

Question #1

Part 1:

Alice public key  $(N,e) = (21,5)$

- choose  $p$  and  $q$   
so  $p \neq q$   
 $p=3, q=17$
- compute  $\phi N$   
 $(3-1)(17-1) = 32$
- choose  $e$  relatively  
prime to  $\phi N$   
 $e=3$
- Find  $d$ , such that  
 $(e)(d) \% \phi N = 1$   
 $3(d) \bmod 32 = 1$   
 $d=11$

public key  $(N,e) = (32,3)$

private key  $(N,e) = (32,11)$

Part 2:

plain text = "Anna Burke"

ASCII = 34, 97, 110, 110, 97, 32, 98, 117, 114, 107, 101, 34

Encryption:  $C = m^e \bmod N$   
 $(N,e) = (21,5)$

DIGIT	C	$C = m^e \bmod N$
0	0	$= 0^5 \bmod 21$
1	1	$= 1^5 \bmod 21$
2	11	$= 2^5 \bmod 21$
3	12	$= 3^5 \bmod 21$
4	16	$= 4^5 \bmod 21$
5	17	$= 5^5 \bmod 21$
6	6	$= 6^5 \bmod 21$
7	7	$= 7^5 \bmod 21$
8	8	$= 8^5 \bmod 21$
9	18	$= 9^5 \bmod 21$

encrypted message = 1216, 187, 110, 110, 187, 1211, 188, 117, 1116, 107, 101, 1216

Part 3:

plain text = "Anna Burke"

ASCII = 34, 97, 110, 110, 97, 32, 98, 117, 114, 107, 101, 34

Sign formula:  $S = m^d \bmod N$

$(N,d)$  Senders private key:  $(32,11)$

DIGIT	S
0	$0 = 0^{11} \bmod 32$
1	$11 = 1^{11} \bmod 32$
2	$25 = 2^{11} \bmod 32$
3	$19 = 3^{11} \bmod 32$
4	$17 = 4^{11} \bmod 32$
5	$27 = 5^{11} \bmod 32$
6	$9 = 6^{11} \bmod 32$
7	$3 = 7^{11} \bmod 32$
8	$1 = 8^{11} \bmod 32$
9	$11 = 9^{11} \bmod 32$

Signed Message = 1917, 113, 110, 110, 113, 1925, 111, 113, 1117, 103, 101, 1917

Part 4:

Encrypted Message: 1216, 187, 110, 110, 187, 1211, 188, 117, 1116, 107, 101, 1216  
Senders Private Key =  $(32,11)$

DIGIT	S
0	$0 = 0^{11} \bmod 32$
1	$11 = 1^{11} \bmod 32$
2	$25 = 2^{11} \bmod 32$
3	$19 = 3^{11} \bmod 32$
4	$17 = 4^{11} \bmod 32$
5	$27 = 5^{11} \bmod 32$
6	$9 = 6^{11} \bmod 32$
7	$3 = 7^{11} \bmod 32$
8	$1 = 8^{11} \bmod 32$
9	$11 = 9^{11} \bmod 32$

$\{M\} = 1917, 113, 110, 110, 113, 1925, 111, 113, 1117, 103, 101, 1917$

Part 5:

Signed Message: 1917, 113, 110, 110, 113, 1925, 111, 113, 1117, 103, 101, 1917  
Recievers Public Key =  $(21,5)$

DIGIT	C	$C = m^e \bmod N$
0	0	$= 0^5 \bmod 21$
1	1	$= 1^5 \bmod 21$
2	11	$= 2^5 \bmod 21$
3	12	$= 3^5 \bmod 21$
4	16	$= 4^5 \bmod 21$
5	17	$= 5^5 \bmod 21$
6	6	$= 6^5 \bmod 21$
7	7	$= 7^5 \bmod 21$
8	8	$= 8^5 \bmod 21$
9	18	$= 9^5 \bmod 21$

$\{M\} = 11817, 1112, 110, 110, 1112, 118117, 111, 1112, 1117, 1012, 101, 11817$

Question #2

$p = 541$  Alice Bob  
 $g = 10$   $A = 11$   $B = 13$

Alice  $\rightarrow$  Bob

$g^a \bmod p$   
 $10^{11} \bmod 541 = 297$

Bob  $\rightarrow$  Alice

$g^b \bmod p$   
 $10^{13} \bmod 541 = 486$

Bob computes:

$297^{13} \bmod 541 = 511$   
 $(g^a \bmod p)^b \bmod p$

Alice Computes:

$486^{11} \bmod 541 = 511$   
 $(g^b \bmod p)^a \bmod p$

Symmetric  
Key