Assessment 1

Unit Number: ENGR6011 (Mechanics of Solids and Numerical Methods)

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Answer to the Question no 1

Given,

E = 200e9 Pa

A = 0.0015 m^2

As my student id = 21446012, ID = 105 for me.

F1 = f6 = 15+(0.2*ID) kN = -36000 N

F2 = f5 =10 - (0.02*ID) kN = 7900 N

x = 5 – (0.02*ID) m = 2.9 m

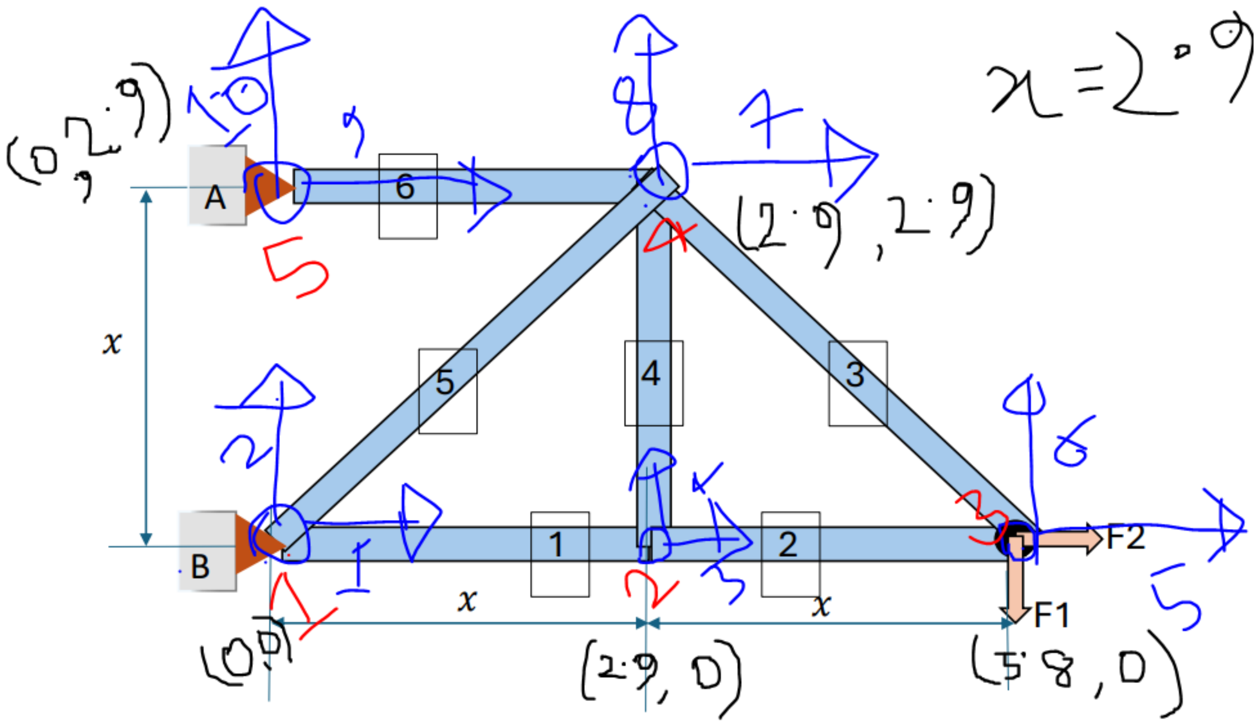

Figure: Co-ordinates, nodes, elements shown in Truss Problem 1.

We know,

$$k = (A*E/L)*\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

```
clear global; clc;
% Introducing a function "elestiff_key" to calculate stiffness matrix (4*4)
function key = elestiff_key(E, A, x)
    lxx = x(3) - x(1);
    lyy = x(4) - x(2);
    lee = sqrt(lxx^2 + lyy^2);
    C = lxx / lee;
    S = lyy / lee;
    CS = C * S;
    fa = A * E / lee;
    key = [C^2, CS, -C^2, -CS;
           CS, S^2, -CS, -S^2;
           -C^2, -CS, C^2, CS;
           -CS, -S^2, CS, S^2] * fa;
```

Figure: Introducing a function "elestiff_key" to calculate stiffness matrix (4*4)

```
% Given values
E = 200e9; A = 0.0015;
% Member 1
x1 = [0, 0, 2.9, 0];
k1 = elestiff_key(E, A, x1);
% Member 2
x2 = [2.9, 0, 5.8, 0];
k2 = elestiff_key(E, A, x2);
% Member 3
x3 = [5.8, 0, 2.9, 2.9];
k3 = elestiff_key(E, A, x3);
% Member 4
x4 = [2.9, 0, 2.9, 2.9];
k4 = elestiff_key(E, A, x4);
% Member 5
x5 = [0, 0, 2.9, 2.9];
k5 = elestiff_key(E, A, x5);
% Member 6
x6 = [0, 2.9, 2.9, 2.9];
k6 = elestiff_key(E, A, x6);
```

Figure: Process of getting the values of stiffness matrix (4*4) for each member.

