdot\mathbf{B}&=0\\ \boldsymbol{\nabla}\times\mathbf{E}&=-\frac{\partial\mathbf{B}}{\partial t}\end{aligned}\quad (22)$$

$$\boldsymbol{\nabla}\times\mathbf{H}=\mathbf{j}+\mathbf{j}_{ext}+\frac{\partial\mathbf{D}}{\partial t}$$

$$\boldsymbol{\nabla}\cdot\mathbf{P}=-\rho_{\mathrm{vez}}\quad (23)$$

$$q=UC=U\epsilon_0\frac{S}{d}\quad (24)$$

$$\frac{d}{dt}\left(\sum_n\varepsilon_n+\underbrace{\int_V\mathrm{d}^3r\left(\frac{1}{2}\mathbf{D}\cdot\mathbf{E}+\frac{1}{2}\mathbf{H}\cdot\mathbf{B}\right)}_{W_{em}}\right)$$

$$=-\oint_{\partial S}\mathbf{S}_pd\mathbf{S}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\sum_n\varepsilon_n)=\int\mathrm{d}^3r\mathbf{j}\cdot\mathbf{E}$$

$$\begin{aligned}\mathbf{S}_p&=\mathbf{E}\times\mathbf{H}\\\mathbf{g}&=\frac{\mathbf{S}_p}{c^2}\quad\text{- gustina impulsa}\end{aligned}\quad (26)$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathbf{L}_{\mathrm{meh}}+\int_V\mathrm{d}^3r\left(\mathbf{r}\times\mathbf{g}\right)\right)&=\oint_{\partial V}\left(\mathbf{r}\times\hat{T}\,\mathrm{d}\mathbf{S}\right)\\ \mathbf{L}_f&=\epsilon_0\int_V\mathrm{d}^3r\left(\mathbf{r}\times\left(\mathbf{E}\times\mathbf{B}\right)\right)\end{aligned}\quad (27)$$

$$\begin{aligned}\frac{d}{dt}\left[\sum_{\alpha}\mathbf{p}_{\alpha}+\int\mathrm{d}^3r\left(\mathbf{D}\times\mathbf{B}\right)\right]\\\hphantom{\frac{d}{dt}}-\frac{1}{2}\int\mathrm{d}^3r\left(\epsilon_0\mathbf{E}^2\boldsymbol{\nabla}\epsilon_r+\mu_0\mathbf{H}^2\boldsymbol{\nabla}\mu_r\right)=\oint_S\hat{T}d\mathbf{S}\end{aligned}\quad (28)$$

$$\hat{T}=|\mathbf{E}\rangle\langle\mathbf{D}|+|\mathbf{H}\rangle\langle\mathbf{B}|-\frac{1}{2}(\mathbf{E}\cdot\mathbf{D}+\mathbf{H}\cdot\mathbf{B})\hat{I}$$

$$V^{\mu}=\left(\frac{\frac{c}{\sqrt{1-\frac{v^2}{c^2}}}}{\frac{\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}}}\right)\quad (29)$$

$$P^{\mu}=mV^{\mu}=\left(\frac{E}{\mathbf{p}}\right)\quad (30)$$

$$\mathcal{A}^{\mu}=\frac{\mathrm{d}V^{\mu}}{\mathrm{d}\tau}=\left(\gamma\frac{c\gamma\frac{\mathrm{d}\gamma}{\mathrm{d}t}}{\gamma\frac{\mathrm{d}\gamma}{\mathrm{d}t}\mathbf{v}+\gamma^2\mathbf{a}}\right)\quad (31)$$

$$\mathcal{F}^{\mu}=\left(\frac{\frac{\mathbf{v}\cdot\mathbf{F}}{c\sqrt{1-\frac{v^2}{c^2}}}}{\frac{\mathbf{F}}{\sqrt{1-\frac{v^2}{c^2}}}}\right)\quad (32)$$

$$\begin{aligned}\mathbf{E}'_{\parallel}&=\mathbf{E}_{\parallel} & \mathbf{E}'_{\perp}&=\frac{\mathbf{E}_{\perp}+\mathbf{v}\times\mathbf{B}_{\perp}}{\sqrt{1-\frac{v^2}{c^2}}}\\ \mathbf{B}'_{\parallel}&=\mathbf{B}_{\parallel} & \mathbf{B}'_{\perp}&=\frac{\mathbf{B}_{\perp}-\frac{1}{c^2}\mathbf{v}\times\mathbf{E}_{\perp}}{\sqrt{1-\frac{v^2}{c^2}}}\end{aligned}\quad (33)$$

