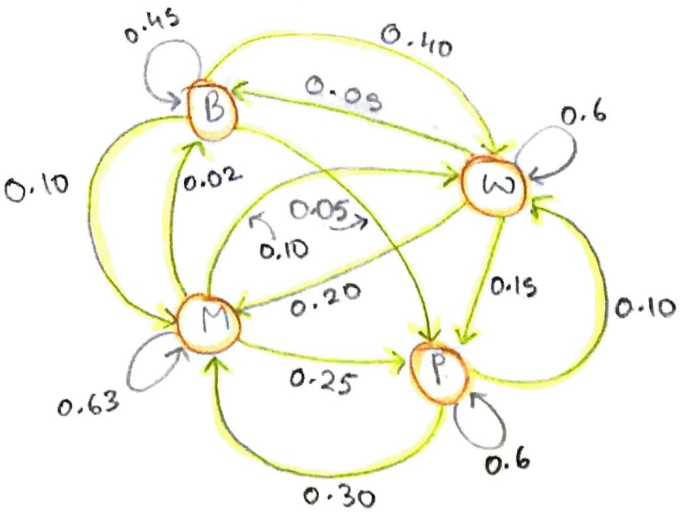
1) Markov Chains

(a)



- States = {B, W, M, P}
- Transitional probabilities along edges

(b) Transitional probability matrix $\Rightarrow$ $P =$ (4×4 matrix)

[where prob of a state to itself is $1 - \sum P_{x_i, x_{i+1}}$ (i.e. sum of row)]

$$P = \begin{array}{c} \\ B \\ W \\ M \\ P \end{array} \begin{array}{c} \begin{array}{cccc} B & W & M & P \end{array} \\ \begin{bmatrix} 0.45 & 0.40 & 0.10 & 0.05 \\ 0.05 & 0.60 & 0.20 & 0.15 \\ 0.02 & 0.10 & 0.63 & 0.25 \\ 0 & 0.10 & 0.30 & 0.6 \end{bmatrix} \end{array}$$

(c) After 4 time steps,

$$P_{t=4} = (P_{t=0})^4 = \begin{array}{c} \\ B \\ W \\ M \\ P \end{array} \begin{array}{c} \begin{array}{cccc} B & W & M & P \end{array} \\ \begin{bmatrix} 0.0818901 & 0.330476 & 0.328083 & 0.259552 \\ 0.0465629 & 0.277854 & 0.370946 & 0.304637 \\ 0.0314819 & \boxed{0.201884} & 0.421355 & 0.345279 \\ 0.0262419 & 0.195219 & 0.41387 & 0.364729 \end{bmatrix} \end{array}$$

$\therefore P_{t=4}(M \to W) = \boxed{0.201884} \Rightarrow \approx \boxed{20\%}$

(d) After 10 time steps,

$$P_{t=10} = (P_{t=0})^{10} = \begin{array}{c} \\ B \\ W \\ M \\ P \end{array} \begin{array}{c} \begin{array}{cccc} B & W & M & P \end{array} \\ \begin{bmatrix} 0.0362684 & 0.226037 & 0.40122 & 0.336475 \\ 0.0353536 & 0.222926 & \boxed{0.403153} & 0.338567 \\ 0.0346149 & 0.220237 & 0.404807 & 0.340341 \\ 0.0344663 & 0.219761 & 0.405086 & 0.340686 \end{bmatrix} \end{array}$$

$\therefore P_{t=10}(W \to M) = \boxed{0.403153} \approx \boxed{40\%}$

(e) Steady state prob vector $v = vM$ (M = P i.e. the transitional prob. matrix from (b))

$\therefore v = v \begin{bmatrix} 0.45 & 0.40 & 0.10 & 0.05 \\ 0.05 & 0.60 & 0.20 & 0.15 \\ 0.02 & 0.10 & 0.63 & 0.25 \\ 0 & 0.10 & 0.30 & 0.6 \end{bmatrix} \Rightarrow v(I - P) = 0$

$\hookrightarrow P$

$v = [x_1\ x_2\ x_3\ x_4]$

$I = I_{4\times 4}$

$\therefore \boxed{v = \begin{bmatrix} 0.035 \\ 0.2210 \\ 0.4040 \\ 0.3400 \end{bmatrix}^T}$

(using a program to calculate)

2) Decision tree classification

- Attributes = Time, Game type, Weather
- Target = Outcome (A, B)
- 16 examples total

→ Root node with distribution (A, B) = (11, 5)

Attr	Values
• Time :	Morning, Afternoon, Night (3)
• GT :	Regular, playoff, preseason (3)
• W :	Rainy, windy, cloudy, clear, snowy (5)

