Modified version

EECS 592: Project 4

1) (R & N 13.10)

→ Slot machine with 3 independent wheels → Bar, bell, Lemon, cherry 50% (4 symbols: BR, BL, L, C)

↳ blank, blank, blank, blank 50% (4 blanks: BK)

⇓ equal % ∴ prob of each = $\frac{1}{8}$

→ Payout scheme: (bet = 1 coin)

BR / BR / BR = 20 coins

BL / BL / BL = 15

L / L / L = 5

C / C / C = 3

C / C / ? = 2

C / ? / ? = 1

(a) "Payback" % = ? for each coin played, expected coin return

Ans

→ Total outcomes = $8 \times 8 \times 8 = 512$ ∴ each case prob = $\frac{1}{512}$

→ P(symbol / symbol / symbol) = $\frac{4}{512}$, $P(C/C/?) = \left(\frac{1}{8}\right)^2 - \frac{1}{512} = \frac{7}{512}$

$P(BR/BR/BR) = \frac{1}{512} \times 20 = \frac{20}{512}$ $P(C/?/?) = \left(\frac{1}{8}\right) - \frac{1}{512} - \frac{7}{512} = \frac{56}{512}$

$P(BL/BL/BL) = \frac{1}{512} \times 15 = \frac{15}{512}$ P(all other results) = $\frac{512 - 56 - 7 - 4}{512} = \frac{445}{512}$

$P(L/L/L) = \frac{1}{512} \times 5 = \frac{5}{512}$

$P(C/C/C) = \frac{1}{512} \times 3 = \frac{3}{512}$ ∴ Payback in $\frac{67}{512}$ outcomes only

$P(C/C/?) = \frac{7}{512} \times 2 = \frac{14}{512}$

$P(C/?/?) = \frac{56}{512} \times 1 = \frac{56}{512}$

∴ Payback % = $\frac{20 + 15 + 5 + 3 + 14 + 56}{512} = \frac{113}{512} \times 100 = \boxed{22.07}\,\%$

(b) P(win at 1 play) = Sum of outcomes for payback scheme = $\frac{56+7+4}{512} = \frac{67}{512}$

(c) Mean and median no. of plays until broke (starting with 10 coins).

→ Using python simulation to compute mean and median.

[* code attached for simulation] ↳ "prob1_sim.py"

Mean ≈ 18
Median = 16

Original version (in project document)

1) (R&N 13.10)

equal probability $\therefore$ 1/4 each

Slot machine with 3 independent wheels $\xrightarrow{4}$ BAR, BELL, LEMON, CHERRY
$\downarrow$ BR, $\downarrow$ BL, $\downarrow$ L, $\downarrow$ C

Payout scheme:

$$\left.\begin{array}{l} \bullet\ BR/BR/BR = 20 \text{ coins} \\ \bullet\ BL/BL/BL = 15 \\ \bullet\ L/L/L = 5 \\ \bullet\ C/C/C = 3 \\ \bullet\ C/C/? = 2 \\ \bullet\ C/?/? = 1 \end{array}\right\} \text{for 1 coin}$$

- anything else = 0 assumed

(a) "Payback" % = ? for each coin played, expected coin return

Total outcomes = $4 \times 4 \times 4 = 64$ $\therefore$ P(each outcome) = $\frac{1}{4}$

Prob

(i) $P(\text{symbol} \wedge \text{symbol} \wedge \text{symbol}) = \frac{4}{64}$

(ii) $P(C \wedge C \wedge \text{symbol} \wedge \text{symbol} \neq C) = \frac{4}{64} - \underset{(i)}{\frac{1}{64}} = \frac{3}{64}$

(iii) $P(C \wedge \text{symb1} \wedge \text{symb2} \wedge \text{symb1} \neq C \wedge \text{symb2} \neq C) = \frac{4 \times 4}{64} - \underset{(i)}{\frac{1}{64}} - \underset{(ii)}{\frac{3}{64}} = \frac{12}{64}$

(iv) $P(\text{all other non-scheme}) = \frac{64 - 12 - 3 - 4}{64} = \frac{48}{64}$

$P(\text{some } C) = 1 - P(\text{no } C) = \frac{61}{64}$

$\therefore$ Payback in $\frac{19}{64}$ outcomes only.

