Rotating Steel Shaft

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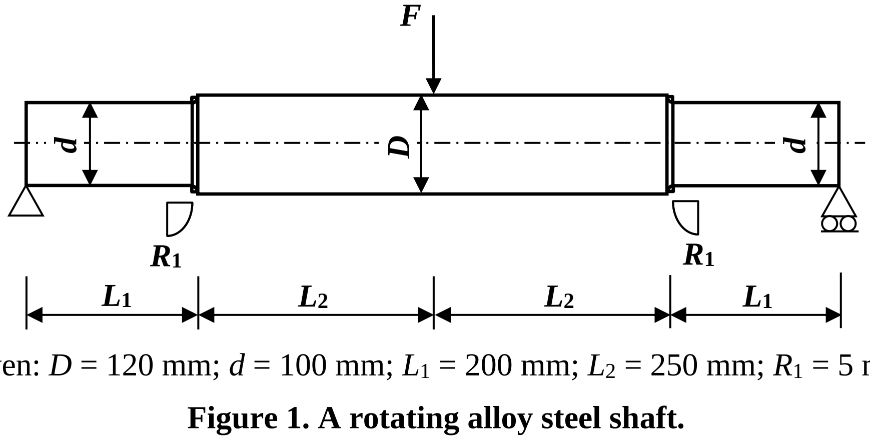
1. INTRODUCTION

This study aims to analyze the stress in a shaft subjected to a bending load in the middle span. The goal is to calculate the stress concentration and the factor of safety required to obtain an infinite fatigue life. In the first step, we need to calculate the stress concentration factor when the shaft is not transferring any power. Manual calculations must be performed using the chart table. In the second part, the stress concentration factor will be determined using the Finite Element Analysis (FEA) program ANSYS. This number will then be compared to the previous portion in order to assess the difference between manual calculations and the results produced from the FEA software. In the third phase, we will utilize the stress concentration factor obtained from the finite element analysis (FEA) program to calculate the factor of safety in relation to achieving an infinite fatigue lifetime. In the fourth phase, the FEA model will be modified by incorporating a groove in order to mitigate the stress concentration factor. The extent to which the stress concentration factor is reduced will be substantiated. In the fifth section, the factor of safety will be determined based on the new design, while maintaining the same conditions, in order to attain an infinite fatigue lifetime. In the sixth section, the shaft is continuously spun with a consistent transmission power. A bending load is applied to the center of the newly designed model, where a groove has been incorporated to decrease the stress concentration factor. The maximum permissible load will be calculated with a safety factor of 2.5 in order to achieve an unlimited fatigue life.

2. PROBLEM STATEMENT

Figure 1 illustrates the first design. A shaft with two fillets symmetrically distributed on both sides is experiencing a bending force of 100 kN at its center. The shaft was manufactured using Alloy steel and possesses a hardness of 320 BHN. The object undergoes rotation at a speed of 2000 revolutions per minute. When the shaft begins to rotate, it will experience both torsion due to torque and bending loads.



Given: D = 120 mm; d = 100 mm; L_1 = 200 mm; L_2 = 250 mm; R_1 = 5 mm.

Figure 1. A rotating alloy steel shaft.

3. RESULTS

3.1. Part (a)

Given,

D = 120 mm, d = 100 mm, r = 5 mm

Here,

D/d = 1.2, r/d = 0.05

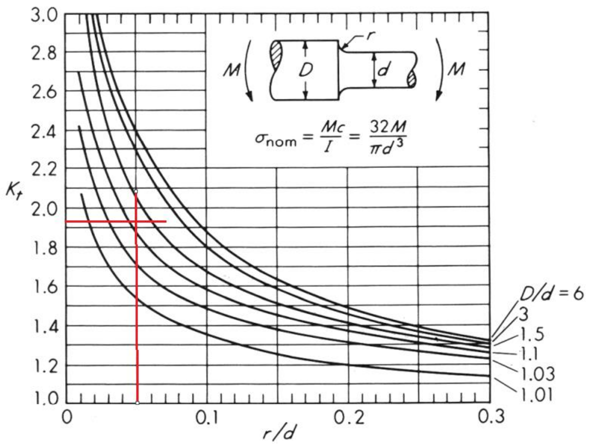


Figure 2: Determination of Stress Concentration Factor from chart table

The value of K_t is slightly higher than 1.9.