(a) Decision tree attached with submission file

(b) Decision tree attached with submission file

(c) From our trees, we can observe that -

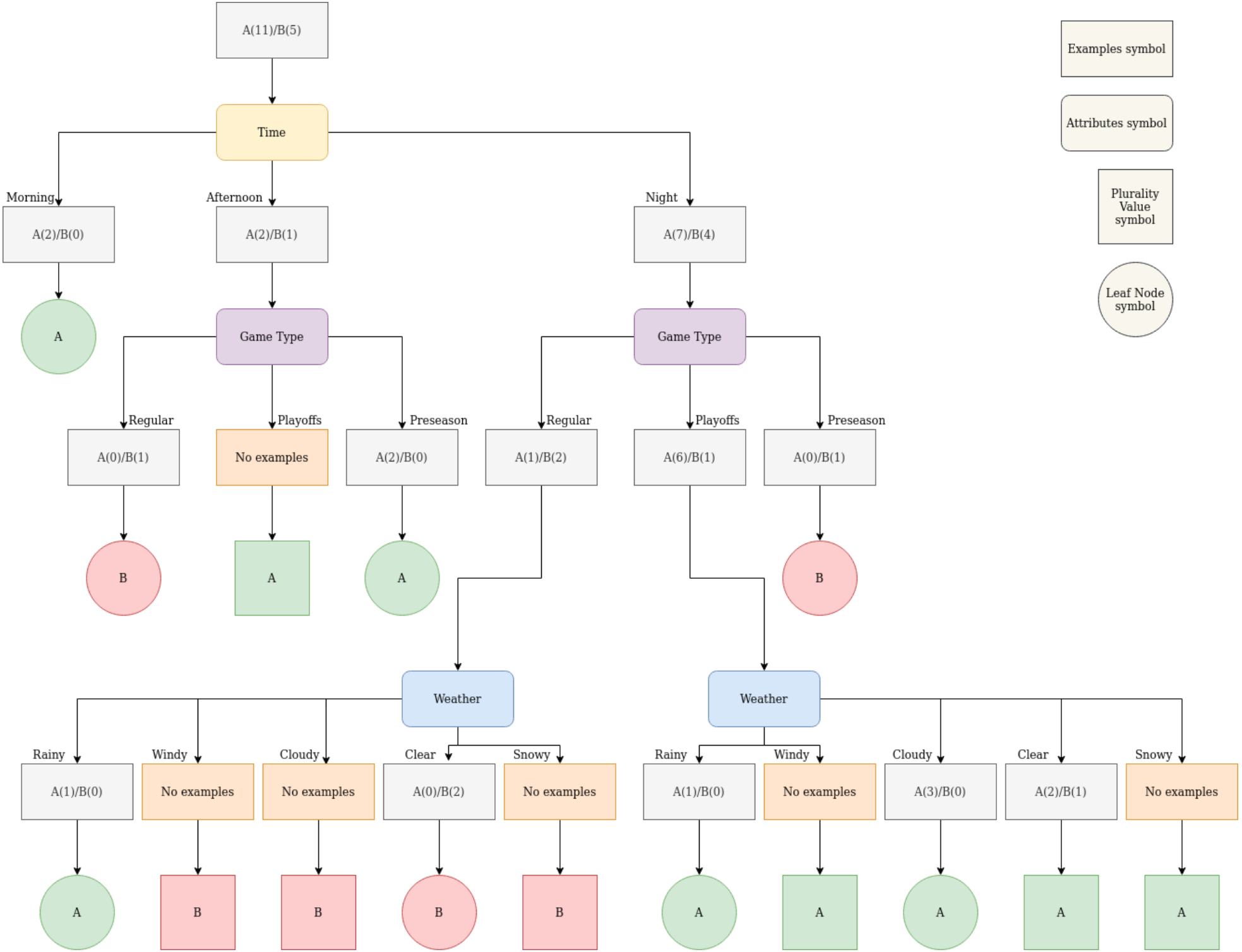
→ for (a), Night → Playoff → Clear ⇒ Team A wins!

