Data visualization

COSC 480B

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Lecture 21

Hidden Markov models

Overview

- Defining interpretive models
- Using Markov chains to model data
- Inferring hidden state using a hidden Markov model

Overview

Exercise 1

What makes a model interpretable may be slightly subjective. What’s your criteria for an interpretable model?

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ANSWER

We like to refer to mathematical proofs as the de facto explanation technique. If one were to convince another about the truth of a mathematical theorem, then a proof that irrefutably traces the steps of reasoning is sufficient.

Example of a not-so-interpretable model

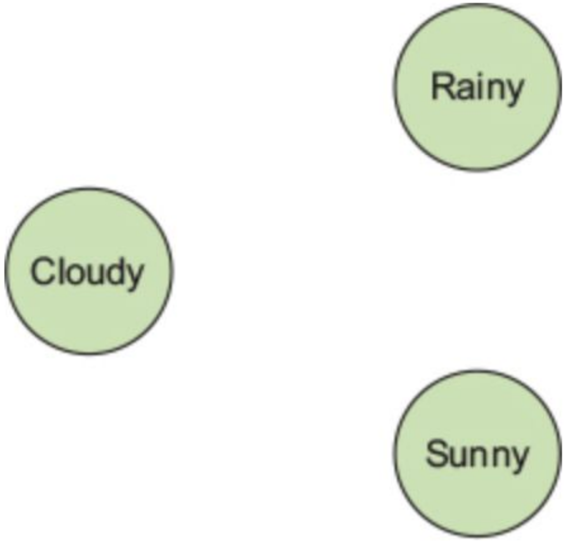
- One classic example of a black-box machine-learning algorithm that’s difficult to interpret is image classification.
- You’ll learn how to solve the problem of classifying images in next lectures
- It’s difficult to ask an image classifier why it made the decision that it did.
- Machine learning sometimes gets the notoriety of being a black-box tool that solves a specific problem without revealing how it arrives at its conclusion.
- The purpose of this chapter is to unveil an area of machine learning with an interpretable model.
- Specifically, you’ll learn about the HMM and use TensorFlow to implement it.

Markov model

- Andrey Markov was a Russian mathematician who studied the ways systems change over time in the presence of randomness.
- For example, maybe a gas particle in Europe has barely any effect on a particle in the United States. So why not ignore it?
- The mathematics is simplified when you look only at a nearby neighborhood instead of the entire system.
- This notion is now referred to as the Markov property.

Markov model

Weather conditions (states) represented as nodes in a graph



Markov model

Exercise 2

A robot that decides which action to perform based on only its current state is said to follow the Markov property. What are the advantages and disadvantages of such a decision-making process?

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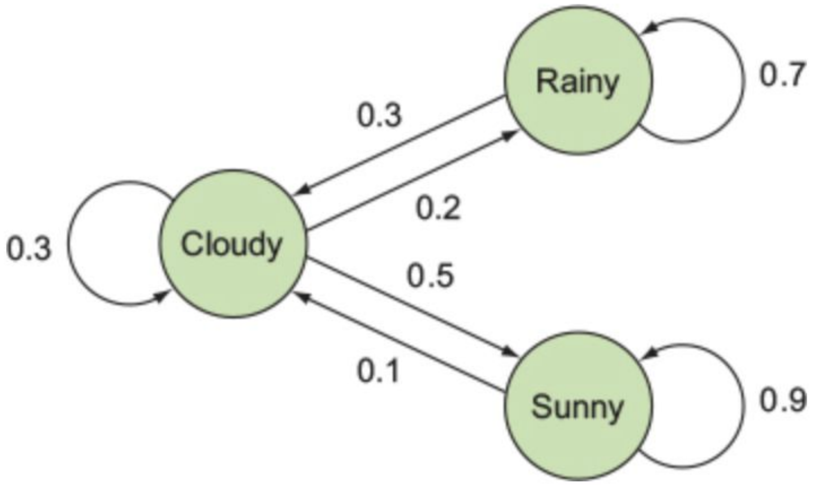
ANSWER

The Markov property is computationally easy to work with. But these models aren’t able to generalize to situations that require accumulating a history of knowledge. Examples of these are models in which a trend over time is important, or in which knowledge of more than one past state gives a better idea of what to expect next.

