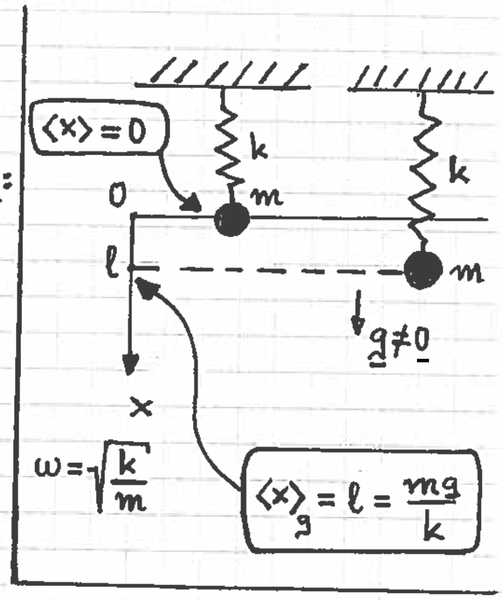
Lecture 13

Ground state as the lowest energy state of a dynamical system.

Classical Mechanics



Spring in homogeneous gravitational field:

$$L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2 + mgx \;\; ; \;\; V = \frac{1}{2} k x^2 - mgx$$

Look for $x = \text{const}$ solutions to the EoM:

$$\underbrace{\frac{d}{dt}\frac{\partial L}{\partial \dot{x}}}_{=0} - \frac{\partial L}{\partial x} = \frac{\partial V}{\partial x} = 0 \rightsquigarrow kx - mg = 0$$

$$\rightsquigarrow \langle x \rangle_g = \frac{mg}{k} \neq 0$$

Study of the dynamical system about the new equilibrium point or the new ground state $\langle x \rangle_g \neq \langle x \rangle_0 = 0$: $x = \langle x \rangle_g + y$.

This leads to an equivalent Lagrangian

$$L = \frac{1}{2} m \dot{y}^2 - \frac{1}{2} k y^2 + \frac{m^2 g^2}{2k} \;\; , \text{ with } \langle y \rangle_g = 0 \, .$$

Quantum Mechanics

Harmonic charged oscillator in $\underline{E} = \text{const}$ electric field.

1-dim model:
$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} k \hat{x}^2 + eE\hat{x} = \hbar\omega\left[\hat{a}^\dagger \hat{a} + \jmath(\hat{a} + \hat{a}^\dagger) + \frac{1}{2}\right],$$

with $\hat{p} = \frac{\hbar}{i}\frac{d}{dx}$, $\omega \hat{=} \sqrt{\frac{k}{m}}$, $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\,(\hat{a} + \hat{a}^\dagger)$, $\hat{p} = -i\sqrt{\frac{m\omega\hbar}{2}}\,(\hat{a} - \hat{a}^\dagger)$,

$\jmath = \frac{eE}{\hbar\omega}\sqrt{\frac{\hbar}{2m\omega}} \neq 0$, and $[\hat{a}, \hat{a}^\dagger] = 1$, with $\hat{a}|0\rangle_{\jmath=0} = 0$.

Define $\hat{b} = \hat{a} + \jmath$ to eliminate the terms linear in $\hat{a}, \hat{a}^\dagger$ from $\hat{H}$.

$$\hat{H} = \hbar\omega\left[\hat{b}^\dagger \hat{b} + \frac{1}{2} - \jmath^2\right].$$

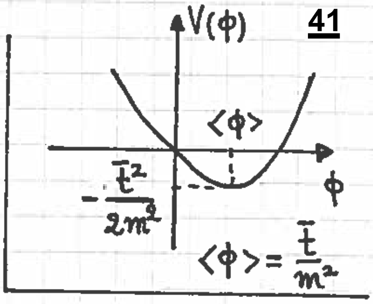
True ground state $|\Omega\rangle \hat{=} |0\rangle_{\jmath \neq 0}$, with $\hat{b}|\Omega\rangle = 0 \rightsquigarrow \hat{a}|\Omega\rangle = -\jmath|\Omega\rangle$.

Energies: $E_n = \hbar\omega\left(n + \frac{1}{2} - \jmath^2\right)$

True ground state : $|\Omega\rangle = N e^{-\jmath \hat{a}^\dagger} |0\rangle_{\jmath=0}$,
with $[\hat{b}, \hat{b}^\dagger] = 1$

$|\Omega\rangle$ is a coherent state. See WIKIPEDIA for animations!

Ground state in (Quantum) Field Theory



Simple model with a real scalar field ϕ:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 + \bar{t}\phi \quad ; \quad V(\phi) = \frac{1}{2}m^2\phi^2 - \bar{t}\phi \,.$$

Look for $\phi = \text{const}$ (t-independent and homogeneous) solutions to its EoM,

$$\partial_\mu \overbrace{\cancel{\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}}}^{0} - \frac{\partial\mathcal{L}}{\partial\phi} = \frac{\partial V}{\partial\phi} = 0 \quad \rightsquigarrow \quad \langle\phi\rangle = \frac{\bar{t}}{m^2}$$

Vacuum expectation value (VEV) of ϕ

Expand ϕ about its VEV ($\hat{=}$ its ground state),

$$\phi(x) = \langle\phi\rangle + h(x) \quad , \text{ with } \quad \langle h(x)\rangle = 0 \,.$$

Then, $\mathcal{L}$ becomes

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m^2 h^2 + \frac{\bar{t}^2}{2m^2}$$

Note that m_h does not depend on the tadpole parameter $\bar{t}$.

Mass spectrum:

$$m_h = \sqrt{|m^2|} = |m| > 0 \,,$$

one massive scalar field h.

Spontaneous Symmetry Breaking (SSB)

Consider an $SO(2)$ model, with $\underline{\Phi} = \begin{pmatrix} \Phi_1(x) \\ \Phi_2(x) \end{pmatrix}$; $\Phi_1(x), \Phi_2(x) \in \mathbb{R}$:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\underline{\Phi})^2 - V(\underline{\Phi}) \,,$$

where

$$V(\underline{\Phi}) = \frac{m^2}{2}(\Phi_1^2 + \Phi_2^2) + \frac{\lambda}{4}(\Phi_1^2 + \Phi_2^2)^2 .$$

To have a convex potential, such that $V(\underline{\Phi}) \mapsto +\infty$ as $|\underline{\Phi}| \mapsto +\infty$, one must have $\lambda > 0$.