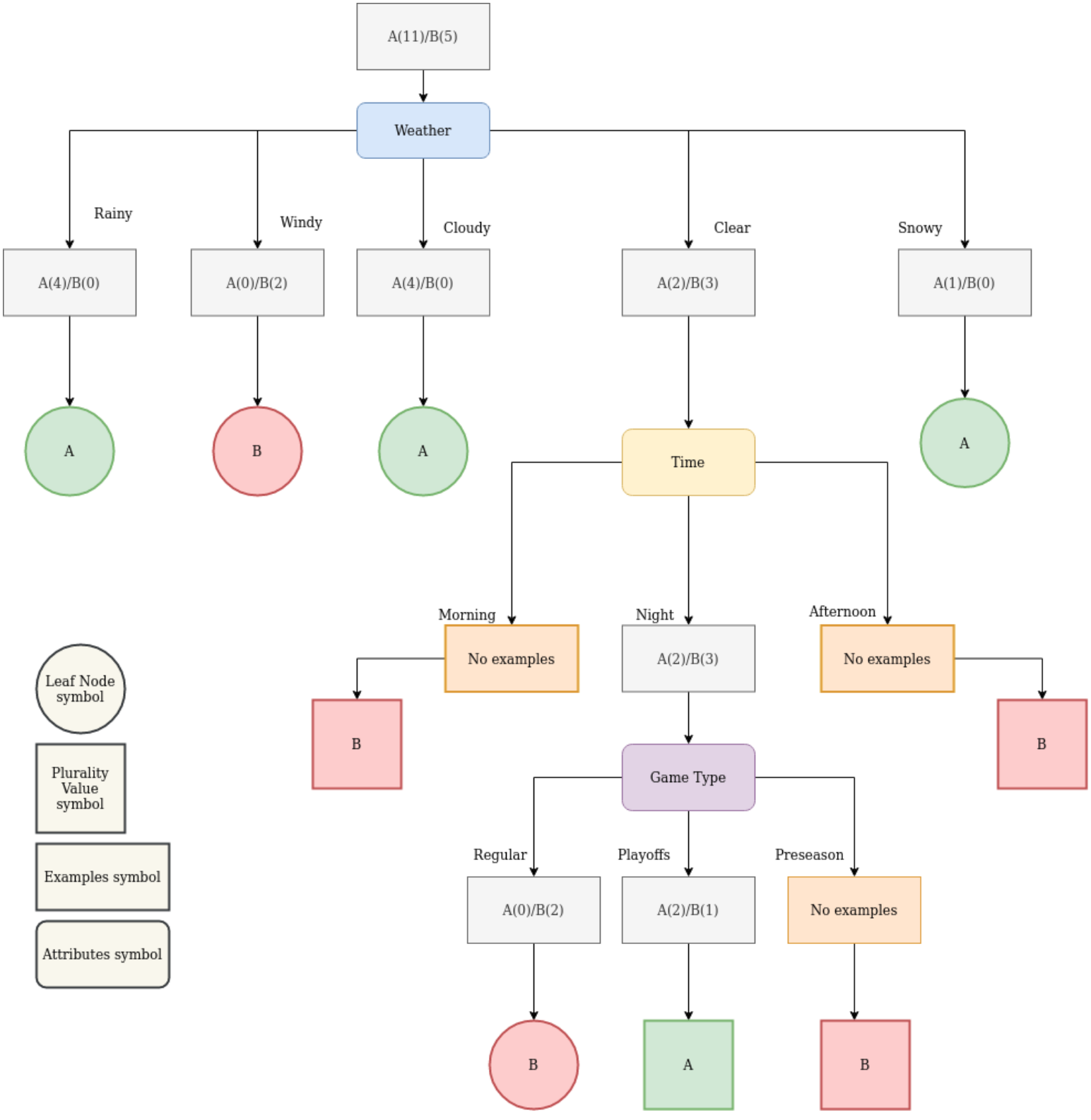
→ for (b), Clear → Night → Playoff ⇒ Team A wins!

∴ Team A's win is the most likely outcome ☺

3) Program attached with submission files

4) Program attached with submission files.


A(11)/B(5)
Time
Morning
Afternoon
Night
A(2)/B(0)
A(2)/B(1)
A(7)/B(4)
A
Game Type
Game Type
Regular
Playoffs
Preseason
Regular
Playoffs
Preseason
A(0)/B(1)
No examples
A(2)/B(0)
A(1)/B(2)
A(6)/B(1)
A(0)/B(1)
B
A
A
B
Weather
Weather
Rainy
Windy
Cloudy
Clear
Snowy
Rainy
Windy
Cloudy
Clear
Snowy
A(1)/B(0)
No examples
No examples
A(0)/B(2)
No examples
A(1)/B(0)
No examples
A(3)/B(0)
A(2)/B(1)
No examples
A
B
B
B
B
A
A
A
A
A
Examples symbol
Attributes symbol
Plurality Value symbol
Leaf Node symbol


A(11)/B(5)
Weather
Rainy
Windy
Cloudy
Clear
Snowy
A(4)/B(0)
A(0)/B(2)
A(4)/B(0)
A(2)/B(3)
A(1)/B(0)
A
B
A
Time
A
Morning
Night
Afternoon
No examples
A(2)/B(3)
No examples
B
B
Game Type
Regular
Playoffs
Preseason
A(0)/B(2)
A(2)/B(1)
No examples
B
A
B
Leaf Node symbol
Plurality Value symbol
Examples symbol
Attributes symbol