Expectation

(i) $P(BR/BR/BR) = \frac{1}{64} \times 20 = \frac{20}{64}$

(i) $P(BL/BL/BL) = \frac{1}{64} \times 15 = \frac{15}{64}$

(i) $P(L/L/L) = \frac{1}{64} \times 5 = \frac{5}{64}$

(i) $P(C/C/C) = \frac{1}{64} \times 3 = \frac{3}{64}$

(ii) $P(C/C/?) = \cancel{\frac{2}{6}}\ \frac{3}{64} \times 2 = \frac{6}{64}$

(iii) $P(C/?/?) = \frac{12}{64} \times 1 = \frac{12}{64}$

$\therefore$ Payback % $= \frac{20 + 15 + 5 + 3 + 6 + 12}{64} \times 100$

$= \boxed{95.3125\%}$

(because expectation $= \frac{61}{64} = 0.953125$)

(b) P(win at 1 play) = Sum of outcomes for payback scheme = $\frac{19}{64}$ =

(c) Mean =
Median =
} Simulation

2) (R&N 13.13).

2 medical tests → A, → B

↳ independent methods

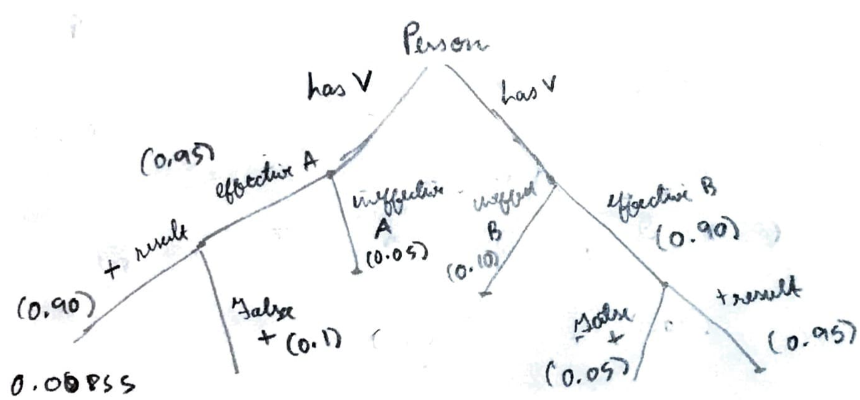
	Effective	False Pos
A	0.95	0.1
B	0.90	0.05

$\therefore$ $P(V) = 0.01$

$P(A|V) = 0.95$

$P(B|V) = 0.90$

People carrying virus = 1% = $\frac{1}{100}$ $\therefore$ P(no virus ∧ person) = $1 - \frac{1}{100} = \frac{99}{100}$



2) (R&N 13.13)

2 medical tests $\to$ A, B

$\hookrightarrow$ independent methods

People carrying virus $= \frac{1}{100} = P(V)$ $\therefore$ $P(\neg V) = \frac{99}{100}$

$P(\text{test } A \mid \text{virus}) = \frac{95}{100} = P(A \mid V)$ $\quad$ $P(\text{test } B \mid \text{virus}) = \frac{90}{100} = P(B \mid V)$

$P(\text{test } A \mid \neg \text{virus}) = \frac{10}{100}$ (false +) $= P(A \mid \neg V)$ $\quad$ $P(\text{test } B \mid \neg \text{virus}) = \frac{5}{100}$ (false +) $= P(B \mid \neg V)$

→ Which test is better indicator of having the virus? $\therefore$

$P(V \mid A) = ?$ $\quad P(V \mid B) = ?$

$$\therefore \; P(V \mid A) = \frac{P(A \mid V)\,P(V)}{P(A \mid V)\,P(V) + P(A \mid \neg V)\,P(V)} = \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.1 \times 0.99} = \underline{0.088}$$

$$\therefore \; P(V \mid B) = \frac{P(B \mid V)\,P(V)}{P(B \mid V)\,P(V) + P(B \mid \neg V)\,P(V)} = \frac{0.9 \times 0.01}{0.9 \times 0.01 + 0.05 \times 0.99} = \underline{0.154}$$