Extrema of the potential $V(\underline{\Phi})$:

$$\left.\begin{aligned} \frac{\partial V}{\partial \Phi_1} &= \Phi_1\left[m^2 + \lambda\left(\Phi_1^2+\Phi_2^2\right)\right] = 0 \\ \frac{\partial V}{\partial \Phi_2} &= \Phi_2\left[m^2 + \lambda\left(\Phi_1^2+\Phi_2^2\right)\right] = 0 \end{aligned}\right\} :$$

Minimization or vacuum equations

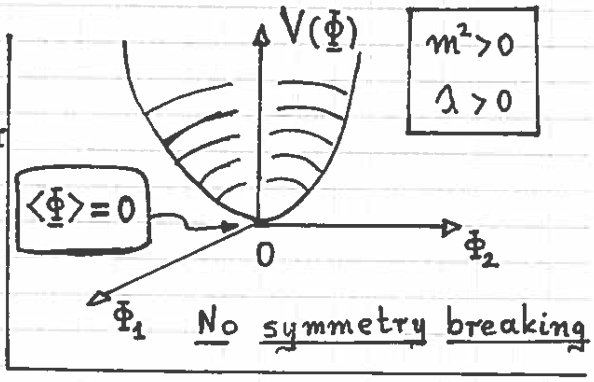
For $\lambda > 0$, there are now two distinct cases:

(i) $m^2 > 0$.

Real solution: $\Phi_1^2 + \Phi_2^2 = 0 \rightsquigarrow \langle\Phi_1\rangle = \langle\Phi_2\rangle = 0$ ← Single vacuum solution

No breaking of SO(2) symmetry.

Note $\langle\underline{\Phi}\rangle = \begin{pmatrix}\langle\Phi_1\rangle\\ \langle\Phi_2\rangle\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ is invariant under SO(2) rotations.



No symmetry breaking

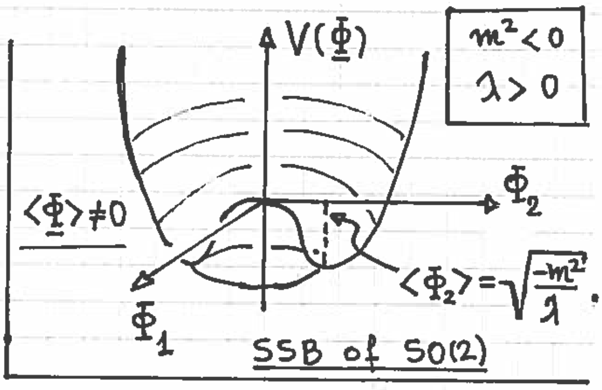
$e^{i\theta\sigma_2}\langle\underline{\Phi}\rangle = \langle\underline{\Phi}\rangle$, since $\sigma_2\langle\underline{\Phi}\rangle = \underline{0}$,

where $e^{i\theta\sigma_2} \in SO(2)$ and σ_2 is the SO(2) generator.

(ii) $m^2 < 0$.

Infinitely many vacuum solutions:

$$\langle\Phi_1\rangle^2 + \langle\Phi_2\rangle^2 = v^2 = -\frac{m^2}{\lambda} > 0$$



SSB of SO(2)

Spontaneous breaking of SO(2) by the ground state: $e^{i\theta\sigma_2}\langle\underline{\Phi}\rangle = \langle\underline{\Phi}'\rangle \neq \underline{0}$, with $\langle\underline{\Phi}\rangle \neq \langle\underline{\Phi}'\rangle$, since $\sigma_2\langle\underline{\Phi}\rangle \neq \underline{0}$. All vacuum solutions are degenerate in energy, and they form a vacuum manifold $\mathcal{M}$ in $\underline{\Phi}$-space homeomorphic to S^1:

$$SO(2) \xmapsto{\langle\underline{\Phi}\rangle \neq \underline{0}} \mathrm{I} \quad \rightsquigarrow \mathcal{M} = SO(2)/\mathrm{I} \cong SO(2) \sim S^1 .$$

Lecture 14

Physical mass spectra of the global SO(2) model:

(i) $m^2 > 0$. Two scalar fields, Φ_1 and $\Phi_2 \in \mathbb{R}$, with equal masses: $M_{\Phi_1} = M_{\Phi_2} = m$.

(ii) $m^2 < 0$.

To determine the spectrum, we first pick one point from $\mathcal{M} \sim S^1$, e.g. $\langle \Phi_1 \rangle = 0$ and $\langle \Phi_2 \rangle = v = \sqrt{-\frac{m^2}{\lambda}}$.

Then, we expand linearly about $\langle \Phi \rangle$:

$$\Phi_1(x) = \langle \Phi_1 \rangle + \pi(x) = \pi(x) \quad , \quad \Phi_2(x) = \langle \Phi_2 \rangle + \sigma(x) = v + \sigma(x) \, ,$$

where $\pi(x), \sigma(x) \in \mathbb{R}$ are the physical fields.

In terms of $\pi(x)$ and $\sigma(x)$, the Lagrangian $\mathcal{L}$ reads

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \underline{\Phi})^2 - \frac{1}{2} m^2 \underline{\Phi}\cdot\underline{\Phi} - \frac{\lambda}{4}(\underline{\Phi}\cdot\underline{\Phi})^2 \qquad ; \quad m^2 = -\lambda v^2$$

$$= \frac{1}{2}\left[(\partial_\mu \pi)^2 + (\partial_\mu \sigma)^2\right] + \frac{1}{2}\lambda v^2 \left[\pi^2 + (v+\sigma)^2\right] - \frac{\lambda}{4}\left[\pi^2 + (v+\sigma)^2\right]^2$$

$$= \frac{1}{2}\left[(\partial_\mu \pi)^2 + (\partial_\mu \sigma)^2\right] + \sigma \overbrace{\left(\frac{\lambda}{2} 2v^3 - \frac{\lambda}{4} 4 v^3\right)}^{=0} + \pi^2 \overbrace{\left(\frac{\lambda v^2}{2} - \frac{\lambda}{4} 2v^2\right)}^{=0}$$

$$+ \sigma^2 \underbrace{\left(\frac{\lambda v^2}{2} - \frac{\lambda}{4} 6v^2\right)}_{= -\lambda v^2} - \lambda v \sigma (\pi^2 + \sigma^2) - \frac{\lambda}{4}(\pi^2+\sigma^2)^2$$

$$\rightsquigarrow \quad \mathcal{L} = \frac{1}{2}\left[(\partial_\mu \pi)^2 + (\partial_\mu \sigma)^2\right] - \lambda v^2 \sigma^2 - \lambda v \sigma (\pi^2 + \sigma^2) - \frac{\lambda}{4}(\pi^2+\sigma^2)^2$$