$$\begin{pmatrix}x'^0\\x'^1\\x'^2\\x'^3\end{pmatrix}=\underbrace{\begin{pmatrix}\gamma&-\beta\gamma&0&0\\-\beta\gamma&\gamma&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}}_{\Lambda^{\mu}_{\nu}}\begin{pmatrix}x^0\\x^1\\x^2\\x^3\end{pmatrix}\quad (34)$$

$$g_{\mu\nu}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\quad (35)$$

$$\frac{\partial}{\partial x^{\mu}}=\left(\frac{1}{c}\frac{\partial}{\partial t},\boldsymbol{\nabla}\right)\quad (36)$$

$$j^{\mu}=(c\rho,\mathbf{j}=\rho\mathbf{v})\quad (37)$$

$$A^{\mu}=(\frac{\phi}{c},\mathbf{A})\quad (38)$$

$$\square A^{\mu}=\mu_0 j^{\mu}\quad (39)$$

$$F^{\mu\nu}=\partial^{\mu}A^{\nu}-\partial^{\nu}A^{\mu}\quad (40)$$

$$F^{\mu\nu}=\begin{pmatrix}0&-\frac{E_x}{c}&-\frac{E_y}{c}&-\frac{E_z}{c}\\\frac{E_x}{c}&0&-B_z&B_y\\\frac{E_y}{c}&B_z&0&-B_x\\\frac{E_z}{c}&-B_y&B_x&0\end{pmatrix}\quad (41)$$

$$F^{'\mu\nu}=(\Lambda F\Lambda^T)^{\mu\nu}\quad (42)$$

$$\begin{aligned}\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}&=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})\\\frac{\mathrm{d}}{\mathrm{d}t}\left(\underbrace{\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}}_E\right)&=\mathbf{v}\cdot\mathbf{F}=q\mathbf{v}\cdot\mathbf{E}\end{aligned}\quad (43)$$

$$E^2=c^2p^2+m^2c^4\quad (44)$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\partial\tilde{L}}{\partial u^{\alpha}}\right)-\frac{\partial\tilde{L}}{\partial x^{\alpha}}=0\quad (45)$$

$$H=p_i\dot{q}_i-L\quad (46)$$

$$\begin{aligned}\frac{\partial u^{\mu}}{\partial u^{\alpha}}&=\delta^{\mu}_{\alpha}\\ \frac{\partial u^{\mu}}{\partial u_{\alpha}}&=g^{\mu\alpha}\end{aligned}\quad (47)$$

$$S=\int(-mc\mathrm{d}s-qA^{\mu}\mathrm{d}x_{\mu})+S_f$$

$$\mathrm{d}s=\sqrt{g_{\mu\nu}\frac{\mathrm{d}x^{\mu}}{\mathrm{d}\tau}\frac{\mathrm{d}x^{\nu}}{\mathrm{d}\tau}}\mathrm{d}\tau\quad (48)$$

$$\begin{aligned}\mathrm{d}s&=c\mathrm{d}\tau=c\sqrt{1-\frac{v^2}{c^2}}\mathrm{d}t\\ A^{\mu}\mathrm{d}x_{\mu}&=(\phi-\mathbf{A}\cdot\mathbf{v})\mathrm{d}t\end{aligned}$$

$$j_{\mathrm{vez}}^{\mu}=(c\rho_{\mathrm{vez}},\mathbf{j}_{\mathrm{vez}})=(-c\boldsymbol{\nabla}\cdot\mathbf{P},\boldsymbol{\nabla}\times\mathbf{M}+\frac{\partial\mathbf{P}}{\partial t})\quad (49)$$

$$\mathbf{P}'_{\parallel}=\mathbf{P}_{\parallel}\quad \mathbf{P}'_{\perp}=\frac{\mathbf{P}_{\perp}-\frac{1}{c^2}\mathbf{v}\times\mathbf{M}}{\sqrt{1-\frac{v^2}{c^2}}}\quad (50)$$