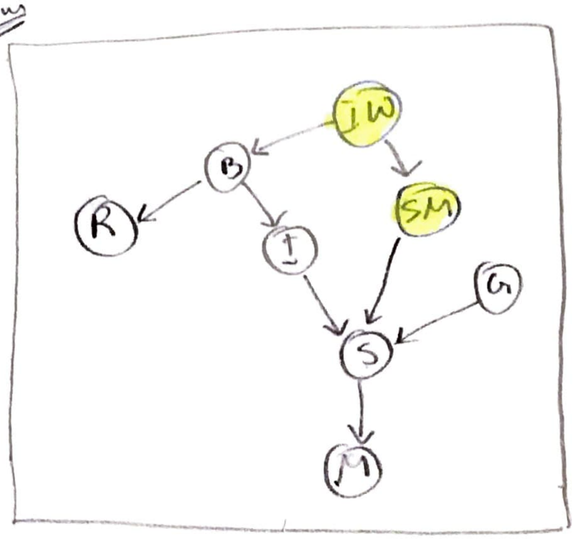
$\therefore$ <u>test B</u> is more indicative of carrying the virus!

3) (R&N 14.8) Modified

Car diagnosis figure 14.21

(a) Extend BN with 'IcyWeather' and 'StarterMotor'

Ans



- R = Radio
- B = Battery
- I = Ignition
- S = Starts
- G = Gas
- M = Moves
- IW = IcyWeather
- SM = StarterMotor

(b) How many independent values are contained in the full JPD for 8 boolean nodes? (No C.I relations amongst them)

Ans

→ 8 bool nodes ∴ $2^8 - 1 = 256 - 1 = \boxed{255}$ independent values.

(c) How many indep. prob. values does BN contain?

Ans

→ $\underset{IW}{2^0} + \underset{B}{2^1} + \underset{SM}{2^1} + \underset{R}{2^1} + \underset{I}{2^1} + \underset{G}{2^0} + \underset{S}{2^3} + \underset{M}{2^1} = \boxed{20}$ indep prob values

(for IW, B, SM, R, I, G, S, M)

(d) What nodes are C I of S given M and B?

$P(X \mid S, M, B) = P(X \mid M, B)$ for C.I between X and S

Ans

→ B is C.I of S given I

IW is C.I of S given SM

~~I is ⫫ G given M as evidence~~

→ Node ⫫ S given M

Rule 1 : None (Moves not breaking dependence chain)

Rule 2 : None (Moves ≠ parent of S)

Rule 3: None (None)

→ Node ⫫ S given B, M

Rule 1 : None

Rule 2 : R

Rule 3: None

∴ $\boxed{\text{only Radio ⫫ Starts | Moves, Battery}}$

4) (R&N 14.11)

Bool. variables ; A = Alarm Sounds

F_A = Faulty Alarm

F_G = Faulty gauge

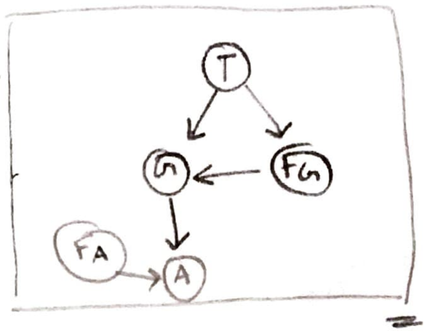
G = gauge reading

T : Actual core temp.

(a) ~~..~~ $F_G \propto T \uparrow$

Ans

$\longrightarrow$

∴ 

(b) Is this BN a polytree?