For member 1,

$$k1 = \begin{bmatrix} 1.03e+08 & 0 & -1.03e+08 & 0 \\ 0 & 0 & 0 & 0 \\ -1.03e+08 & 0 & 1.03e+08 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

	1	2	3	4	
1	1.0345e+08	0	-1.0345e+08	0	
2	0	0	0	0	
3	-1.0345e+08	0	1.0345e+08	0	
4	0	0	0	0	
5					
6					

Figure: Values of k1 (output from MATLAB)

For member 2,

$$k2 = \begin{bmatrix} 1.03e+08 & 0 & -1.03e+08 & 0 \\ 0 & 0 & 0 & 0 \\ -1.03e+08 & 0 & 1.03e+08 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

	1	2	3	4	
1	1.0345e+08	0	-1.0345e+08	0	
2	0	0	0	0	
3	-1.0345e+08	0	1.0345e+08	0	
4	0	0	0	0	
5					

Figure: Values of k2 (output from MATLAB)

For member 3,

$$k3 = \begin{bmatrix} 3.66e+07 & -3.66e+07 & -3.66e+07 & 3.66e+07 \\ -3.66e+07 & 3.66e+07 & 3.66e+07 & -3.66e+07 \\ -3.66e+07 & 3.66e+07 & 3.66e+07 & -3.66e+07 \\ 3.66e+07 & -3.66e+07 & -3.66e+07 & 3.66e+07 \end{bmatrix}$$

	1	2	3	4	
1	3.6574e+07	-3.6574e+07	-3.6574e+07	3.6574e+07	
2	-3.6574e+07	3.6574e+07	3.6574e+07	-3.6574e+07	
3	-3.6574e+07	3.6574e+07	3.6574e+07	-3.6574e+07	
4	3.6574e+07	-3.6574e+07	-3.6574e+07	3.6574e+07	
5					

Figure: Values of k3 (output from MATLAB)

For member 4,

k4 = $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1.03e+08 & 0 & -1.03e+08 \\ 0 & 0 & 0 & 0 \\ 0 & -1.03e+08 & 0 & 1.03e+08 \end{bmatrix}$

0	0	0	0	
0	1.0345e+08	0	-1.0345e+08	
0	0	0	0	
0	-1.0345e+08	0	1.0345e+08	

Figure: Values of k4 (output from MATLAB)

For member 5,

k5 = $\begin{bmatrix} 3.66e+07 & 3.66e+07 & -3.66e+07 & -3.66e+07 \\ 3.66e+07 & 3.66e+07 & -3.66e+07 & -3.66e+07 \\ -3.66e+07 & -3.66e+07 & 3.66e+07 & 3.66e+07 \\ -3.66e+07 & -3.66e+07 & 3.66e+07 & 3.66e+07 \end{bmatrix}$

	1	2	3	4	
1	3.6574e+07	3.6574e+07	-3.6574e+07	-3.6574e+07	
2	3.6574e+07	3.6574e+07	-3.6574e+07	-3.6574e+07	
3	-3.6574e+07	-3.6574e+07	3.6574e+07	3.6574e+07	
4	-3.6574e+07	-3.6574e+07	3.6574e+07	3.6574e+07	
5					

Figure: Values of k5 (output from MATLAB)

For member 6,

$$k6 = \begin{bmatrix} 1.03e+08 & 0 & -1.03e+08 & 0 \\ 0 & 0 & 0 & 0 \\ -1.03e+08 & 0 & 1.03e+08 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

	1	2	3	4	
1	1.0345e+08	0	-1.0345e+08	0	
2	0	0	0	0	
3	-1.0345e+08	0	1.0345e+08	0	
4	0	0	0	0	
5					

Figure: Values of k6 (output from MATLAB)

The stiffness matrix is a 10x10 matrix due to the presence of 10 degrees of freedom. Initially, the stiffness matrix was computed for each individual member. Next, combine those elements to construct the global stiffness matrix. In my MATLAB code, the program displays the data for member 1 initially. Then, it combines a common row and column with member 3. The resulting common row and column are then added to member 3. This process continues iteratively. This is the method I used to obtain the assembled stiffness matrix.

```
% Assembly
K = zeros(10,10);
F = zeros(10,1);

K(1:4,1:4) = k1(1:4, 1:4);
K(3:6,3:6) = K(3:6,3:6) + k2(1:4, 1:4);
K(5:8, 5:8) = K(5:8, 5:8) + k3(1:4, 1:4);
K([3, 4, 7, 8],[3, 4, 7, 8]) = K([3, 4, 7, 8], [3, 4, 7, 8])+ k4(1:4, 1:4);
K([1, 2, 7, 8],[1, 2, 7, 8]) = K([1, 2, 7, 8], [1, 2, 7, 8]) + k5(1:4, 1:4);
K(7:10, 7:10) = K(7:10, 7:10)+ k6(1:4, 1:4);
```

Figure: Code to get assembled stiffness matrix (10*10).

