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Divide and Conquer | Set 5 (Strassen's Matrix Multiplication)

Given two square matrices A and B of size n x n each, find their multiplication matrix.

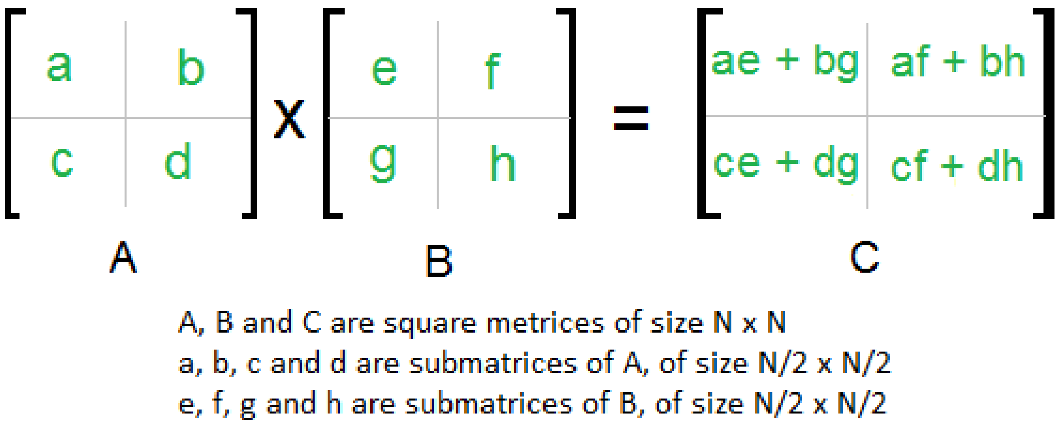
Naive Method
Following is a simple way to multiply two matrices.

```
void multiply(int A[][N], int B[][N], int C[][N])
{
    for (int i = 0; i < N; i++)
    {
        for (int j = 0; j < N; j++)
        {
            C[i][j] = 0;
```

```
            for (int k = 0; k < N; k++)
            {
                C[i][j] += A[i][k]*B[k][j];
            }
        }
    }
}
```

Time Complexity of above method is $O(N^3)$.

Divide and Conquer
Following is simple Divide and Conquer method to multiply two square matrices.
1) Divide matrices A and B in 4 sub-matrices of size N/2 x N/2 as shown in the below diagram.
2) Calculate following values recursively. ae + bg, af + bh, ce + dg and cf + dh.



In the above method, we do 8 multiplications for matrices of size N/2 x N/2 and 4 additions. Addition of two matrices takes $O(N^2)$ time. So the time complexity can be written as

```
T(N) = 8T(N/2) + O(N^2)

From Master's Theorem, time complexity of above method is O(N^3)
which is unfortunately same as the above naive method.
```

Simple Divide and Conquer also leads to $O(N^3)$, can there be a better way?
In the above divide and conquer method, the main component for high time complexity is 8 recursive calls. The idea of **Strassen's method** is to reduce the number of recursive calls to 7. Strassen's method is similar to above simple divide and conquer method in the sense that this method also divide matrices to sub-matrices of size N/2 x N/2 as shown in the above diagram, but in Strassen's method, the four sub-matrices of result are calculated using following formulae.

$$p1 = a(f - h) \qquad p2 = (a + b)h$$
$$p3 = (c + d)e \qquad p4 = d(g - e)$$
$$p5 = (a + d)(e + h) \qquad p6 = (b - d)(g + h)$$
$$p7 = (a - c)(e + f)$$

The A x B can be calculated using above seven multiplications.
Following are values of four sub-matrices of result C

$$\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \times \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{C}{\begin{bmatrix} p5 + p4 - p2 + p6 & p1 + p2 \\ p3 + p4 & p1 + p5 - p3 - p7 \end{bmatrix}}$$

A, B and C are square metrices of size N x N
a, b, c and d are submatrices of A, of size N/2 x N/2
e, f, g and h are submatrices of B, of size N/2 x N/2
p1, p2, p3, p4, p5, p6 and p7 are submatrices of size N/2 x N/2

Time Complexity of Strassen's Method

Addition and Subtraction of two matrices takes $O(N^2)$ time. So time complexity can be written as

```
T(N) = 7T(N/2) +  O(N^2)
```

```
From Master's Theorem, time complexity of above method is
O(N^Log7) which is approximately O(N^2.8074)
```

Generally Strassen's Method is not preferred for practical applications for following reasons.
1) The constants used in Strassen's method are high and for a typical application Naive method works better.
2) For Sparse matrices, there are better methods especially designed for them.
3) The submatrices in recursion take extra space.
4) Because of the limited precision of computer arithmetic on noninteger values, larger errors accumulate in Strassen's algorithm than in Naive Method (Source: CLRS Book)

References:

Introduction to Algorithms 3rd Edition by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest
https://www.youtube.com/watch?v=LOLebQ8nKHA
https://www.youtube.com/watch?v=QXY4RskLQcI

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Tags: Divide and Conquer

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kadir · 4 months ago

matrix multiplication recursion method using java code ?

Ethan Glover · 8 months ago

Fantastic, this simplified things a great bit.

Gaurav Patil · 9 months ago

```
#include <stdio.h>

int num;

void strassen(int a[][num], int b[][num], int c[][num], int size) {

int p1[size/2][size/2], p2[size/2][size/2], p3[size/2][size/2], p4[size/2][size/2], p5[size/2][size/2],
p6[size/2][size/2], p7[size/2][size/2];

int temp1[size/2][size/2], temp2[size/2][size/2];

int q1, q2, q3, q4, q5, q6, q7, i, j;

if(size >= 2) { //give recursive calls

//p1

for(i = 0; i < size / 2; i++) {
```

```
for(j = 0; j < size / 2; j++) {
```

see more

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Twinkle → Gaurav Patil • 10 days ago

Really very nice

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nice post.. @admin could you post multiplications of 3*3 matrix by this method.. every where there is only explanation of those formulas and time complexity is explained

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can be done bro, just fill the remaining elements by zero to make it the nearest 2^n*2^n matrix

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cannot be done as n has to be exact power of 2

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