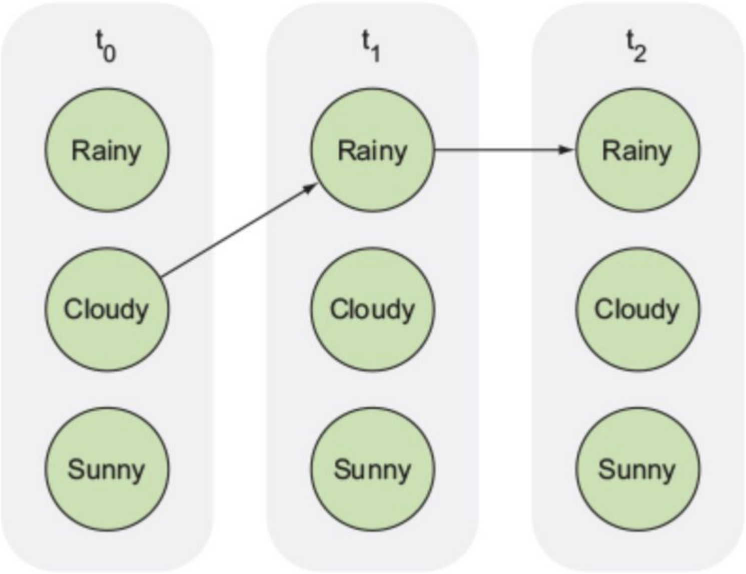
Markov model

Transition probabilities between weather conditions are represented as directed edges.



Markov model

A trellis representation of the Markov system changing states over time



Markov model

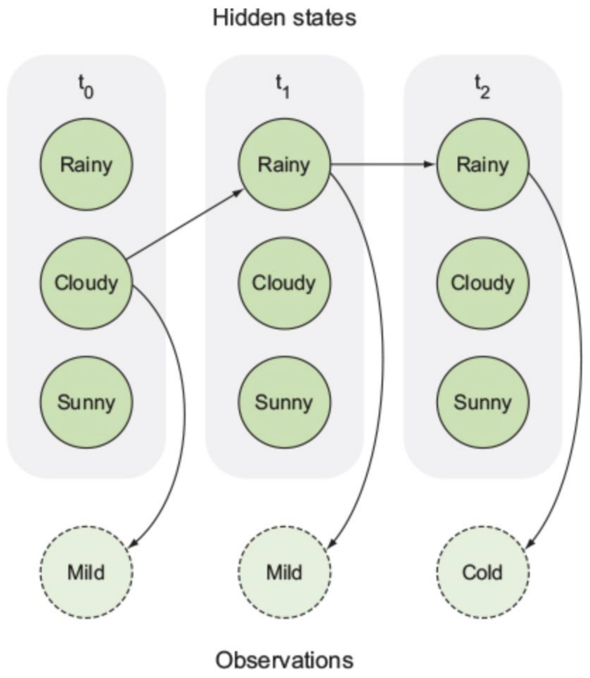
A transition matrix conveys the probabilities of a state from the left (rows) transitioning to a state at the top (columns).

	Cloudy	Rainy	Sunny
Cloudy	0.3	0.2	0.5
Rainy	0.3	0.7	0.0
Sunny	0.1	0.0	0.9

3 × 3 transition matrix

Hidden Markov model

A hidden Markov model trellis showing how weather conditions might produce temperature readings



Forward algorithm

Defining the HMM class

```
import numpy as np                                    1
import tensorflow as tf                              1

class HMM(object):
    def __init__(self, initial_prob, trans_prob, obs_prob):
        self.N = np.size(initial_prob)                   2
        self.initial_prob = initial_prob                2
        self.trans_prob = trans_prob                      2
        self.emission = tf.constant(obs_prob)             2

        assert self.initial_prob.shape == (self.N, 1)       3
        assert self.trans_prob.shape == (self.N, self.N)
3
        assert obs_prob.shape[0] == self.N                3

        self.obs_idx = tf.placeholder(tf.int32)           4
        self.fwd = tf.placeholder(tf.float64)           4
```

1 Imports the required libraries
2 Stores the parameters as method variables
3 Double-checks that the shapes of all the matrices make sense
4 Defines the placeholders used for the forward algorithm

Forward algorithm

Creating a helper function to access emission probability of an observation

```
def get_emission(self, obs_idx):
        slice_location = [0, obs_idx]                                    1
        num_rows = tf.shape(self.emission)[0]
        slice_shape = [num_rows, 1]                                      2
        return tf.slice(self.emission, slice_location, slice_shape)  3
```

1 The location of where to slice the emission matrix
2 The shape of the slice
3 Performs the slicing operation

Forward algorithm

Initializing the cache

```
def forward_init_op(self):
    obs_prob = self.get_emission(self.obs_idx)
    fwd = tf.multiply(self.initial_prob, obs_prob)
    return fwd
```