$$\mathbf{M}'_{\parallel}=\mathbf{M}_{\parallel}\quad \mathbf{M}'_{\perp}=\frac{\mathbf{M}_{\perp}-\mathbf{v}\times\mathbf{P}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\mathbf{D}'_{\parallel}=\mathbf{D}_{\parallel}\quad \mathbf{D}'_{\perp}=\frac{\mathbf{D}_{\perp}-\frac{1}{c^2}\mathbf{v}\times\mathbf{H}}{\sqrt{1-\frac{v^2}{c^2}}}\quad (51)$$

$$\mathbf{H}'_{\parallel}=\mathbf{H}_{\parallel}\quad \mathbf{H}'_{\perp}=\frac{\mathbf{H}_{\perp}-\mathbf{v}\times\mathbf{D}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\begin{aligned}\sin\frac{\alpha}{2} &= \pm\sqrt{\frac{1-\cos\alpha}{2}} \\ \cos\frac{\alpha}{2} &= \pm\sqrt{\frac{1+\cos\alpha}{2}} \\ \tan\frac{\alpha}{2} &= \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\end{aligned}\tag{55}$$

$$\begin{aligned}\sin(\alpha\pm\beta) &= \sin\alpha\cos\beta\pm\cos\alpha\sin\beta \\ \cos(\alpha\pm\beta) &= \cos\alpha\cos\beta\mp\sin\alpha\sin\beta \\ \tan(\alpha\pm\beta) &= \frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}\end{aligned}\tag{52}$$

$$\begin{aligned}\sin(2\alpha) &= 2\sin\alpha\cos\alpha \\ \cos(2\alpha) &= \cos^2\alpha-\sin^2\alpha \\ \tan(2\alpha) &= \frac{2\tan\alpha}{1-\tan^2\alpha} \\ \cot(2\alpha) &= \frac{\cot^2\alpha-1}{2\cot\alpha}\end{aligned}\tag{53}$$

$$\begin{aligned}\sin\alpha\pm\sin\beta &= 2\sin\frac{\alpha\pm\beta}{2}\cos\frac{\alpha\mp\beta}{2} \\ \cos\alpha+\cos\beta &= 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\ \cos\alpha-\cos\beta &= -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\ \tan\alpha\pm\tan\beta &= \frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta} \\ \cot\alpha\pm\cot\beta &= \frac{\sin(\alpha\pm\beta)}{\sin\alpha\sin\beta} \\ \sin\alpha\cos\beta &= \frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta)) \\ \cos\alpha\cos\beta &= \frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta)) \\ \sin\alpha\sin\beta &= -\frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta))\end{aligned}\tag{54}$$

$$\begin{aligned}\sin(\operatorname{acos}(x)) &= \cos(\operatorname{asin}(x)) = \sqrt{1-x^2} \\ \sin(\operatorname{atan}(x)) &= \frac{x}{\sqrt{1+x^2}} \\ \cos(\operatorname{atan}(x)) &= \frac{1}{\sqrt{1+x^2}} \\ \tan(\operatorname{asin}(x)) &= \cot(\operatorname{acos}(x)) = \frac{x}{\sqrt{1-x^2}} \\ \tan(\operatorname{acos}(x)) &= \cot(\operatorname{asin}(x)) = \frac{\sqrt{1-x^2}}{x}\end{aligned}\tag{56}$$

$$\int_0^b\sin\frac{k\pi y}{b}\sin\frac{m\pi y}{b}=\frac{b}{2}\delta_{km}\tag{57}$$

$$\phi(r,\theta)=\sum_{l=0}^{\infty}\left(A_lr^l+\frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)\tag{58}$$

$$\begin{aligned}V(\mathbf{r}) &= \int_V G(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}')\mathrm{d}V'+ \\ \epsilon_0\oint_{\partial V}\mathrm{d}S'\left(G(\mathbf{r},\mathbf{r}')\frac{\partial V(\mathbf{r}')}{\partial n'}-V(\mathbf{r}')\frac{\partial G(\mathbf{r},\mathbf{r}')}{\partial n'}\right)\end{aligned}\tag{59}$$