Ans

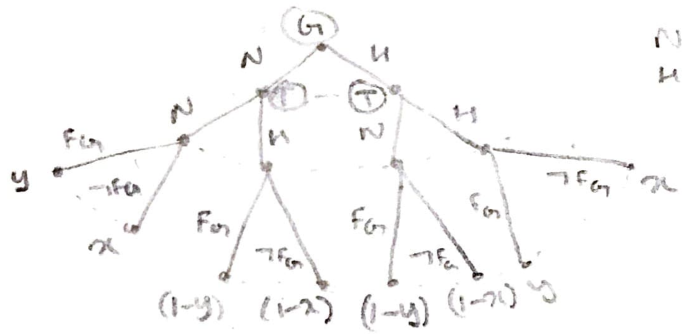
→ $\boxed{\text{No}}$ because a polytree has at most one undirected path between any 2 nodes in a network and the above network has T affecting G in more than one way.

(c) T → Normal, High } actual measurement   G → Normal, High } measured maintenance

P(G is correct ∧ ¬FG) = x

P(G is correct ∧ FG) = y

Ans CPT associated with G?



N = Normal

H = High

→ ∴ CPT as follows:

	G=N		G=H	
	FG	¬FG	FG	¬FG
T = N	y	x	1-y	1-x
T = H	1-y	1-x	y	x

(d) Alarm works correctly unless its faulty ($F_A \rightarrow$ no sound)

Ans CPT for A?

alarm is sound or no sound ∴ binary.

F = false

T = true

→ ∴

	G=N		G=H	
	FA	¬FA	FA	¬FA
A	F	F	F	T
¬A	T	T	T	F

(e) A and G $\rightarrow$ working and A $\rightarrow$ sounds 7

$P(T = high)$ in terms of various cond. prob in the BN.

Ans

$\rightarrow$ $\neg FA$ and $\neg FG$ (given in question)

A $\rightarrow$ alarm sounds

$\therefore$ $P(T = high \mid A, \neg FA, \neg FG)$

$\rightarrow$ From (c) and (d), since alarm sounds and not faulty alarm, gauge reading should be high (from observing previous CPT's)

$\therefore$ $P(T = high \mid A, \neg FA, G = high, \neg Fg)$

Bayes Rule

$$\Rightarrow P(T \mid A, \neg FA, G, \neg FG) = \frac{P(T, A, \neg FA, G, \neg FG)}{P(A, \neg FA, G, \neg FG)}$$

[let T = high be T
let G = high be G]

Chain Rule

$$\Rightarrow P(T \mid A, \neg FA, G, \neg FG) = \frac{P(A \mid \neg \cancel{FA}, G)\, P(\neg \cancel{FA}) \cdot P(G \mid \neg FG, T)\, P(\neg FG \mid T) P(T)}{P(A \mid \neg \cancel{FA}, G)\, P(\neg \cancel{FA})\, P(G, \neg FG)\, P(\neg FG)}$$

$$= \frac{P(T)\, P(\neg FG \mid T)\, P(G \mid \neg FG, T)}{P(\neg FG)\, P(G, \neg FG)}$$

$$= \frac{P(T)\, P(\neg FG \mid T)\, P(G \mid \neg FG, T)}{P(G, \neg FG)}$$ using Bayes's Rule

$$= \frac{P(T)\, P(\neg FG \mid T)\, P(G \mid \neg FG, T)}{P(G, \neg FG, T) + P(G, \neg FG, \neg T)}$$

$$= \frac{P(T)\, P(\neg FG \mid T)\, P(G \mid \neg FG, T)}{P(T)\, P(\neg FG \mid T)\, P(G \mid \neg FG, T) + P(\neg T)\, P(\neg FG \mid \neg T)\, P(G \mid \neg FG, \neg T)}$$

$$\boxed{P(T \mid A, \neg FA, G, \neg FG) = \frac{s \cdot p \cdot (1 - q)}{s \cdot p \cdot (1 - q) + (1 - s)(1 - r)(1 - p)}}$$