$$K = \begin{bmatrix} 14 & 3.66 & -10.35 & 0 & 0 & 0 & -3.66 & -3.66 & 0 & 0 \\ 3.66 & 3.66 & 0 & 0 & 0 & 0 & -3.66 & -3.66 & 0 & 0 \\ -10.35 & 0 & 20.69 & 0 & -10.35 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10.35 & 0 & 0 & 0 & -10.35 & 0 & 0 \\ 0 & 0 & -10.35 & 0 & 14 & -3.66 & -3.66 & 3.66 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.66 & 3.66 & 3.66 & -3.66 & 0 & 0 \\ -3.66 & -3.66 & 0 & 0 & -3.66 & 3.66 & 17.66 & 0 & -10.35 & 0 \\ -3.66 & -3.66 & 0 & -10.35 & 3.66 & -3.66 & 0 & 17.66 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -10.35 & 0 & 10.35 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} * 10^7$$

	1	2	3	4	5	6	7	8	9	10	
1	1.4002e+08	3.6574e+07	-1.0345e+08	0	0	0	-3.6574e+07	-3.6574e+07	0	0	
2	3.6574e+07	3.6574e+07	0	0	0	0	-3.6574e+07	-3.6574e+07	0	0	
3	-1.0345e+08	0	2.0690e+08	0	-1.0345e+08	0	0	0	0	0	
4	0	0	0	1.0345e+08	0	0	0	-1.0345e+08	0	0	
5	0	0	-1.0345e+08	0	1.4002e+08	-3.6574e+07	-3.6574e+07	3.6574e+07	0	0	
6	0	0	0	0	-3.6574e+07	3.6574e+07	3.6574e+07	-3.6574e+07	0	0	
7	-3.6574e+07	-3.6574e+07	0	0	-3.6574e+07	3.6574e+07	1.7660e+08	0	-1.0345e+08	0	
8	-3.6574e+07	-3.6574e+07	0	-1.0345e+08	3.6574e+07	-3.6574e+07	0	1.7660e+08	0	0	
9	0	0	0	0	0	0	-1.0345e+08	0	1.0345e+08	0	
10	0	0	0	0	0	0	0	0	0	0	
11											
12											

Figure: Value of K (output from MATLAB)

We know,

Q= K*D

$$\begin{bmatrix} Q1 \\ Q2 \\ 0 \\ 0 \\ 7900 \\ -36000 \\ 0 \\ 0 \\ Q9 \\ Q10 \end{bmatrix} = \begin{bmatrix} 14 & 3.66 & -10.35 & 0 & 0 & 0 & -3.66 & -3.66 & 0 & 0 \\ 3.66 & 3.66 & 0 & 0 & 0 & 0 & -3.66 & -3.66 & 0 & 0 \\ -10.35 & 0 & 20.69 & 0 & -10.35 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10.35 & 0 & 0 & 0 & -10.35 & 0 & 0 \\ 0 & 0 & -10.35 & 0 & 14 & -3.66 & -3.66 & 3.66 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.66 & 3.66 & 3.66 & -3.66 & 0 & 0 \\ -3.66 & -3.66 & 0 & 0 & -3.66 & 3.66 & 17.66 & 0 & -10.35 & 0 \\ -3.66 & -3.66 & 0 & -10.35 & 3.66 & -3.66 & 0 & 17.66 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -10.35 & 0 & 10.35 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{*10^7*} \begin{matrix} 0 \\ 0 \\ D1 \\ D2 \\ D3 \\ D4 \\ D5 \\ D6 \\ 0 \\ 0 \end{matrix}$$

Now, node 1 and node 5 are fixed, so **according to boundary condition**, there is no displacement. Hence, D1 = D2 = D9 = D10 = 0.

But there is reaction force acting on node 1 and node 5.

```
%using boundary condition and making matrix small (6*6) to calculate displacements
Ksmall = K([3:8],[3:8]);
Fsmall = F([3:8]);
% Equations for getting the values of all unknown displacements
usmall = inv(Ksmall)* Fsmall;
uno = zeros(10,1);
uno([3:8]) = usmall;
```

Figure: using boundary conditions to get the values of unknown displacements

By matrix partition, Qk = K11*Du + K12*Dk

$$\begin{bmatrix} 0 \\ 0 \\ 7900 \\ -36000 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 20.69 & 0 & -10.35 & 0 & 0 & 0 \\ 0 & 10.35 & 0 & 0 & 0 & -10.35 \\ -10.35 & 0 & 14 & -3.66 & -3.66 & 3.66 \\ 0 & 0 & -3.66 & 3.66 & 3.66 & -3.66 \\ 0 & 0 & -3.66 & 3.66 & 17.66 & 0 \\ 0 & -10.35 & 3.66 & -3.66 & 0 & 17.66 \end{bmatrix} * 10\text{^}7 * \begin{bmatrix} D3 \\ D4 \\ D5 \\ D6 \\ D7 \\ D8 \end{bmatrix}$$

	1	
1	0	
2	0	
3	-0.0003	
4	-0.0017	
5	-0.0005	
6	-0.0039	
7	0.0007	
8	-0.0017	
9	0	
10	0	
11		

