CHITTAGONG UNIVERSITY OF ENGINEERING & TECHNOLOGY

B. Sc. Engineering Level-1 Term-I, Final Examination 2020

Subject : Civil Engineering

Paper : Engineering Mathematics – I (Math-101)

Time : 3 Hours Full Marks : 200

Answer any TWO questions from EACH section. Use separate script for EACH section. The figures in the right margin indicate full marks. Use standard value if needed.

SECTION-A

Q.1. (a) Define Continuity and Differentiability of a function. Let, (20)

$$f(x) = \begin{cases} 0 & if\ x \le 0 \\ 5 - x & if\ 0 < x < 4 \\ \dfrac{1}{5-x} & if\ x \ge 4 \end{cases}$$

i) Where is f discontinuous?

ii) Where is f not differentiable?

(b) Find the nth derivative of $y = \cos(ax + b)$ (12)

(c) State Leibnitz's theorem. If $y = \tan^{-1} x$, prove that $(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$ (18)

Q.2. (a) State the Mean Value Theorem. Suppose that $f(0) = -3$ and $f^{/}(x) \le 5$ for all values of x. Use the Mean Value Theorem to find the largest possible value of $f(2)$. (15)

(b) Find the Taylor series for $f(x) = \ln x$ centered at $a = -2$. Also find the radius of convergence. (20)

(c) Evaluate $\lim_{x\to 1^+} x^{1/(1-x)}$ (15)

Q.3. (a) If $p = x\cos\alpha + y\sin\alpha$ touches the curve $\left(\frac{x}{a}\right)^{n/n-1} + \left(\frac{y}{b}\right)^{n/n-1} = 1$, prove that $p^n = (a\cos\alpha)^n + (b\sin\alpha)^n$ (20)

(b) Prove that the curves $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{a'} + \frac{y^2}{b'} = 1$, will cut orthogonally if $a - b = a' - b'$ (15)

(c) If $g(s,t) = f(s^2 - t^2, t^2 - s^2)$ and f is differentiable, show that g satisfies the equation $t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = 0$ (15)

SECTION-B

Q.4. (a) Integrate any three of the following (30)

i) $\int \cos 2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} dx$

ii) $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$

iii) $\int \frac{dx}{(1+x^2)\sqrt{(1-x^2)}}$

iv) $\frac{dx}{(2x-3)\sqrt{(2x^2-3x+4)}}$

(b) Establish a reduction formula for $\int sin^m x cos^n x dx$, and hence evaluate $\int sin^5 x cos^3 x dx$ (20)

Q.5. (a) Evaluate any three of the following: (30)

i) $\int_2^3 \frac{dx}{\sqrt{(x-1)(5-x)}}$

ii) $\int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx$

iii) $\int_0^{\pi/4} \ln(1 + \tan\theta) d\theta$

iv) $\int_0^1 x^2(1-x^2)^{3/2} dx$

(b) If $I_n = \int_0^{\pi/4} tan^n \theta d\theta$, show that $I_n = \frac{1}{n-1} - I_{n-2}$ and hence evaluate $\int_0^{\pi/4} tan^6 \theta d\theta$ (20)

Q.6. (a) Define Gamma function and Beta function. Show that $\beta(m, n) = \frac{\Gamma(\mathrm{m})\Gamma(\mathrm{n})}{\Gamma(\mathrm{m+n})}$ (20)

(b) Find the area of the segment cut off from the parabola $y^2 = 2x$ by the straight line $y = 4x - 1$ (18)

(c) Find the length of one arc of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ (12)

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