Project #5 Solutions

EECS 592

WN 2020

1. Markov Chains

a. Draw the Markov Chain directed graph corresponding to the above uncertain transition dynamics. Label all states {B, W, M, P} and specify all transition probabilities along the edges.

Figure 1 shows the Markov Chain directed graph.

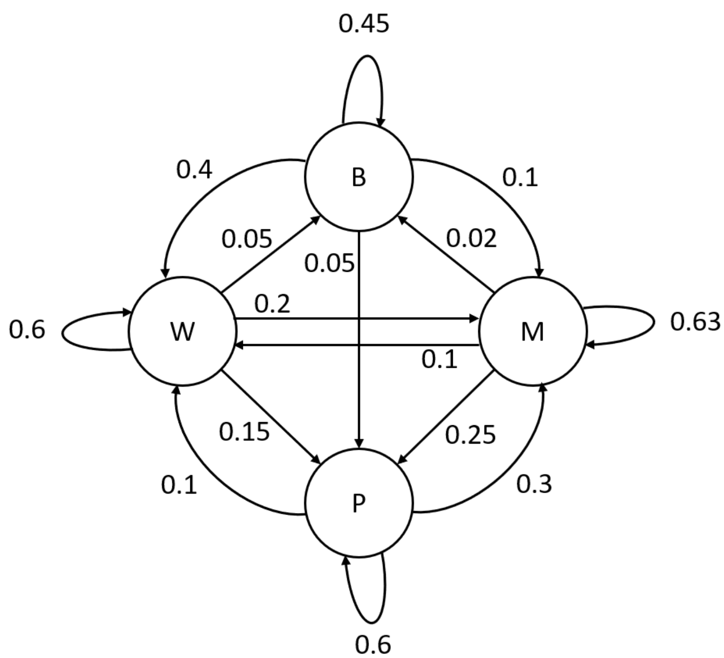

Figure 1

b. Specify the transition probability matrix M such that $X_{i+1} = X_i M$ given a mapping $X = \{X_1, X_2, X_3, X_4\} = \{B, W, M, P\}$.

$$M = \begin{bmatrix} 0.45 & 0.4 & 0.1 & 0.05 \\ 0.05 & 0.6 & 0.2 & 0.15 \\ 0.02 & 0.1 & 0.63 & 0.25 \\ 0 & 0.1 & 0.3 & 0.6 \end{bmatrix}$$

c. What is the probability a person that is initially middle-class will be wealthy after four time steps?

$$\begin{aligned} X_0 &= \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \\ X_4 &= X_0 M^4 \\ X_4 &= \begin{bmatrix} 0.0315 & 0.2019 & .4214 & 0.3453 \end{bmatrix} \end{aligned}$$

Therefore, a person that is initially middle-class will have probability $\boxed{0.2019}$ of becoming wealthy after four time steps.

d. What is the probability a person that is initially wealthy will be middle-class after ten time steps?

$$\begin{aligned} X_0 &= \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \\ X_{10} &= X_0 M^{10} \\ X_{10} &= \begin{bmatrix} 0.0354 & 0.2229 & .4032 & 0.3386 \end{bmatrix} \end{aligned}$$

Therefore, a person that is initially wealthy will have probability $\boxed{0.4032}$ of becoming middle-class after ten time steps.

e. Specify the steady state probability vector X for this Markov Chain if such a solution exists.

A steady state probability vector will have the property that $X_{ss} = X_{ss}M$. X_{ss} can then be solved as the left eigenvector of M that has corresponding eigenvalue 1 (i.e. $X_{ss}(1) = X_{ss}M$). In this case, that eigenvector is $X_{ss} = \begin{bmatrix} 0.0606 & 0.385 & 0.705 & 0.5926 \end{bmatrix}$. However, we additionally need to normalize it so that the probabilities sum to 1 (we can multiply both sides with a scaling factor α to get $(\alpha X_{ss}) = (\alpha X_{ss})M$ without changing the stationary property). Finally, after normalizing we obtain $\boxed{X_{ss} = \begin{bmatrix} 0.0348 & 0.2209 & 0.4044 & 0.3399 \end{bmatrix}}$

Alternatively, you could have chosen a large number of steps to forward propagate the state to determine the same steady state value.

2. Decision Tree Classification

a. Manually create a decision tree using attribute ordering Time → Game Type → Weather. Label each leaf node; terminate a branch when all examples have been classified correctly (if possible).