Figure: values of all displacements

By calculations from MATLAB we get,
D3 = -0.0003m, D4 = -0.0017m, D5 = -0.0005m, D6 = -0.0039m, D7 = 0.0007m, D8 = -0.0017m
Now,

$$\begin{bmatrix} F1 \\ F2 \\ F9 \\ F10 \end{bmatrix} = \begin{bmatrix} -10.35 & 0 & 0 & 0 & -3.66 & -3.66 \\ 0 & 0 & 0 & 0 & -3.66 & -3.66 \\ 0 & 0 & 0 & 0 & -10.35 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} * 10\text{^}7 * \begin{bmatrix} -0.0003 \\ -0.0017 \\ -0.0005 \\ -0.0039 \\ 0.0007 \\ -0.0017 \end{bmatrix}$$

```
usmall = inv(Ksmall)* Fsmall;
uno = zeros(10,1);
uno([3:8]) = usmall;
% Equations for getting the values of support reactions
Frf = K([1,2,9,10],[3:8])*usmall;
```

Figure: Equations to get the values of support reactions

	1	
1	6.4100e+04	
2	3.6000e+04	
3	-7.2000e+04	
4	0	
5		

Figure: the output values of support reactions

By calculations from MATLAB, support reaction forces are:
Q1 = 64100 N, Q2 = 36000 N, Q9 = -72000 N, Q10 = 0 **[Ans]**

qf denotes as a tension exert in the member. So that's the force for members.

We know,

qf = (AE/L)*$[-C \quad -S \quad C \quad S]$*$\begin{bmatrix} Dnx \\ Dny \\ Dfx \\ Dfy \end{bmatrix}$

```
% Introducing a function "elestiff_Q" to calculate stiffness matrix (1*4)
function Q = elestiff_Q(E, A, x)
    lxx = x(3) - x(1);
    lyy = x(4) - x(2);
    lee = sqrt(lxx^2 + lyy^2);
    C = lxx / lee;
    S = lyy / lee;
    fa = A * E / lee;
    Q = [-C, -S, C, S] * fa;
end
```

Figure: Building a function "elestiff_Q" to calculate new stiffness matrix (1*4)

```
% Member 1
x1 = [0, 0, 2.9, 0];
Q1 = elestiff_Q(E, A, x1);
% Member 2
x2 = [2.9, 0, 5.8, 0];
Q2 = elestiff_Q(E, A, x2);
% Member 3
x3 = [5.8, 0, 2.9, 2.9];
Q3 = elestiff_Q(E, A, x3);
% Member 4
x4 = [2.9, 0, 2.9, 2.9];
Q4 = elestiff_Q(E, A, x4);
% Member 5
x5 = [0, 0, 2.9, 2.9];
Q5 = elestiff_Q(E, A, x5);
% Member 6
x6 = [0, 2.9, 2.9, 2.9];
Q6 = elestiff_Q(E, A, x6);
```

Figure: calculating ((AE/L)*$[-C \quad -S \quad C \quad S]$) for each member. Then each member will be multiplied by displacements to get force acting on each member

```
% caculating force value for member 1
q1 = Q1*uno([1:4]);
% caculating force value for member 2
q2 = Q2*uno([3:6]);
% caculating force value for member 3
q3 = Q3*uno([5:8]);
% caculating force value for member 4
q4 = Q4*uno([3, 4, 7, 8]);
% caculating force value for member 5
q5 = Q5*uno([1, 2, 7, 8]);
% caculating force value for member 6
q6 = Q6*uno([7:10]);
```

Figure: equations to get the value of force for each member

For member 1,

q1 = [(0.0015*200e+09)/2.9]* $[-1 \quad 0 \quad 1 \quad 0]$*$\begin{bmatrix} 0 \\ 0 \\ -0.0003 \\ -0.0017 \end{bmatrix}$

= -2.81e+04 N

For member 2,

q2 = [(0.0015*200e+09)/2.9]* $[-1 \quad 0 \quad 1 \quad 0]$* $\begin{bmatrix}-0.0003\\-0.0017\\-0.0005\\-0.0039\end{bmatrix}$

= -2.81e+04 N

For member 3,

q3 = [(0.0015*200e+09)/2.9]* $[5.17 \quad -5.17 \quad -5.17 \quad 5.17]$* $10^7 *$ $\begin{bmatrix}-0.0005\\-0.0039\\0.0007\\-0.0017\end{bmatrix}$

= 5.09e+04 N

For member 4,

q4 = [(0.0015*200e+09)/2.9]* $[0 \quad -1 \quad 0 \quad 1]$* $\begin{bmatrix}-0.0003\\-0.0017\\0.0007\\-0.0017\end{bmatrix}$

= 0

For member 5,

q5 = [(0.0015*200e+09)/2.9]* $[-5.17 \quad -5.17 \quad 5.17 \quad 5.17]$* $10^7 *$ $\begin{bmatrix}0\\0\\0.0007\\-0.0017\end{bmatrix}$

= -5.09e+04 N

For member 6,

q6 = [(0.0015*200e+09)/2.9]* $[-1 \quad 0 \quad 1 \quad 0]$* $\begin{bmatrix}-0.0003\\-0.0017\\0\\0\end{bmatrix}$