Hence, K_t **= 1.92 (Approx.) [Ans]**

Now, the value of fatigue stress concentration factor will be determined.

We know,

K_f = 1 + (K_t - 1) qC_s

To use this formula, we need to calculate q & C_s.

Here, Hardness = 320 BHN

Hardness is equal or less than 350 BHN

S_U (MPa) = 3.45*320 = 1104 MPa

C_s = 0.68 [found from figure 3 chart table]

q = 0.954 [found from figure 4 chart table]

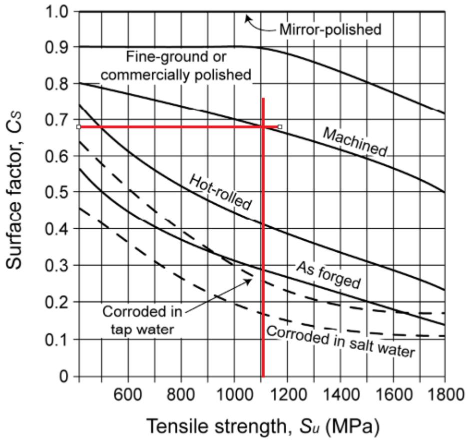


Figure 3: Determination of Surface Factor from Chart Table

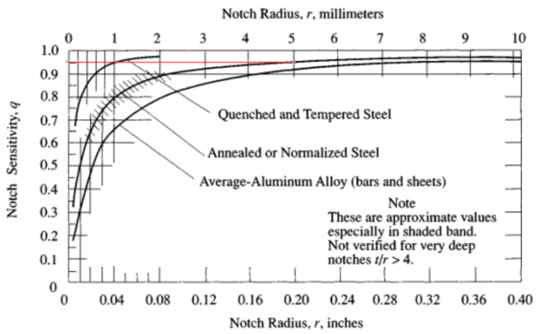


Figure 4: Determination of Notch Sensitivity from Chart Table

Now, K_f = 1 +(1.92-1) * 0.954* 0.68

Hence, K_f **= 1.597 [Ans]**

3.2. Part (b)

From FEA software ANSYS, the stress concentration factor is calculated and the value of K_t is equal to 1.9972.

So, K_t = 1.9971 [From FEA Software]

We know,

K_f = 1 + (K_t - 1) qC_s

K_f = 1 +(1.9971-1) * 0.954* 0.68 = 1.65

The difference between FEA and chart Table = (K_t value from FEA Software ANSYS – chart value of K_t) = (1.9971- 1.92) = 0.0771

Percentage = (The difference between FEA and chart Table/ chart value of K_t) * 100

= (0.0771/1.92) *100 = 4.02%

The values are nearly identical, with about a 4% discrepancy between them. There was a slight discrepancy because we manually estimated it based on the chart table. The manually calculated K_t value closely aligns with the value obtained from the FEA software. That is the reason why we are able to provide support for it.

3.3. Part (c)

In this part, Factor of safety will be determined. First Start with by calculating 10^6 (infinite) cycle stress without (fatigue limit) stress concentration, $S_n = C_L C_D C_S (0.5 S_U)$

C_S & S_U are calculated in part a & C_L = 1 [found from Figure 5]

Factor	Load type		
	Bending	Torsion	Axial
C_L	1.0	0.58	0.9*
C_D	1.0 for $D \leq 10$ mm 0.9 for $10 \leq D \leq 50$ mm		1.0
C_S	From the respective figure		

Figure 5: Determination of Load Factor

Size factor, $C_D = 1.189 * d^{-0.097}$ [when, $8 \leq d \leq\ 250$] (Milella, 2012)

$= 1.189 * (120)^{-0.097}$

$= 0.747$

Different formula is used because shaft diameter is 120 mm.