Figure 2 shows the decision tree

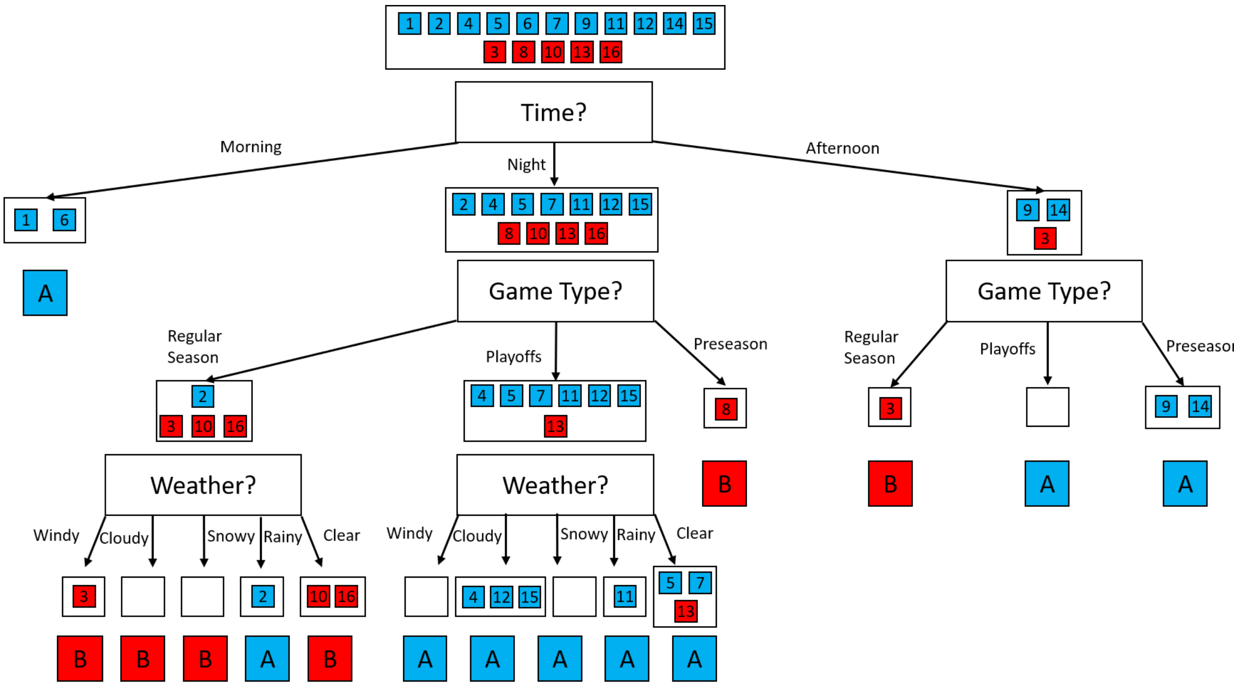

Figure 2: Decision Tree for 2a

b. Now create a decision tree using attribute ordering Weather → Time → Game Type. Label each leaf node; terminate a branch when all examples have been classified correctly (if possible).

Figure 3 shows the decision tree

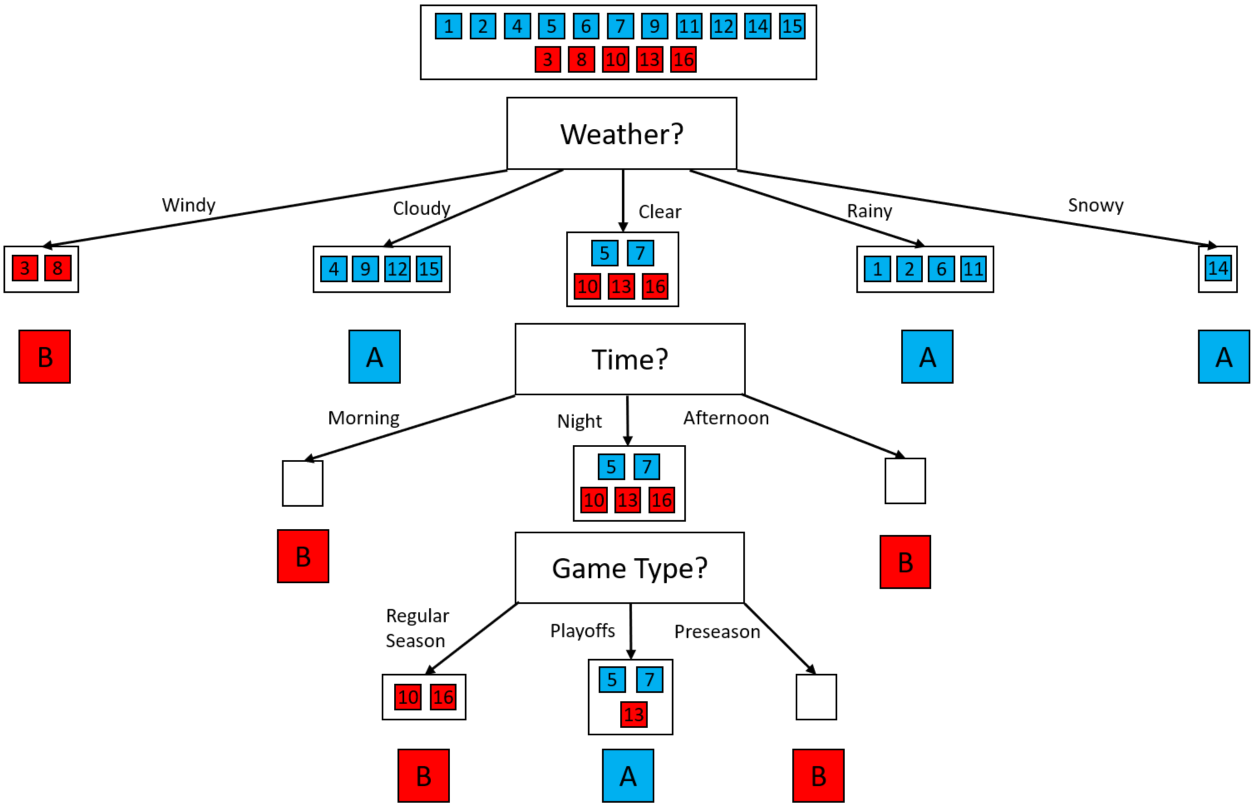

Figure 3: Decision Tree for 2b

c. What is the most likely outcome of a Playoff game on a clear night, given your decision tree(s)?

The most likely outcome is that team A will win.