= -7.2e+04 N

	1
1	-2.8100e+04
2	-2.8100e+04
3	5.0912e+04
4	0
5	-5.0912e+04
6	-7.2000e+04

Figure: Force values for each member

Forces for member 1, member 2, member 3, member 4, member 5 & member 6 are respectively -2.81e+04 N, -2.81e+04 N, 5.09e+04 N, 0, -5.09e+04 N, -7.2e+04 N. [Answer]

Result and discussion

Based on the diagram, there are a total of 5 nodes and 6 members. Each node has two degrees of freedom. Nodes 2, 3, and 4 are connected by pinned joints, while nodes 1 and 5 are fixed. Therefore, node 1 and node 5 do not experience any displacement since they are fixed nodes according to boundary condition. Subsequently, the value for the displacement is obtained.

D1 has a value of 0, D2 has a value of 0, D3 has a value of -0.0003m, D4 has a value of -0.0017m, D5 has a value of -0.0005m, D6 has a value of -0.0039m, D7 has a value of 0.0007m, D8 has a value of -0.0017m, D9 has a value of 0, and D10 has a value of 0.

Node 5 is experiencing two types of forces: axial force, which has a magnitude of 7900 N, and shear force, which has a magnitude of -36000 N. There are no additional forces acting on the other nodes. Therefore, the values of f3, f4, f7, and f8 are all equal to zero. Node 3 is experiencing axial and shear stresses, with f5 being 7900 N and f6 being -36000 N. Node 1 and Node 5 are immovable. Therefore, there are reactive forces at work. The support reaction forces are as follows: Q1 = 64100 N, Q2 = 36000 N, Q9 = -72000 N, Q10 = 0.

In this particular question, we are required to compute the forces acting on each individual member. Already calculated the displacements of each node, there exists a direct method to calculate the force for each member. By performing computations in MATLAB, we obtain the displacements, which in turn provide us with the values of the force members. The forces acting on member 1, member 2, member 3, member 4, member 5, and member 6 are -2.81e+04 N, -2.81e+04 N, 5.09e+04 N, 0 N, -5.09e+04 N, and -7.2e+04 N, respectively.

Answer to Question no. 2

Here, W = 40.5e+03 N/m
F2 = 39.5e+03
E = 200e+09 Pa
A = 0.012 m^2
I = 450e-09 m^4
h = 2.9 m
b = 4.05 m