Mass spectrum: One massless real scalar $\pi(x)$: $m_\pi = 0$;

One massive real scalar $\sigma(x)$: $m_\sigma = \sqrt{2\lambda}\, v = \sqrt{2}\, |m| > 0$.

The field $\pi(x)$ is called the Goldstone boson associated with the SO(2) breaking.

The Goldstone theorem

If a theory described by a Lagrangian $\mathcal{L}$ possesses a global symmetry group G which breaks spontaneously to a smaller symmetry group $H \subset G$, then there exists one massless Goldstone boson for each broken generator of G.

Note that theorem only holds for continuous global symmetries in theories with more than 1+1 dimensions

Proof:

Consider a theory with n real scalars $\underline{\Phi} = (\Phi_1, \Phi_2, \ldots, \Phi_n)$.
Note that a complex field φ can always decomposed into two real scalar fields, e.g. $\varphi = \frac{h + ia}{\sqrt{2}}$, with $h, a \in \mathbb{R}$.
The Lagrangian of the theory with n real scalars reads

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \underline{\Phi})^2 - V(\underline{\Phi})$$

Now, $\mathcal{L}$ is invariant under the symmetry group G, which acts on Φ_i as follows:

$$\Phi_i \longmapsto \Phi_i' = \Phi_i + i\theta^a T^a_{ij} \Phi_j \; ; \qquad T^a_{ij} \equiv T^{a\,j}_{\;i}\text{, e.g. for } G = SO(N).$$

Since $V(\underline{\Phi}) = V(\underline{\Phi}')$, we have

$$\delta V = V(\underline{\Phi}) - V(\underline{\Phi}') = 0 \quad \rightsquigarrow \quad \frac{\partial V}{\partial \Phi_i} \left(-i\theta^a T^a_{ij}\right) \Phi_j = 0 \quad , \; \forall \, \theta^a \in \mathbb{R}$$

$$\text{or} \qquad \underline{\frac{\partial V}{\partial \Phi_i} T^a_{ij} \Phi_j = 0} \qquad \text{(A)}$$

We now expand $\underline{\Phi}$ about <u>one</u> <u>vacuum solution</u>, say $\langle\underline{\Phi}\rangle$, from the set of <u>all vacuum solutions</u>, $\mathcal{M}$, called the <u>vacuum manifold</u>

$$\underline{\Phi} = \underline{\phi} + \underline{v} \iff \Phi_i = \varphi_i + v_i \ ,$$

with $\langle\underline{\Phi}\rangle = \underline{v} = (v_1, v_2, \ldots, v_n)$.

The <u>kinetic part</u> of $\mathcal{L}$ remains <u>invariant</u>: $\frac{1}{2}(\partial_\mu\underline{\Phi})^2 = \frac{1}{2}(\partial_\mu\underline{\varphi})^2$.
Instead, $V(\underline{\Phi})$ can be rewritten as

$$V(\underline{\Phi}) = V(\underline{v}) + \underline{\varphi}\cdot\underline{\nabla}_{\underline{\Phi}}V(\underline{\Phi})\Big|_{\underline{\Phi}=\underline{v}} + \frac{1}{2}\varphi_i\varphi_j \frac{\partial^2 V}{\partial\Phi_i\partial\Phi_j}\Big|_{\underline{\Phi}=\underline{v}} + \ldots$$

<u>But</u>, $\underline{\nabla}_{\underline{\Phi}} V(\underline{\Phi})\Big|_{\underline{\Phi}=\underline{v}} = \underline{0}$ ⟵ <u>vacuum equations</u> for $\underline{\Phi} = \underline{v}$,

<u>and</u>

$$\frac{\partial^2 V}{\partial\Phi_i\partial\Phi_j}\Big|_{\underline{\Phi}=\underline{v}} = M^2_{ij}$$ ⟵ <u>Mass matrix</u> (squared) for the <u>physical scalar</u> <u>fields</u> φ_i.

Differentiate <u>(A)</u> w.r.t. Φ_k and then setting $\underline{\Phi} = \underline{v}$ gives

$$\underbrace{\frac{\partial^2 V}{\partial\Phi_k\partial\Phi_i}}_{=M^2_{ki}} T^a_{ij}\,\Phi_j\Big|_{\underline{\Phi}=\underline{v}} + \underbrace{\frac{\partial V}{\partial\Phi_i}}_{=0,\ \text{due to vacuum eqs.}} T^a_{ij}\,\delta_{jk}\Big|_{\underline{\Phi}=\underline{v}} = 0$$

$$\rightsquigarrow \underline{M^2_{ki}\,T^a_{ij}\,v_j = 0} \qquad \underline{(B)}$$

Equation (B) implies two sets of generators $T^a = (X^b, Y^c)$ of the group G:

(i) The broken generators X^b of G, for which $X^b \underline{v} \neq \underline{0}$, with $\{X^b\} = (T^1, T^2, \ldots, T^\nu)$ and $\nu \leq n_G$.

(ii) The unbroken generators Y^c of G, for which $Y^c \underline{v} = \underline{0}$, with $\{Y^c\} = (T^{\nu+1}, \ldots, T^{n_G})$.