We know,

10^6 Infinite cycle stress without (fatigue limit) stress concentration,

$S_n = C_L C_D C_S (0.5 S_U)$ = 1*0.747*0.68*(0.5*1104) = 280 MPa

For 5 mm radius,

10^6 (infinite) cycle stress, $S_n = \frac{C_L C_D C_S (0.5 S_U)}{Kf} = \frac{280}{1.65}$ = 170 MPa

We know,

$\sigma_{nominal} = \frac{32M}{\pi d^3} = \frac{32(0.5 F_f L_1)}{\pi (d)^3} = \frac{32(0.5 * F_f * 200)}{\pi (100)^3} = 0.00102 F_f$

$F_f = \frac{170}{0.00102}$ = 166.897 kN

Now, checking the point of calculation,

S_n =280 MPa, $\sigma_{nominal} = \frac{32M}{\pi D^3} = \frac{32 F_f (L_1 + L_2)}{\pi D^3} = \frac{32 F_f * 450}{\pi (120)^3} = 0.0013 F_f$

$F_f = \frac{280}{0.0013}$ = 211.115 kN

The lower failure load is 166.897 kN

Factor of Safety, FOS = $\frac{166.897}{100}$ = 1.67 [Answer]

3.4. Part (d)

Table 1. Results of Stress Concentration Factor at fillet

Revision Number	Max Stress at fillet (MPa)	Nominal Stress at fillet (MPa)	K_t at fillet	% of reduction from previous value
0	203.42	101.86	1.9971	
1	193.72	101.86	1.9018	4.77%
2	191.11	101.86	1.8762	1.35%

Based on the data in table 1, revision number 3 had the greatest reduction, which was around 6.05% ($\frac{1.9971-1.8762}{1.9971}$) compared to revision number 1. In the third revision, the adjustment resulted in the lowest value of K_t at the groove, which is 1.704. In the second revision, K_t was 1.72. In revision 2 and revision 3, the groove diameter is 20 mm. However, the difference between them lies in their location. The modification in revision 2 is positioned 21.05 mm away from the fillet, whilst the modification in revision 3 is positioned 20 mm away from the fillet. The optimal value for K_t is located at the groove and is equal to 1.704.

The positions of the groove, as well as the diameters and fillets, are illustrated in figures 6 to 8.

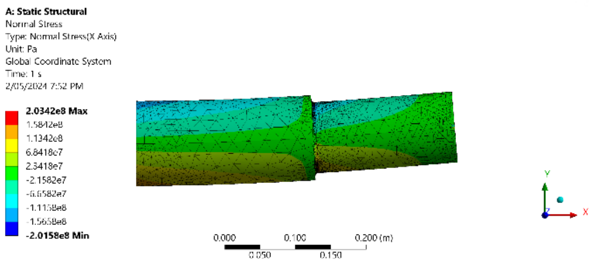


Figure 6: Maximum Normal Stress of Revision number 1. Only fillets without groove.

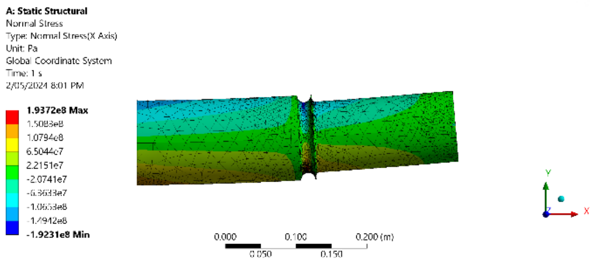


Figure 7: Maximum Normal Stress of Revision number 1. The radius of the groove is adjusted to 10 mm and positioned 21.05 mm distant from the fillet.

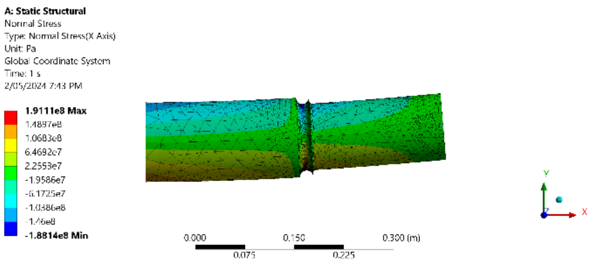


Figure 8: Maximum Normal Stress of Revision number 1. The radius of the groove, originally 10 mm, has been adjusted and positioned 20 mm away from the fillet.