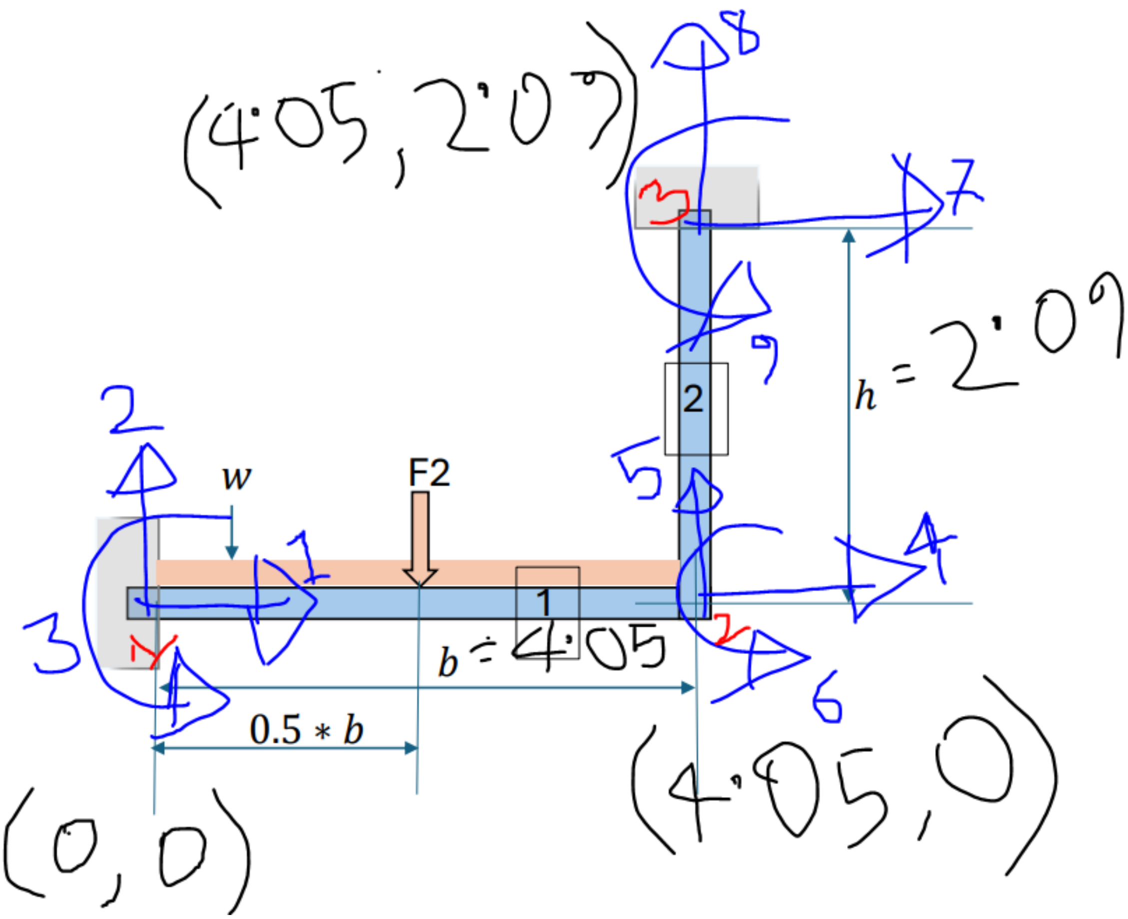

Figure: Co-ordinates, nodes, elements shown in Framed Structure.

For member 1,

$$k1 = \begin{bmatrix} 592.6 & 0 & 0 & -592.6 & 0 & 0 \\ 0 & 0.016 & 0.033 & 0 & -0.016 & 0.033 \\ 0 & 0.033 & 0.089 & 0 & -0.033 & 0.044 \\ -592.6 & 0 & 0 & 592.6 & 0 & 0 \\ 0 & -0.016 & -0.033 & 0 & 0.016 & -0.033 \\ 0 & 0.033 & 0.044 & 0 & -0.033 & 0.089 \end{bmatrix} * 10\text{^}6$$

Got this value from MATLAB. Codes and output are given below:

```
clear global; clc;
% Introducing a function "elestiff_key" to calculate stiffness matrix (6*6)
function key = elestiff_key(E, A, I, x)
    lxx = x(3) - x(1);
    lyy = x(4) - x(2);
    lee = sqrt(lxx^2 + lyy^2);
    C = lxx / lee;
    S = lyy / lee;
    CS = C * S;
    key = [(A*E/lee)*C^2,  (A*E/lee)*CS,      -(6*E*I/lee^2)*S,  -(A*E/lee)*C^2,   -(A*E/lee)*CS,       -(6*E*I/lee^2)*S;
          (A*E/lee)*CS,    (12*E*I/lee^3)*C^2, (6*E*I/lee^2)*C,  -(A*E/lee)*CS,    -(12*E*I/lee^3)*C^2, (6*E*I/lee^2)*C;
         -(6*E*I/lee^2)*S, (6*E*I/lee^2)*C,    (4*E*I/lee),       (6*E*I/lee^2)*S, -(6*E*I/lee^2)*C,    (2*E*I/lee);
         -(A*E/lee) * C^2,-(A*E/lee)*CS,       (6*E*I/lee^2)*S,   (A*E/lee)*C^2,    (A*E/lee)*CS,        (6*E*I/lee^2)*S;
         -(A*E/lee)*CS,   -(12*E*I/lee^3)*C^2, -(6*E*I/lee^2)*C,   (A*E/lee)*CS,    (12*E*I/lee^3)*C^2,  -(6*E*I/lee^2)*C;
         -(6*E*I/lee^2)*S, (6*E*I/lee^2)*C,    (2*E*I/lee),      (6*E*I/lee^2)*S, -(6*E*I/lee^2)*C,    (4*E*I/lee)];
end
```

Figure: Build a function "elestiff_key" to calculate stiffness matrix (6*6)

```
E = 200e+09; I = 450e-09; A = 0.012;
% Member 1
x1 = [0, 0, 4.05, 0];
k1 = elestiff_key(E, A, I, x1);
```

Figure: Codes to get the values of stiffness matrix (6*6) for member 1.

	1	2	3	4	5	6	
1	5.9259e+08	0	0	-5.9259e+08	0	0	
2	0	1.6258e+04	3.2922e+04	0	-1.6258e+04	3.2922e+04	
3	0	3.2922e+04	8.8889e+04	0	-3.2922e+04	4.4444e+04	
4	-5.9259e+08	0	0	5.9259e+08	0	0	
5	0	-1.6258e+04	-3.2922e+04	0	1.6258e+04	-3.2922e+04	
6	0	3.2922e+04	4.4444e+04	0	-3.2922e+04	8.8889e+04	
7							

Figure: Output from Matlab code for K1.