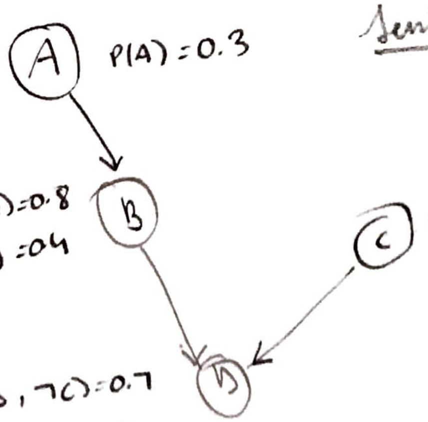
$p = P(t)$
$q = P(\neg FG \mid T)$
$r = P(\neg FG \mid \neg T)$
$s = P(G \mid \neg FG, T)$

5) $P(A) = 0.3$

Simple BN

$*\; P(\text{Var}) = \cancel{\text{ke}}\; P(\text{var} = \text{true})$

$P(\neg \text{Var}) = P(\text{var} = \text{false})$



$P(B \mid \neg A) = 0.8$
$P(B \mid A) = 0.4$

$P(C) = 0.8$

$P(D \mid \neg B, \neg C) = 0.7$
$P(D \mid \neg B, C) = 0.2$
$P(D \mid B, \neg C) = 0.6$
$P(D \mid B, C) = 0.3$

(a) $P(A, \neg B, \neg C, D) = ?$

Ans

$$P(A, \neg B, \neg C, D) = P(A)\, P(\neg C)\, P(\neg B \mid A)\, P(D \mid \neg B, \neg C)$$

$$= 0.3 \times (1-0.8) \times (1-0.6) \times 0.7 = \boxed{0.0168}$$

(b) $$P(D \mid \neg A, B, C) = \frac{P(D, \neg A, B, C)}{P(\neg A, B, C)} = \frac{\cancel{P(\neg A)}\, \cancel{P(C)}\, P(B \mid \cancel{A})\, P(D \mid B, C)}{P(\cancel{\neg A})\, P(\cancel{C})\, P(B \cancel{\mid} \neg A)}$$

$$= P(D \mid B, C) = \boxed{0.3}$$

(c) $$P(\neg A \mid \neg B, C, \neg D) = \frac{P(\neg A, \neg B, C, \neg D)}{P(\neg B, \neg C, \neg D)} = \frac{P(\neg A)\, P(\cancel{C})\, P(\neg B \mid \neg A)\, P(\neg \cancel{D} \mid \neg B, C)}{P(\neg D \mid \cancel{\neg B}, C)\, P(\cancel{C})\, P(\neg B)}$$

$$= \frac{P(\neg A)\, P(\neg B \mid \neg A)}{P(\neg B)} = \frac{1-0.3 \cdot 1-0.8}{0.32 \text{ (from (e))}}$$

$$= \boxed{0.4375}$$

(d) $$P(B \mid A, \neg C) = \frac{P(B, A, \neg C)}{P(A, \neg C)} = \frac{P(\cancel{A})\, P(\cancel{\neg C})\, P(B \mid A)}{P(\cancel{A}) \cancel{\neg C})\, P(\cancel{\neg C})} = \boxed{0.4}$$

$P(A) \because A \perp\!\!\!\perp \neg C$
from rule 3!

(e) $P(\neg B) = \cancel{\$}\; 1 - P(B)$ where $P(B) = P(B \mid \neg A)\, P(\neg A) + P(B \mid A)\, P(A)$

$= 0.8 \cdot 0.7 + 0.4 \cdot 0.3$

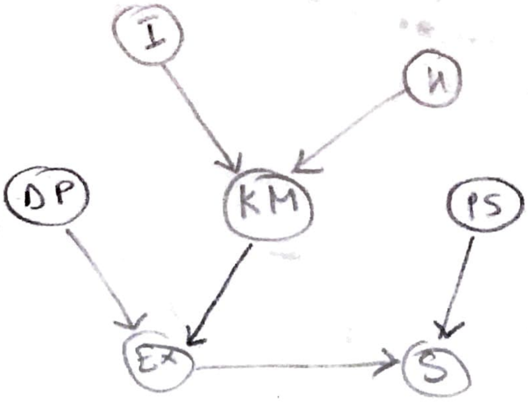
$= 0.56 + 0.12 = 0.68$

$= 1 - 0.68$

$= \boxed{0.32}$

6) Exam BN

- I : Intelligent
- H : hard-working
- DP : doesn't panic
- Ex : high exam score
- KM : knows material
- PS : gains practical skill
- S : Success!