The $\{Y^c\}$ generators produce a little group or a subgroup H of G, i.e. $H \subseteq G$. Indeed, given that

$$[Y^a, Y^b] = i f^{abc} Y^c + i f'^{abc} X^c$$

in general, and $[Y^a, Y^b]\underline{v} = \underline{0}$, one must have

$$i\left(f^{abc} Y^c + f'^{abc} X^c\right)\underline{v} \overset{!}{=} \underline{0} \rightsquigarrow f'^{abc} X^c \underline{v} = 0 \rightsquigarrow \underline{f'^{abc} = 0},$$

since $X^c \underline{v} \neq \underline{0}$. Hence, $[Y^a, Y^b] = i f^{abc} Y^c$, $\forall\, Y^a \in H$, where f^{abc} are the structure constants of H.

The non-null eigenvectors derived from (B) are obtained from (B) correspond to the massless Goldstone bosons

$$G^b(x) = \frac{(i X^b \underline{v})_j}{\| X^b \underline{v} \|} \varphi_j \quad , \text{ with } b = 1, 2, \ldots, \nu .$$

Consequently, there are number ν of Goldstone bosons, each associated with a broken generator X^b. q.e.d.

SUMMARY from the lecture notes.

Lecture 15

The Higgs mechanism in an Abelian U(1) Model.

Consider a gauged U(1) model with a complex scalar field Φ. Its Lagrangian is given by

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi),$$

where $D_\mu \Phi = (\partial_\mu + \frac{i}{2} e A_\mu)\Phi$, A_μ is the U(1) gauge field and Φ has U(1) hyper-charge $Y(\Phi) = \frac{1}{2}$. Finally, the scalar potential is given by

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \; ; \text{ with } \underline{\mu^2 > 0} \text{ and } \underline{\lambda > 0}.$$

Expand linearly Φ about its physical vacuum $\langle \Phi \rangle = \frac{v}{\sqrt{2}}$ as

$$\Phi = \frac{1}{\sqrt{2}} (v + H + iG),$$

where $v^2 = \frac{\mu^2}{\lambda}$, derived from $\frac{\partial V}{\partial \Phi^\dagger} = \Phi(-\mu^2 + 2\lambda \Phi^\dagger \Phi) = 0$, i.e. from the vacuum equation.

and $G(x)$ is a massless would-be Goldstone boson (in the absence of A_μ) and $H(x)$ is another real massive scalar field 'orthogonal' to $G(x)$.

An equivalent non-linear expansion of Φ is

$$\Phi = \frac{1}{\sqrt{2}} (v + H') e^{iG'/v} = \frac{1}{\sqrt{2}} \Big[v + \underbrace{v\left(\cos\tfrac{G'}{v} - 1\right) + H' \cos\tfrac{G'}{v}}_{\hat{=} H} + i \underbrace{\sin\tfrac{G'}{v}(v + H')}_{\hat{=} G} \Big]$$

Unitary gauge. Under U(1), Φ transforms as

$$U(1):\; \Phi \mapsto \Phi' = e^{i\frac{1}{2}\theta} \frac{(v + H')}{\sqrt{2}} e^{iG'/v} = \frac{v + H}{\sqrt{2}}, \text{ for } \theta(x) = -2\frac{G'(x)}{v}.$$

This specific gauge transf. for which G' (and G) gets eliminated, i.e. $G' \mapsto 0$, is called the unitary gauge.

The mass of A_μ in the unitary gauge

$$(D_\mu\phi)^\dagger(D^\mu\phi) = \left[\left(\partial_\mu - \frac{i}{2}eA_\mu\right)\left(\frac{v+H}{\sqrt{2}}\right)\right]\left[\left(\partial^\mu + \frac{i}{2}eA_\mu\right)\left(\frac{v+H}{\sqrt{2}}\right)\right]$$

symbol for subset of terms

$$\supset +\frac{1}{4}A_\mu A^\mu\left(e^2\frac{v^2}{2}\right) = \frac{1}{2}\left(\frac{e^2v^2}{4}\right)A_\mu A^\mu \hat{=} \frac{1}{2}M_A^2 A_\mu A^\mu$$

The gauge field A_μ now becomes massive, with mass $M_A = \frac{ev}{2}$. Hence, the would-be Goldstone boson has been absorbed by the longitudinal polarization of the massive A_μ in the unitary gauge. This mass generation for A_μ is called the Higgs-Englert-Brout mechanism, or in short the Higgs mechanism.

The Higgs mechanism predicts a massive scalar boson, the Higgs boson, with mass $M_H = \sqrt{2\lambda}\,v$.

Gauge independence of the mass of A_μ

Let us consider the R_ξ gauge (sometimes called the Fermi gauge):

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu - \xi\frac{ev}{2}G\right)^2$$

Our aim is to compute the gauge-boson propagator in the R_ξ gauge. After observing the absence of mixing terms, like $(\partial_\mu A^\mu)G$ in the R_ξ gauge (see Ex IV.2 (iii), for example), The Lagrangian containing terms quadratic in A_μ is given by

$$\mathcal{L}_{A_\mu A_\nu} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_A^2 A_\mu A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \quad ; \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

This is similar to QED with the addition of a mass term for the gauge boson $\propto M_A^2 = \frac{e^2v^2}{4}$.

Hence, the E-L eqn of motion for A_ν is given by

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} - \frac{\partial \mathcal{L}}{\partial A_\nu} = 0 \rightsquigarrow \left[-\eta_{\mu\nu}\left(\partial_\kappa \partial^\kappa + M_A^2\right) + \left(1-\frac{1}{\xi}\right)\partial_\mu \partial_\nu\right] A^\nu = 0$$

The gauge-field propagator $\Delta^{(\xi)}_{\mu\nu}(x-y)$ is the Green's function of the above linear differential operator, i.e.

$$\left[-\eta^{\mu\nu}\left(\Box + M_A^2\right) + \left(1-\frac{1}{\xi}\right)\partial^\mu \partial^\nu\right] \Delta^{(\xi)}_{\nu\lambda}(x-y) = -\delta^\mu_{\ \lambda}\, \delta^{(4)}(x-y) .$$

Following the same approach as in Lecture 9, we find

$$\tilde{\Delta}^{(\xi)}_{\mu\nu}(k) = \left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{M_A^2}\right)\frac{i}{k^2 - M_A^2 + i\varepsilon} - \frac{k_\mu k_\nu}{M_A^2}\,\frac{i}{k^2 - \xi M_A^2 + i\varepsilon}$$

$$= \left[-\eta_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2 - \xi M_A^2 + i\varepsilon}\right]\frac{i}{k^2 - M_A^2 + i\varepsilon}$$

Observe that there are two poles at $k^2 = M_A^2$ independent of ξ and a second one at $k^2 = \xi M_A^2$ that depends on ξ. The first one is physical related to the mass of A_μ generated by the Higgs mechanism. The second one is unphysical and cancels against similar poles that occur in the would-be G-propagator when computing physical observables, such as S-matrix elements.