Similarly for member 2,

$$k2 = \begin{bmatrix} 0.044 & 0 & -0.064 & -0.044 & 0 & -0.064 \\ 0 & 827.6 & 0 & 0 & -827.6 & 0 \\ -0.064 & 0 & 0.12 & 0.064 & 0 & 0.062 \\ -0.044 & 0 & 0.064 & 0.044 & 0 & 0.064 \\ 0 & -827.6 & 0 & 0 & 827.6 & 0 \\ -0.064 & 0 & 0.062 & 0.064 & 0 & 0.12 \end{bmatrix} * 10\text{^}6$$

```
% Introducing a function "elestiff_kee" to calculate stiffness matrix (6*6)
function kee = elestiff_kee(E, A, I, x)
    lxx = x(3) - x(1);
    lyy = x(4) - x(2);
    lee = sqrt(lxx^2 + lyy^2);
    C = lxx / lee;
    S = lyy / lee;
    CS = C * S;
    kee = [(12*E*I/lee^3)*S^2,   -(12*E*I/lee^3)*CS,  -(6*E*I/lee^2)*S, -(12*E*I/lee^3)*S^2,  (12*E*I/lee^3)*CS,   -(6*E*I/lee^2)*S;
          -(12*E*I/lee^3)*CS,   (A*E/lee)*S^2,        (6*E*I/lee^2)*C,  (12*E*I/lee^3)*CS,    -(A*E/lee)*S^2,      (6*E*I/lee^2)*C;
          -(6*E*I/lee^2)*S,     (6*E*I/lee^2)*C,      (4*E*I/lee),      (6*E*I/lee^2)*S,      -(6*E*I/lee^2)*C,    (2*E*I/lee);
          -(12*E*I/lee^3)*S^2,  12*E*I/lee^3*CS,      (6*E*I/lee^2)*S,  (12*E*I/lee^3)*S^2,   -(12*E*I/lee^3)*CS,  (6*E*I/lee^2)*S;
          (12*E*I/lee^3)*CS,    -(A*E/lee)*S^2,       -(6*E*I/lee^2)*C, -(12*E*I/lee^3)*CS,   (A*E/lee)*S^2,       -(6*E*I/lee^2)*C;
          -(6*E*I/lee^2)*S,     (6*E*I/lee^2)*C,      (2*E*I/lee),      (6*E*I/lee^2)*S,      -(6*E*I/lee^2)*C,    (4*E*I/lee)];
end
```

Figure: Introducing a function "elestiff_kee" to calculate stiffness matrix (6*6)

```
% Member 2
x2 = [4.05, 0, 4.05, 2.9];
k2 = elestiff_kee(E, A, I, x2);
```

Figure: Codes to get the values of stiffness matrix (6*6) for member 2.

	1	2	3	4	5	6	
1	4.4282e+04	0	-6.4209e+04	-4.4282e+04	0	-6.4209e+04	
2	0	8.2759e+08	0	0	-8.2759e+08	0	
3	-6.4209e+04	0	1.2414e+05	6.4209e+04	0	6.2069e+04	
4	-4.4282e+04	0	6.4209e+04	4.4282e+04	0	6.4209e+04	
5	0	-8.2759e+08	0	0	8.2759e+08	0	
6	-6.4209e+04	0	6.2069e+04	6.4209e+04	0	1.2414e+05	
7							

Figure: output from MATLAB code for k2

The size of the global stiffness matrix is 9 by 9. Initially, obtain the values of k1 and k2. Proceed to sum these values based on the common row and column, resulting in the value of K.

```
% Assembly
K = zeros(9,9);
F = zeros(9,1);
K(1:6,1:6) = k1(1:6, 1:6);
K(4:9,4:9) =  K(4:9,4:9) + k2(1:6, 1:6);
```

Figure: Code to get assembled stiffness matrix (9*9).

$$K = \begin{bmatrix} 592.6 & 0 & 0 & -592.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.016 & 0.033 & 0 & -0.016 & 0.033 & 0 & 0 & 0 \\ 0 & 0.033 & 0.088 & 0 & -0.033 & 0.044 & 0 & 0 & 0 \\ -592.6 & 0 & 0 & 592.6 & 0 & -0.064 & -0.044 & 0 & -0.064 \\ 0 & -0.016 & -0.033 & 0 & 827.6 & -0.033 & 0 & -827.6 & 0 \\ 0 & 0.033 & 0.044 & -0.064 & -0.033 & 0.21 & 0.064 & 0 & 0.062 \\ 0 & 0 & 0 & -0.044 & 0 & 0.064 & 0.044 & 0 & 0.064 \\ 0 & 0 & 0 & 0 & -827.6 & 0 & 0 & 827.6 & 0 \\ 0 & 0 & 0 & -0.064 & 0 & 0.062 & 0.064 & 0 & 0.12 \end{bmatrix} * 10\text{^}6$$

	1	2	3	4	5	6	7	8	9
1	5.9259e+08	0	0	-5.9259e+08	0	0	0	0	0
2	0	1.6258e+04	3.2922e+04	0	-1.6258e+04	3.2922e+04	0	0	0
3	0	3.2922e+04	8.8889e+04	0	-3.2922e+04	4.4444e+04	0	0	0
4	-5.9259e+08	0	0	5.9264e+08	0	-6.4209e+04	-4.4282e+04	0	-6.4209e+04
5	0	-1.6258e+04	-3.2922e+04	0	8.2760e+08	-3.2922e+04	0	-8.2759e+08	0
6	0	3.2922e+04	4.4444e+04	-6.4209e+04	-3.2922e+04	2.1303e+05	6.4209e+04	0	6.2069e+04
7	0	0	0	-4.4282e+04	0	6.4209e+04	4.4282e+04	0	6.4209e+04
8	0	0	0	0	-8.2759e+08	0	0	8.2759e+08	0
9	0	0	0	-6.4209e+04	0	6.2069e+04	6.4209e+04	0	1.2414e+05
10									

Figure: Output from MATLAB code for K

Node 1 & Node 3 are fixed. **According to boundary condition,** there is no displacement due to fixed points at node 1 and node 3. Hence, D1 = D2 = D3 = D7 = D8 = D9 = 0.