Forward algorithm

Updating the cache

```
def forward_op(self):
    transitions = tf.matmul(self.fwd,
 tf.transpose(self.get_emission(self.obs_idx)))
    weighted_transitions = transitions * self.trans_prob
    fwd = tf.reduce_sum(weighted_transitions, 0)
    return tf.reshape(fwd, tf.shape(self.fwd))
```

Forward algorithm

Defining the forward algorithm given an HMM

```
def forward_algorithm(sess, hmm, observations):
    fwd = sess.run(hmm.forward_init_op(),
feed_dict={hmm.obs_idx:
     observations[0]})
    for t in range(1, len(observations)):
        fwd = sess.run(hmm.forward_op(),
feed_dict={hmm.obs_idx:
     observations[t], hmm.fwd: fwd})
    prob = sess.run(tf.reduce_sum(fwd))
    return prob
```

Forward algorithm

Screenshot of HMM example scenario from Wikipedia

```
states = ('Rainy', 'Sunny')

observations = ('walk', 'shop', 'clean')

start_probability = {'Rainy': 0.6, 'Sunny': 0.4}

transition_probability = {
   'Rainy  : {'Rainy': 0.7, 'Sunny': 0.3},
   'Sunny' : {'Rainy': 0.4, 'Sunny': 0.6},
}

emission_probability = {
   'Rainy  : {'walk': 0.1, 'shop': 0.4, 'clean': 0.5},
   'Sunny' : {'walk': 0.6, 'shop': 0.3, 'clean': 0.1},
}
```

Forward algorithm

Defining the HMM and calling the forward algorithm

```
if __name__ == '__main__':
    initial_prob = np.array([[0.6],
                             [0.4]])

    trans_prob = np.array([[0.7, 0.3],
                           [0.4, 0.6]])

    obs_prob = np.array([[0.1, 0.4, 0.5],
                         [0.6, 0.3, 0.1]])

    hmm = HMM(initial_prob=initial_prob, trans_prob=trans_prob,
     obs_prob=obs_prob)

    observations = [0, 1, 1, 2, 1]
    with tf.Session() as sess:
        prob = forward_algorithm(sess, hmm, observations)
        print('Probability of observing {} is {}'.format(observations, prob))
```

Viterbi decoding

- Many permutations may cause a particular observation
- So enumerating all possibilities the naive way will take an exponentially long time to compute
- Instead, you can solve the problem by using dynamic programming
- The Viterbi decoding algorithm finds the most likely sequence of hidden states, given a sequence of observations.

Viterbi decoding

Adding the Viterbi cache as a member variable

```
def __init__(self, initial_prob, trans_prob,
obs_prob):
  ...
  ...
  ...
  self.viterbi = tf.placeholder(tf.float64)
```

Viterbi decoding

Defining an op to update the forward cache

```
def decode_op(self):
        transitions = tf.matmul(self.viterbi,
      tf.transpose(self.get_emission(self.obs_idx)))
        weighted_transitions = transitions * self.trans_prob
        viterbi = tf.reduce_max(weighted_transitions, 0)
        return tf.reshape(viterbi, tf.shape(self.viterbi))
```

Viterbi decoding

Defining an op to update the back pointers

```
def backpt_op(self):
    back_transitions = tf.matmul(self.viterbi, np.ones((1, self.N)))
    weighted_back_transitions = back_transitions * self.trans_prob
    return tf.argmax(weighted_back_transitions, 0)
```

Viterbi decoding

Defining the Viterbi decoding algorithm

```
def viterbi_decode(sess, hmm, observations):
    viterbi = sess.run(hmm.forward_init_op(), feed_dict={hmm.obs:
     observations[0]})
    backpts = np.ones((hmm.N, len(observations)), 'int32') * -1
    for t in range(1, len(observations)):
        viterbi, backpt = sess.run([hmm.decode_op(),
hmm.backpt_op()], feed_dict={hmm.obs: observations[t],
hmm.viterbi: viterbi})
        backpts[:, t] = backpt
    tokens = [viterbi[:, -1].argmax()]
    for i in range(len(observations) - 1, 0, -1):
        tokens.append(backpts[tokens[-1], i])
    return tokens[::-1]
```

Viterbi decoding

Running the Viterbi decode

```
seq = viterbi_decode(sess, hmm, observations)
print('Most likely hidden states are {}'.format(seq))
```

Uses of hidden Markov models

- Modeling a video
- Modeling DNA
- Modeling an image
- Sequencing words in a sentence
- Recognizing part of speech