As a final remark, we note that the G-propagator is given by

$$\tilde{\Delta}^{(\xi)}_G(k) = \frac{i}{k^2 - \xi M_A^2 + i\varepsilon} \qquad \text{(Why?)} \qquad \text{in the } R_\xi \text{ gauge} .$$

Observe that the unitary gauge corresponds to $\xi \mapsto +\infty$ in the R_ξ gauge, since $\tilde{\Delta}^{(\infty)}_G(k) \mapsto 0$, i.e. the would-be Goldstone boson decouples from the theory.

Lecture 16

Glashow '61, Salam & Weinberg '67

The Higgs Mechanism in the Standard Model (SM)

The SM is based on SSB pattern

For direct product of groups, see Lecture 8.

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \xmapsto{\langle\Phi\rangle} SU(3)_c \otimes U(1)_{em} \,,$$

where Φ is a scalar doublet in the fundamental rep of $SU(2)_L$.

subscript L stands for the weak isospin of Left-handed fermions

The SM Higgs potential is given by

$$V(\Phi) = -\mu^2\,\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \,,$$

and for $\mu^2>0$ ($\lambda>0$), the ground state or VEV of Φ is

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \quad \text{derived from the vacuum eqn: } \frac{\partial V}{\partial \Phi^\dagger} = 0 .$$

(see also Lecture 15)

with $v = \sqrt{\dfrac{\mu^2}{\lambda}} \simeq 245\ \text{GeV}$.

The doublet Φ is singlet under $SU(3)_c$ (:it has no colour), but it has hypercharge $y_\Phi = \frac{1}{2}$ under the hypercharge group $U(1)_Y$.

The field Φ may be expanded about its VEV $\langle\Phi\rangle$ as follows:

$$\Phi = \underbrace{\begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+H+iG)\end{pmatrix}}_{\text{linear expansion}} = \underbrace{e^{i\frac{G'^i}{v}\frac{\sigma^i}{2}}}_{\text{non-linear expansion}} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+H')\end{pmatrix}; \quad i=1,2,3.$$

In the unitary gauge, Φ takes on the form

$$\Phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+H)\end{pmatrix},$$

obtained by a gauge transf. $\Phi \mapsto \Phi' = e^{i\theta^i\frac{\sigma^i}{2}}\Phi$, with $\theta^i(x) = -\dfrac{G'^i(x)}{v}$.

Given that the generators of $SU(2)_L$ are $T^i = \frac{\sigma^i}{2}$ and of $U(1)_Y$: $Y = y_\Phi \mathbb{1}_2 = \frac{1}{2}\mathbb{1}_2$, it is not difficult to see that the linear combination

$$Q = T^3 + Y = \frac{\sigma^3}{2} + \frac{1}{2}\mathbb{1}_2 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

is <u>not</u> broken by $\langle \Phi \rangle$: $\quad Q\langle\Phi\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \underline{0}$.

The generator Q is identified with the generator of $U(1)_{em}$, since the <u>electroweak vacuum</u> $\langle\Phi\rangle$ has to be <u>electrically neutral</u>.

$$\therefore \quad SU(2)_L \otimes U(1)_Y \xmapsto{\langle\Phi\rangle} U(1)_{em} .$$

<u>Gauge bosons in the SM</u> (we ignore the gluons resulting from $SU(3)_c$)

$$\mathcal{L}_{YM} = -\frac{1}{4} W^i_{\mu\nu} W^{i,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} , \longleftarrow \text{gauge kinetic terms.}$$

where $W^i_{\mu\nu} \hat{=} \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\,\varepsilon^{ijk} W^j_\mu W^k_\nu$ is the $SU(2)_L$ field strength tensor and $B_{\mu\nu} \hat{=} \partial_\mu B_\nu - \partial_\nu B_\mu$ is the $U(1)_Y$ field strength tensor.

The <u>scalar-kinetic part</u> of the SM Lagrangian is

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) ,$$

where $D_\mu \Phi = \left(\mathbb{1}_2 \partial_\mu + i g \frac{\sigma^i}{2} W^i_\mu + i \frac{g'}{2} B_\mu \mathbb{1}_2\right)\Phi$.

From the first term of $\mathcal{L}_\Phi$, we may evaluate the masses of the electroweak gauge bosons. In detail, we have

$$(D_\mu\Phi)^\dagger (D^\mu \Phi) \supset \langle\Phi^\dagger\rangle\left[-ig\frac{\sigma^i}{2} W^i_\mu - i\frac{g'}{2}\mathbb{1}_2 B_\mu\right]\left[ig\frac{\sigma^j}{2} W^{j,\mu} + i\frac{g'}{2}\mathbb{1}_2 B^\mu\right]\langle\Phi\rangle$$

We first consider the mass terms from the non-diagonal generators $\frac{\sigma^{1,2}}{2}$, i.e.

$$\langle \Phi^{+} \rangle \, \frac{g^2}{4} \left(\sigma^1 W^1_\mu + \sigma^2 W^2_\mu \right) \left(\sigma^1 W^{1,\mu} + \sigma^2 W^{2,\mu} \right) \langle \Phi \rangle$$

$$= \frac{g^2}{4} \left(0, \frac{v}{\sqrt{2}}\right) \underbrace{\begin{pmatrix} 0 & W^1_\mu - i W^2_\mu \\ W^1_\mu + i W^2_\mu & 0 \end{pmatrix} \begin{pmatrix} 0 & W^{1,\mu} - i W^{2,\mu} \\ W^{1,\mu} + i W^{2,\mu} & 0 \end{pmatrix}}_{= (W^1_\mu - i W^2_\mu)(W^{1,\mu} + i W^{2,\mu}) \mathbb{1}_2} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \frac{g^2 v^2}{4} W^+_\mu W^{-}$$

where $W^\pm_\mu = \frac{1}{\sqrt{2}} \left(W^1_\mu \mp i W^2_\mu \right)$ are the charged $W^\pm$ bosons, with mass $M_W = \frac{gv}{2} \simeq 80$ GeV.