→ $P(I) = 0.75$

$P(H) = 0.6$

$P(DP) = 0.4$

$P(PS) = 0.8$

→ $P(KM \mid I, H) = 1$

$P(KM \mid I, \sim H) = 0.4$

$P(KM \mid \sim I, H) = 0.6$

$P(KM \mid \sim I, \sim H) = 0.05$

→ $P(S \mid PS, Ex) = 0.8$

$P(S \mid \sim PS, Ex) = 0.7$

$P(S \mid PS, \sim Ex) = 0.7$

$P(S \mid \sim PS, \sim Ex) = 0.3$

→ $P(Ex \mid DP, KM) = 0.85$

$P(Ex \mid \sim DP, KM) = 0.7$

$P(Ex \mid DP, \sim KM) = 0.2$

$P(Ex \mid \sim DP, \sim KM) = 0.1$

(a) $DP \not\perp\!\!\!\perp I \mid Ex$

$\boxed{PS \perp\!\!\!\perp I \mid Ex}$

$\boxed{S \perp\!\!\!\perp I \mid Ex}$

$H \not\perp\!\!\!\perp I \mid Ex$

$KM \not\perp\!\!\!\perp I \mid Ex$

(b) $\boxed{I \perp\!\!\!\perp S \mid KM}$

$\boxed{H \perp\!\!\!\perp S \mid KM}$

$DP \not\perp\!\!\!\perp S \mid KM$

$PS \not\perp\!\!\!\perp S \mid KM$

$Ex \not\perp\!\!\!\perp S \mid KM$

(c) $PS \not\perp\!\!\!\perp Ex \mid S$

$I \not\perp\!\!\!\perp Ex \mid S$

$H \not\perp\!\!\!\perp Ex \mid S$

$DP \not\perp\!\!\!\perp Ex \mid S$

$KM \not\perp\!\!\!\perp Ex \mid S$

$\therefore$ $\boxed{\text{None}}$

(d) $P(KM) = P(KM \mid I, H)P(I)P(H) + P(KM \mid \sim I, H)P(H)P(\sim I) + P(KM \mid I, \sim H)P(I)P(\sim H) + P(KM \mid \sim I, \sim H)P(\sim H)P(\sim I)$

$= (1 \cdot 0.75 \cdot 0.6) + (0.6 \cdot 0.6 \cdot 1 - 0.75) + (0.4 \cdot 0.75 \cdot 1 - 0.6) + (0.05 \cdot 1 - 0.75 \cdot 1 - 0.6)$

$= 0.45 + 0.09 + 0.12 + 0.005$

$= \boxed{0.665}$

(e) $P(S \mid KM) = \dfrac{P(S, KM)}{P(KM)} = \cancel{\dfrac{P(S \mid Ex, PS)\, P(PS)\, P(Ex \mid DP, KM)\, P(DP)}{P(KM)}} = \cancel{\dfrac{0.8 \cdot 0.8 \cdot 0.85 \cdot 0.4}{0.665}} = \cancel{0.327}$

$= \boxed{0.694}$

[using code from P7]

(f) $P(PS \mid S) = \cancel{\dfrac{P(S \mid PS)\, P(PS)}{P(S)}}$ where $\cancel{P(S) = P(S \mid KM)\, P(KM) = (0.327) \cdot (0.665) = 0.2176}$

$= \boxed{0.864}$

[using code from P7]

(g) $P(KM \mid S) = \boxed{0.671}$

[using code from P7]

[* code files attached —
"p6_bn.txt"
"p6_in.txt"]