3.5. **Part (e)**

For 5 mm fillet,

K_t = 1.8762

$K_f = 1 + (K_t - 1)qC_s$ = 1+ (1.8762- 1) *0.954*0.68 = 1.568

Here, q & C_s are calculated in part a. The value of K_t is taken from revision 3 modification, where we found the greatest reduction in stress concentration factor.

10^6 (infinite) cycle stress, $S_n = \frac{C_L C_D C_S (0.5 S_u)}{K_f} = \frac{280}{1.568}$ = 178.571 MPa.

10^6 Infinite cycle stress without (fatigue limit) stress concentration = 280MPa; That is measured in part c.

$\sigma_{nominal} = 178.571 = \frac{32M}{\pi(d)^3} = \frac{32(0.5F_f L_1)}{\pi(d)^3} = \frac{32*0.5*F_f*200}{\pi(100)^3}$ = 0.00101 F_f

$F_f = \frac{178.571}{0.00101}$ = 176.802 kN

For 10 mm groove,

K_t = 1.704

$K_f = 1 + (K_t - 1)qC_s$ = 1+ (1.704- 1) *0.975*0.68 = 1.467

Here, the value of C_s is calculated in part a. K_t is determined by analyzing the third revision alteration made to the groove, which resulted in the most significant decrease in stress concentration factor and q is found from figure 9 chart table.

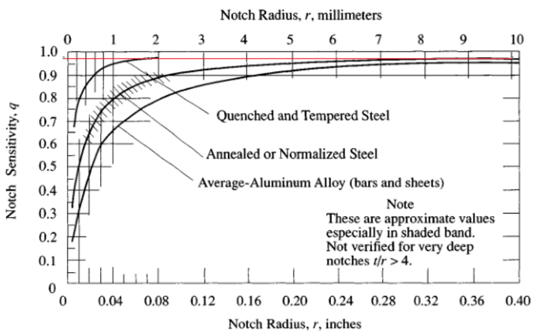


Figure 9: Determination of q for 10mm radius

10^6 (infinite) cycle stress, $S_n = \frac{C_L C_D C_S (0.5 S_u)}{K_f} = \frac{280}{1.467}$ = 190.866 MPa.

Now, $\sigma_{nominal} = 190.866 = \frac{32M}{\pi(d)^3} = \frac{32(0.5F_f L_3)}{\pi(d)^3} = \frac{32*0.5*F_f*220}{\pi(100)^3}$ = 0.0011 F_f

$F_f = \frac{190.866}{0.0011}$ = 173.514 kN

The lower failure load is 173.514 kN.

So, Factor of Safety, FOS = $\frac{173.514}{100}$ = 1.73 [Answer]

3.6. Part (f)

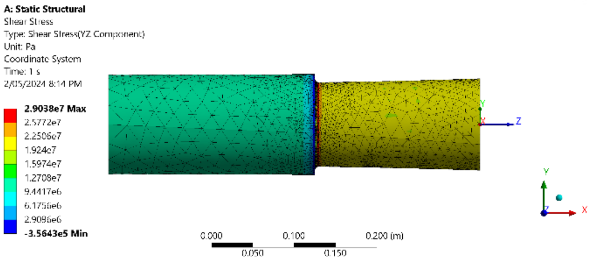


Figure 10: Maximum Shear Stress executed at fillet. Only fillet without groove.

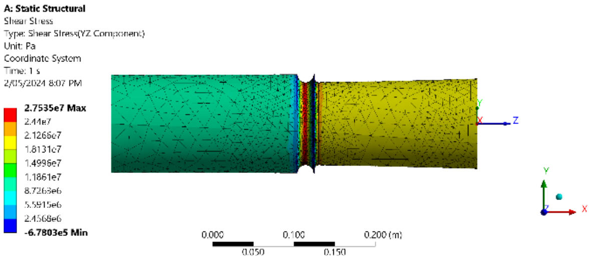


Figure 11: Maximum Shear Stress is working at the groove. The radius of the groove is adjusted to 10 mm and positioned 21.05 mm distant from the fillet.