Next, we consider the mass terms resulting from the diagonal generators $\frac{\sigma^3}{2}$ and $\frac{1}{2}\mathbb{1}_2$:

$$\left(0, \frac{v}{\sqrt{2}}\right) \left[\frac{g}{2} \sigma^3 W^3_\mu + \frac{g'}{2} \mathbb{1}_2 B_\mu \right] \left[\frac{g}{2} \sigma^3 W^{3,\mu} + \frac{g'}{2} \mathbb{1}_2 B^\mu \right] \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{g^2 + g'^2}{8} (0, v) \begin{bmatrix} \frac{g W^3_\mu + g' B_\mu}{\sqrt{g^2 + g'^2}} & 0 \\ 0 & \frac{-g W^3_\mu + g' B_\mu}{\sqrt{g^2 + g'^2}} \end{bmatrix}^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{g^2 + g'^2}{8} v^2 Z_\mu Z^\mu \;\hat{=}\; \frac{1}{2} M_Z^2 Z_\mu Z^\mu ,$$

where $Z_\mu \;\hat{=}\; \frac{g W^3_\mu - g' B_\mu}{\sqrt{g^2 + g'^2}}$ is the Z boson, with mass $M_Z = \frac{\sqrt{g^2 + g'^2}}{2} v \simeq 91$ GeV

whilst $A_\mu \;\hat{=}\; \frac{1}{\sqrt{g^2 + g'^2}} \left(g' W^3_\mu + g B_\mu \right) \perp Z_\mu$ is the photon, which is consistently predicted to be massless.

Alternatively, the Z_μ and A_μ gauge may be written down as

$$Z_\mu = c_W W^3_\mu - s_W B_\mu \,, \quad A_\mu = s_W W^3_\mu + c_W B_\mu \,,$$

$\boxed{\text{unification of EM and weak forces.}}$

with $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, and $t_W = \frac{s_W}{c_W} = \frac{g'}{g}$, $e = \frac{g g'}{\sqrt{g^2 + g'^2}} = g \sin\theta_W$, and θ_W is the weak mixing angle introduced by Glashow in 1961.

Lecture 17

Fermions in the SM

$$L_{iL} = \begin{pmatrix} \nu_{iL} \\ l_{iL} \end{pmatrix} \quad , \quad l_{iR} \quad (\underline{\text{possibly}}\ \nu_{iR}) \quad ; \quad i = e, \mu, \tau = 1, 2, 3$$

generations or families

$$Q^{\alpha}_{iL} = \begin{pmatrix} u^{\alpha}_{iL} \\ d^{\alpha}_{iL} \end{pmatrix} \quad , \quad u^{\alpha}_{iR} \quad , \quad d^{\alpha}_{iR} \quad ; \quad i = 1, 2, 3 \quad , \quad (u_1, u_2, u_3) = (u, c, t)$$

$$(d_1, d_2, d_3) = (d, s, b)$$

<u>colour</u>: $\alpha = r, g, b = 1, 2, 3$.

All fermions are chiral <u>Weyl fermions</u>, e.g. ξ_α or $\bar{\eta}^{\dot{\alpha}}$.

from $\psi = \begin{pmatrix} \xi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$.

Only L_{iL} and Q_{iL} carry weak isospin under the $SU(2)_L$ <u>fundamental rep.</u>

The <u>hypercharge</u> <u>quantum numbers</u> for the SM fermions are

$$y_{L_L} = -1 \quad , \quad y_{Q_L} = \frac{1}{3} \quad , \quad y_{\nu_R} = 0 \quad , \quad y_{l_R} = -2 \quad , \quad y_{u_R} = \frac{4}{3} \quad , \quad y_{d_R} = -\frac{2}{3} \, .$$

The $U(1)_Y$ quantum numbers are independent of <u>colour</u> $\alpha = r, g, b$ and <u>flavour</u> $i = 1, 2, 3$ (lepton or quark).

Gauge-kinetic Lagrangian for a SM fermion f:

$$\mathcal{L}_f = \bar{f}_L i \gamma^\mu D^L_\mu f_L + \bar{f}_R i \gamma^\mu D^R_\mu f_R \quad ,$$

where

$$D^L_\mu f_L = \left(\partial_\mu + i g_s \frac{\lambda^a}{2} G^a_\mu + i g \frac{\sigma^i}{2} W^i_\mu + i g' \frac{y_{f_L}}{2} B_\mu \right) f_L$$

$$D^R_\mu f_R = \left(\partial_\mu + i g_s \frac{\lambda^a}{2} G^a_\mu + i g' \frac{y_{f_R}}{2} B_\mu \right) f_R \quad .$$

The term $g_s \frac{\lambda^a}{2} G^a_\mu$ is <u>only</u> present for <u>quarks</u>, i.e. Q^α_L, u^α_R and d^α_R. Likewise, $g \frac{\sigma^i}{2} W^i_\mu$ is <u>only</u> present for <u>left-handed doublets</u>, i.e. L_L and Q^α_L.