$$\mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P}\tag{60}$$

$$\mathbf{P}=\epsilon_0(\hat{\epsilon}-1)\mathbf{E}=\epsilon_0\hat{\chi}_e\mathbf{E}\tag{61}$$

$$(1+x)^\alpha=\sum_{n=0}^\infty\binom{\alpha}{n}x^n,\,|x|<1\tag{62}$$

$$\int_V\mathrm{d}V(\phi\nabla^2\psi-\psi\nabla^2\phi)=\oint_{\partial V}\mathrm{d}\mathbf{S}(\phi\nabla\psi-\psi\nabla\phi)\tag{63}$$

$$\Delta\ln r=2\pi\delta(r)\tag{64}$$

$$\begin{aligned}\mathbf{p} &= \beta \mathbf{E} \\ \mathbf{p} &= -e \mathbf{r}\end{aligned}\tag{65}$$

$$\omega=\frac{q}{2m}\mathbf{B}_{\mathbf{sp}}\tag{66}$$

$$W=\frac{1}{2}I_1I_2M_{1,2}=\frac{1}{2}I_1I_2\int\frac{\mathrm{d}\mathbf{l}_1\mathrm{d}\mathbf{l}_2}{|\mathbf{r}_1-\mathbf{r}_2|}\underbrace{\frac{\mu_0}{4\pi}}_{M_{1,2}}\tag{67}$$

$$W_{int}=q\phi\tag{68}$$

$$\langle X \rangle = \frac{\int X e^{-\frac{W_{int}}{kT}} \mathrm{d}V}{\int e^{-\frac{W_{int}}{kT}} \mathrm{d}V}\tag{69}$$

$$W_{\mathrm{int}}=Q\phi(0)-\mathbf{p}\cdot\mathbf{E}(0)+\ldots\tag{70}$$

$$\mathbf{F} = Q\mathbf{E}(0) + (\mathbf{p}\cdot\nabla)\mathbf{E}(0) + \ldots\tag{71}$$

$$\mathbf{M}=\mathbf{p}\times\mathbf{E}(0)+\ldots\tag{72}$$

$$W_{\mathrm{int}}=\mathbf{m}\cdot\mathbf{B}(0)+\ldots\tag{73}$$

$$\mathbf{F}=(\mathbf{m}\cdot\nabla)\mathbf{B}\Big|_0\tag{74}$$

$$I=\frac{1}{4\pi\epsilon_0}\left(|\ddot{\mathbf{p}}(\tau)|^2\frac{2}{3c^2}+\frac{2}{3c^5}|\ddot{\mathbf{m}}(\tau)|^2+\frac{1}{180c^5}|\ddot{D}(\tau)|^2\right)\tag{75}$$

$$\frac{\mathrm{d}I}{\mathrm{d}\Omega}=\frac{1}{(4\pi)^2\epsilon_0c^3}\left[\dot{\mathbf{p}}(\tau)\times\mathbf{n}+\frac{1}{c}(\dot{\mathbf{m}}(\tau)\times\mathbf{n})\times\mathbf{n}\right.\tag{76}$$

$$\left.+\frac{1}{6c}\ddot{D}(\tau)\mathbf{n}\times\mathbf{n}\right]^2\tag{77}$$

$$\underbrace{\frac{\mathrm{d}\epsilon}{\mathrm{d}t}}_{\text{izračena}}=\frac{1}{6\pi\epsilon_0c^3}\frac{e^2}{m^2}\frac{\mathbf{F}^2-\frac{1}{c^2}(\mathbf{F}\cdot\mathbf{v})^2}{1-\frac{v^2}{c^2}}\tag{78}$$

$$\hat{\epsilon}_r=\hat{I}+\frac{N}{\epsilon_0}\hat{\beta}\tag{79}$$

$$v_f=c\sqrt{\frac{\sin^2\theta}{\epsilon_{\parallel}}+\frac{\cos^2\theta}{\epsilon_{\perp}}}\tag{80}$$

$$\epsilon'(\omega)-1=\frac{1}{\pi}P\int_{-\infty}^{\infty}\mathrm{d}x\frac{\epsilon''(x)}{x-\omega}\tag{81}$$

$$\epsilon''(\omega)=-\frac{1}{\pi}P\int_{-\infty}^{\infty}\mathrm{d}x\frac{\epsilon'(x)\frac{1}{x-\omega}}$$