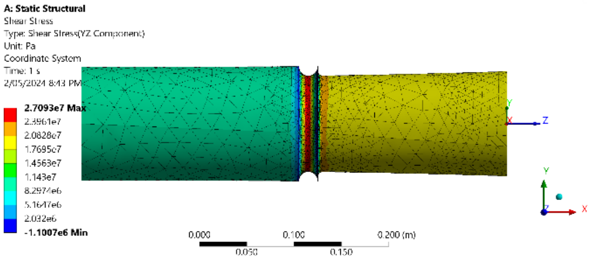


Figure 12: Maximum Shear stress of the optimized model, where the radius of the groove is adjusted to 10 mm and positioned 20 mm distant from the fillet.

Among Figures 10 to 12, the most optimized one is Figure 12 because it shows the least amount of maximum shear stress compared to others. According to figure 12, maximum shear stress is working at the groove. So, FOS will be determined for the groove. After adding the groove, we get the reduced stress concentration factor, $K_{t(t)}$ = 1.3927 [Data collected from FEA Software ANSYS].

Fatigue Stress Concentration Factor due to torsion, $K_{f(t)} = 1 + \left(K_{t(t)} - 1\right)qC_s$

= 1+ (1.3927-1) *0.975*0.68

= 1.26

We know,

Torque, $T = \frac{60P}{2\pi n} = \frac{60*800*10^3}{2\pi*2000}$ = 3819.72 Nm

Here,

Nominal Shear Stress, $\tau_{nominal} = \frac{16T}{\pi(d)^3} = \frac{480}{\pi^3 n d^3}$

So, Nominal Shear Stress, $\tau_{nominal} = \frac{16T}{\pi(d)^3} = \frac{16*3819.72}{\pi(0.1)^3}$ = 19.45 MPa

Mean Shear Stress due to torsion, $\tau_{m(t)} = \frac{480}{\pi^3 n d^3} K_{f(t)}$

$= \tau_{nominal} K_{f(t)}$

= 19.45*1.26 = 24.507 MPa

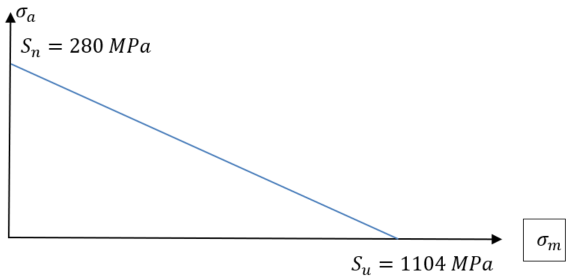


Figure 13: Construction of Goodman Curve

$S_n = 280\ MPa$ [value from part c]

$\sigma_{a(b)} = S_1 - \frac{S_n}{S_u} \tau_{m(t)}$

= 280 - [$\frac{280}{1104} * (24.507)$] = 273.78 MPa

For 10 mm groove, $K_{f(b)} = 1.467$ [Found from part e]

We know,

$\sigma_{a(b)} = 273.78 = \frac{32M}{\pi (d)^3} K_{f(b)} = \frac{32*0.5*F_f*220}{\pi (100)^3} * 1.467 = 0.0016\ F_f$

Hence, $F_f = \frac{273.78}{0.0016}$ = 183.219 kN

Factor of Safety, FOS = $\frac{F_f}{F_{(b)}}$

Hence, $F_{(b)} = \frac{F_f}{FOS} = \frac{183.219}{2.5}$ = 73.288 kN [Answer]

REFERENCES

1. Milella PP. Fatigue and Corrosion in Metals. Springer; 2013
2. Pilkey WD. Peterson's Stress Concentration Factors. 2nd ed. John Wiley & Sons, Inc.; 1997.
3. Stephens RI, Fatemi A, Stephens RR, Fuchs HO. Metal Fatigue in Engineering. 2nd ed. John Wiley; 2000
4. Juvinall RC, Marshek KM. Fundamentals of machine component design. 7th ed. John Wiley; 2019.