EM interactions to SM fermions

We know that

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \rightsquigarrow \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} .$$

From $\mathcal{L}_f$, we have

$$\left.\begin{matrix} \bar{f}_L i \not{D}_L f_L \\ + \\ \bar{f}_R i \not{D}_R f_R \end{matrix}\right\} \supset \bar{f}_L \gamma^\mu \left(-g \frac{\sigma^3}{2} W^3_\mu - g' \frac{y_{f_L}}{2} B_\mu \mathbb{1}_2 \right) f_L + \bar{f}_R \gamma^\mu \left(-g' \frac{y_{f_R}}{2} B_\mu \right) f_R$$

$$\supset -A_\mu \bar{f}_L \gamma^\mu \left(\underbrace{g s_W}_{=e} \frac{\sigma^3}{2} + \underbrace{g' c_W}_{=e} \frac{y_{f_L}}{2} \right) f_L - A_\mu \bar{f}_R \gamma^\mu \underbrace{g' c_W}_{=e} \frac{y_{f_R}}{2} f_R$$

$$\mathcal{L}_{A_\mu \bar{f} f} = -e A_\mu \left(\bar{f}_L \gamma^\mu Q_{f_L} f_L + \bar{f}_R \gamma^\mu Q_{f_R} f_R \right)$$

$$\therefore \underline{\mathcal{L}_{A_\mu \bar{f} f} = -e A_\mu \bar{f} \gamma^\mu Q_f f} \quad ; \quad \begin{array}{|l} Q_{f_L} = \frac{\sigma^3}{2} + \frac{y_{f_L}}{2} = T^3_f + Y_f \\ Q_{f_R} = \frac{y_{f_R}}{2} \ \underline{\text{and}} \ Q_f = Q_{f_L} = Q_{f_R} . \end{array}$$

with $Q_{\ell_{L,R}} = -1$, $Q_{\nu_{L,R}} = 0$, $Q_{u_{L,R}} = \frac{2}{3}$, $Q_{d_{L,R}} = -\frac{1}{3}$.

Z-boson interactions to SM fermions

$$\mathcal{L}_f \supset Z_\mu \bar{f}_L \gamma^\mu \Big(-g c_W T^3_f + \underbrace{g' s_W}_{= g \frac{s_W^2}{c_W}} \underbrace{\frac{y_{f_L}}{2}}_{= Q_{f_L} - T^3_f} \Big) f_L + Z_\mu \bar{f}_R \gamma^\mu \underbrace{g' s_W}_{= g \frac{s_W^2}{c_W}} \underbrace{\frac{y_{f_R}}{2}}_{= Q_{f_R}} f_R$$

$$= -\frac{g}{c_W} Z_\mu \left[\bar{f}_L \gamma^\mu \left(T^3_f - s_W^2 Q_{f_L} \right) f_L + \bar{f}_R \gamma^\mu \left(-s_W^2 Q_{f_R} \right) f_R \right]$$

$$\therefore \underline{\mathcal{L}_{Z_\mu \bar{f} f} = -\frac{g}{c_W} Z_\mu \bar{f} \gamma^\mu \left(T^3_f P_L - s_W^2 Q_f \right) f}$$

where $P_L = \frac{\mathbb{1} - \gamma_5}{2}$ and $P_R = \frac{\mathbb{1} + \gamma_5}{2}$ are the <u>chirality projection</u> operators acting on a <u>Dirac fermion</u> ψ:

$$\psi_{L(R)} = P_{L(R)} \psi \quad , \text{ with } \mathbb{1} \hat{=} \mathbb{1}_4 \text{ and } \gamma_5 = \begin{pmatrix} -\mathbb{1}_2 & 0_2 \\ 0_2 & \mathbb{1}_2 \end{pmatrix} .$$

Lecture 18

Yukawa interactions

Other possible gauge-invariant and renormalizable interactions are given by the Yukawa Lagrangian

$$-\mathcal{L}_Y = \bar{Q}_{iL} Y^d_{ij} \Phi d_{jR} + \bar{Q}_{iL} Y^u_{ij} \tilde{\Phi} u_{jR} + \bar{L}_{iL} Y^l_{ij} \Phi l_{jR} + \bar{L}_{iL} Y^\nu_{ij} \tilde{\Phi} \nu_{jR} + \text{H.c.},$$

with $\tilde{\Phi} \hat{=} i\sigma^2 \Phi^*$, and $\underline{Y}^{u,d,l,\nu}$ are 3×3 Yukawa-(coupling) matrices

Gauge invariance of $\bar{Q}_L \Phi d_R$:

$$Y(Q_L) = \frac{y_{Q_L}}{2} = \frac{1}{6} = -Y(\bar{Q}_L) \quad , \quad Y(\Phi) = y_\Phi = \frac{1}{2} \quad , \quad Y(d_R) = \frac{y_{d_R}}{2} = -\frac{1}{3}$$

$$U(1)_Y : \quad \bar{Q}_L \Phi d_R \mapsto \bar{Q}'_L \Phi' d'_R = e^{i(\underbrace{Y(\bar{Q}_L) + Y(\Phi) + Y(d_R)}_{= -\frac{1}{6} + \frac{1}{2} - \frac{1}{3} = 0})\theta} \bar{Q}_L \Phi d_R$$

$$SU(2)_L : \quad Q_L \mapsto Q'_L = e^{i\theta^a T^a} Q_L = U Q_L \quad , \text{ with } U \in SU(2)_L$$

$$\bar{Q}_L \mapsto \bar{Q}'_L = \bar{Q}_L U^\dagger$$

$$\Phi \mapsto \Phi' = U\Phi$$

$$d_R \mapsto d'_R = d_R$$

$$\bar{Q}_L \Phi d_R \mapsto \bar{Q}'_L \Phi' d'_R = \bar{Q}_L \underbrace{U^\dagger U}_{= \mathbf{1}_2} \Phi d_R = \bar{Q}_L \Phi d_R$$

By analogy, we can prove the gauge invariance of the other terms in $-\mathcal{L}_Y$. We only need to know that

$$Y(\tilde{\Phi}) = -Y(\Phi) = -\frac{1}{2} \quad , \text{ and } \quad SU(2)_L : \tilde{\Phi} \mapsto \tilde{\Phi}' = U\tilde{\Phi},$$

since $\quad U i\sigma_2 U^T = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \varepsilon_{ij}$ (Why?),

i.e. $U \in Sp(2) \cong SU(2)$

2-dim. symplectic group

After SSB, $-\mathcal{L}_Y$ generates 3x3 mass matrices that describe the masses and the mixing between the 3 families of quarks and leptons. In detail, we have, e.g. for the quarks,

$$-\langle \mathcal{L}_Y^q \rangle = \bar{d}_L \underline{Y}^d \frac{v}{\sqrt{2}} d_R + \bar{u}_L \underline{Y}^u \frac{v}{\sqrt{2}} u_R + \text{H.c.},$$

where we used $\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ and $\langle \tilde{\Phi} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}$.