But there will be reaction forces at node 1 and node 3.

We get the values of f4, f5, f6 at node 2 due to F2 & W.

So, f4 = 0, f5 = -101.762e+03 N, f6 = -75.355 N.

$$\begin{bmatrix} 0 \\ -101.76 \\ -75.355 \end{bmatrix} *10\text{^}3 = \begin{bmatrix} 592.64 & 0 & -0.064 \\ 0 & 827.6 & -0.033 \\ -0.064 & -0.033 & 0.21 \end{bmatrix} *10\text{^}6* \begin{bmatrix} D4 \\ D5 \\ D6 \end{bmatrix}$$

```
F(2) = -101.762e+03;
F(3) = 75.355e+03;
F(5) = -101.76e+03;
F(6) = -75.355e+03;
%using boundary condition and making matrix small (3*3) to calculate displacements
Ksmal = K(4:6,4:6);
Fsmall = F(4:6);
% Equations for getting the values of all unknown displacements
usmall = inv(Ksmal)* Fsmall;
```

Figure: making matrix small (3*3) to calculate displacement.

By calculating from MATLAB, get the Displacement values:

D4 = 0, D5 = -0.0001 m , D6 = -0.3538 m.

	1			
1	-0.0000			
2	-0.0001			
3	-0.3538			
4				
5				

Figure: Displacement values from MATLAB output

Now we have to calculate the reaction forces at Node 1 and Node 3.
We know, Q = KD

$$\begin{bmatrix} Q1 \\ Q2 \\ Q3 \\ Q7 \\ Q8 \\ Q9 \end{bmatrix} = \begin{bmatrix} -592.6 & 0 & 0 \\ 0 & -0.016 & 0.033 \\ 0 & -0.033 & 0.044 \\ -0.044 & 0 & 0.064 \\ 0 & -827.6 & 0 \\ -0.064 & 0 & 0.062 \end{bmatrix} *10\text{^}6* \begin{bmatrix} 0 \\ -0.0001 \\ -0.358 \end{bmatrix}$$

```
uno = zeros(9,1);
uno([4:6]) = usmall;
% Equations for getting the values of support reactions
Ksmall = K([1,2,3,7,8,9],[4:6]);
Frf = Ksmall*usmall;
```

Figure: Equations to get the values of support reactions

	1	
1	2.2713e+04	
2	-1.1644e+04	
3	-1.5718e+04	
4	-2.2713e+04	
5	1.1340e+05	
6	-2.1956e+04	

Figure: Values of Support Reaction Forces from MATLAB

Reaction Forces are:
Q1 = 22.713e+03 N, Q2 = -11.64e+03 N, Q3 = -15.718e+03 Nm, Q7 = -22.71e+03 N, Q8 = 113.40e+03 N, Q9 = -21.96e+03 Nm.

Due to superposition, Resultant forces are:
R2 = (-11.644 + 101.762) KN = 90.118 KN
R3 = (-15.718+75.355) KNm = 59.637 KNm
R4 = f4= 0
R5 = f5 = -101.762 KN
R6 = f6 = -75.355 KN
R7 = Q8 = -22.713 KN
R9 = Q9 = 21.956 KN
For member 1,
Shear forces are as follows: R2 = 90.118 KN, R5 = -101.762 KN [Answer]
Bending Moments are as follows: R3 = 59.637 KNm, R6 = -75.355 KNm [Answer]
For member 2,
Shear forces are as follows: R4 = 0, R7 = -22.713 KN [Answer]
Bending Moments are as follows: R6 = -75.355 KNm, R9 = -21.956 KNm [Answer]

Result & Discussion

There are three nodes and two members. Each node possesses three degrees of freedom and experiences three sorts of forces. The three types of forces are axial, shear, and bending moments. Based on the concept of boundary conditions, fixed locations will not experience any displacements. Since node 1 and node 3 are immovable, there will be no displacements in position. The values of D1, D2, D3, D7, D8, and D9 are all equal to zero. However, support reaction forces are present in Node 1 and Node 3.
At Node 1, f2 and f3 are exerting forces in response to the applied point load (F2) and the uniform load (W). Additionally, there will be reaction force at Node 1 due to its fixed nature. The resultant forces are as follows: R2 is equal to the difference between Q2 (the reaction force) and f2 (the force resulting from the specified load and uniform load). The same applies to R3.
At node 2, the forces f5 and f6 are caused by the point load (F2) and the uniform load (W). There are no other loads acting on this node since there is no force present in member 2.
Only reaction forces are present in node 3, as there are no additional forces acting on member 2.

Reference

Hibbeler, R. C., & Tan, K.-H. (2012). Structural analysis (8th ed. in SI Units. ed.). Singapore: Pearson Education