Defining $\underline{M}^u \hat{=} \frac{v}{\sqrt{2}} \underline{Y}^u$ and $\underline{M}^d \hat{=} \frac{v}{\sqrt{2}} \underline{Y}^d$, we may write $-\langle \mathcal{L}_Y^q \rangle$ as

$$-\langle \mathcal{L}_Y^q \rangle = \bar{d}_L \underline{M}^d d_R + \bar{u}_L \underline{M}^u u_R + \text{h.c.}$$

The matrices $\underline{M}^{u,d}$ are 3x3 non-Hermitian matrices and can always be diagonalized, with non-negative diagonal entries, by bi-unitary transformations:

$$U^d \underline{M}^d V^d = \underline{\hat{M}}^d \quad , \quad U^u \underline{M}^u V^u = \underline{\hat{M}}^u \quad , \text{ with } U^{u,d}, V^{u,d} \in U(3)$$

$\hat{M}^u$ and $\hat{M}^d$ are diagonal and contain the physical masses of the up-type and down-type quarks, respectively. Employing these matrix relations, we find

$$-\langle \mathcal{L}_Y \rangle = \bar{\hat{d}}_L \underline{\hat{M}}^d \hat{d}_R + \bar{\hat{u}}_L \underline{\hat{M}}^u \hat{u}_R + \text{H.c.} = \bar{\hat{d}}\, \underline{\hat{M}}^d \hat{d} + \bar{\hat{u}}\, \underline{\hat{M}}^u \hat{u},$$

where $\hat{u}_{L,R}$ and $\hat{d}_{L,R}$ are the mass eigenstates related to flavour states $u_{L,R}$ and $d_{L,R}$ as follows:

$$\left.\begin{array}{ll} \hat{d}_L = U^d d_L , & \hat{u}_L = U^u u_L \\ \hat{d}_R = V^{d\dagger} d_R , & \hat{u}_R = V^{u\dagger} u_R \end{array}\right\} \Longleftrightarrow \left\{\begin{array}{ll} d_L = U^{d\dagger} \hat{d}_L , & u_L = U^{u\dagger} \hat{u}_L \\ d_R = V^d \hat{d}_R , & u_R = V^u \hat{u}_R \end{array}\right.$$

$W^\pm$-boson interactions to quarks

$$\mathcal{L}_f \supset \bar{Q}_L \gamma^\mu \left(-g\frac{\sigma^1}{2} W^1_\mu - g\frac{\sigma^2}{2} W^2_\mu \right) Q_L = -\frac{g}{\sqrt{2}}\, \bar{Q}_L \gamma^\mu \begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix} Q_L$$

$$= -\frac{g}{\sqrt{2}} (\bar{u}_L, \bar{d}_L)\, \gamma^\mu \begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix} = -\frac{g}{\sqrt{2}} W^+_\mu\, \bar{u}_{iL} \gamma^\mu d_{iL} + \text{H.c.},$$

with $W^\pm_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \mp i W^2_\mu)$.

$\boxed{\text{flavour index } i=1,2,3 \text{ restored}}$

Going from flavour states u_{iL} and d_{iL} to mass eigenstates $\hat{u}_{iL}$ and $\hat{d}_{iL}$, we get

$$\mathcal{L}_{Wud} = -\frac{g}{\sqrt{2}} W^+_\mu\, \hat{\bar{u}}_L\, U^u \gamma^\mu U^{d\dagger} \hat{d}_L + \text{H.c.} = -\frac{g}{\sqrt{2}} W^+_\mu\, \hat{\bar{u}}_i (U^u U^{d\dagger})_{ij}\, \gamma^\mu P_L \hat{d}_j + \text{H.c.}$$

$$\therefore\quad \mathcal{L}_{W^\pm ud} = -\frac{g}{\sqrt{2}} W^+_\mu\, \hat{\bar{u}}_i\, \underline{V}_{ij}\, \gamma^\mu P_L \hat{d}_j + \text{H.c.}$$

where $\underline{V} = U^u U^{d\dagger}$ is the so-called Cabbibo-Kobayashi-Maskawa (CKM) matrix describing quark mixing.

Anologous phenomena of mixing of states occur in the neutrino sector, and the respective 3×3 unitary matrix is called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

SM Feynman Rules

A complete list of Feynman rules in the R_ξ gauge is given in the textbook by S. Pokorski on "Gauge Field Theories," Appendix C.

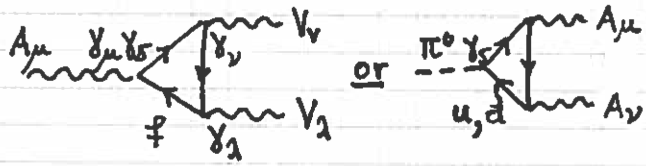
The list includes Feynman rules for gauge and Higgs self-interactions, as well as interactions mediated by the would-be Goldstone bosons $G^{\pm,0}$ and by Fadeev-Popov ghosts.

Further Reading

- The concept of BRS symmetries can be extended to SSB theories, ensuring unitarity and renormalizability of the SM.

- The only unitary non-Abelian theories with massive gauge bosons, such as $W^\pm$ and Z bosons, at high energies $\sqrt{s} \gg M_W, M_Z$, are SSB theories, as shown by Cornwall, Levin & Tiktopoulos '73.

- Chiral fermions may break the underlying gauge symmetries of the theory beyond the tree level, through the so-called chiral anomalies. A typical anomaly graph looks like



as first theoretically observed by Steinberger in 1949 in $\pi^0 \mapsto \gamma\gamma$, and later properly understood by Adler, Bell & Jackiw in 1969 and by Fujikawa '80, in the context of path integrals. All local chiral anomalies vanish in the SM.

- Global chiral anomalies do exist in the SM, giving rise to the θ-term, strong instantons and sphalerons (: topological solutions to EoMs), first studied by 't Hooft '76

- In addition to Dirac fermions and Dirac masses, the possible existence of right-handed neutrinos allows one to introduce Majorana fermions and Majorana masses in the SM Lagrangian. $\boxed{\text{See Ex IV.3 (iv,v)}}$

- Several phenomenological aspects of the SM are studied in Ex IV.4.