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Lower bound for comparison based sorting algorithms

The problem of sorting can be viewed as following.

Input: A sequence of *n* numbers $\langle a_1, a_2, \dots, a_n \rangle$.

Output: A permutation (reordering) $\langle a'_1, a'_2, \dots, a'_n \rangle$ of the input sequence such that $a'_1 \leq a'_2 \dots \leq a'_n$.

A sorting algorithm is comparison based if it uses comparison operators to find the order between two numbers. Comparison sorts can be viewed abstractly in terms of decision trees. A decision tree is a <u>full binary tree</u> that represents the comparisons between elements that are performed by a particular sorting algorithm operating on an input of a given size. The execution of the sorting algorithm corresponds to tracing a path from the root of the decision tree to a leaf. At each internal node, a comparison a_i is

made. The left subtree then dictates subsequent comparisons for $a_i \bowtie a_j$, and the right subtree dictates subsequent comparisons for $a_i > a_j$. When we come to a leaf, the sorting algorithm has established the ordering. So we can say following about the decison tree.

- 1) Each of the n! permutations on n elements must appear as one of the leaves of the decision tree for the sorting algorithm to sort properly.
- 2) Let x be the maximum number of comparisons in a sorting algorithm. The maximum height of the decison tree would be x. A tree with maximum height x has at most 2^x leaves.

After combining the above two facts, we get following relation.

```
n! <= 2^x
Taking Log on both sides.
    [Tex]\log_2n![/Tex] <= x
Since [Tex]\log_2n![/Tex] = [Tex]\Theta(nLogn)[/Tex], we can say
    x = [Tex]\Omega(nLog_2n)[/Tex]</pre>
```

Therefore, any comparison based sorting algorithm must make at least [Tex]\Omega(nLog_2n)[/Tex] comparisons to sort the input array, and Heapsort and merge sort are asymptotically optimal comparison sorts.

References:

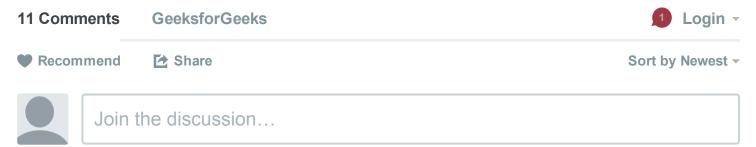
<u>Introduction to Algorithms, by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein</u>

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Heta Vaishnani · 4 months ago

why does it need n! permutationz?



jayasuya_j → Heta Vaishnani • 3 months ago

say for example three numbers: 1,2,3 permutation of them gives the possible arrangements i.e 123,213,231,312,, etc thus one among these must be the answer. So number of possibilities is n! out of which one is the answer. These possibilities form the leaves of the tree,where we choose only one leaf (which gives sorted sequence) by following a path from root to the leaf

1 ^ V • Reply • Share >



sd · 7 months ago

Mention of Stirlings Approximation can be helpful.



GeeksforGeeks • 8 months ago

nice explaination

1 ^ Reply • Share



Kanhaiya Kumawat • a year ago

there is typo in the last line: "Therefore, any comparison based sorting algorithm must make at least comparisons to sort the input array, and Heapsort and merge sort are asymptotically optimal comparison sorts."

its nLogn rather than Logn.



GeeksforGeeks Mod → Kanhaiya Kumawat • a year ago

Thanks for pointing this out. We have corrected the typo.

Reply • Share >



wgpshashank · 4 years ago

More Info ..

http://www.it-c.dk/courses/ITM/F2003/Sorting.pdf

Reply • Share >



lovocas • 4 years ago

" A decision tree is a full binary tree"

I am confused by this statement. :-<

A full binary tree is a tree has 2^k(k+1)-1 nodes ?k is the heright, the root's height is 0?.

I think desision tree is just a binary tree, whose nodes has either 2 or zero nodes?

```
Neply • Snare >
```



GeeksforGeeks → lovocas · 4 years ago

@lovocas: The statement looks correct. Please see the following Wiki definition of Full Binary Tree.

A full binary tree (sometimes proper binary tree or 2-tree or strictly binary tree) is a tree in which every node other than the leaves has two children.

To avoid confusion, we have updated the post and added the Wiki link for full binary tree.

```
2 ^ | V · Reply · Share >
```



Yang → GeeksforGeeks · 2 years ago

What if I write a "really bad" algorithm and it asks the same question every time and gets the same answer every time. Then, the Decision tree will still be full binary tree?

```
∧ | ∨ • Reply • Share >
```



lovocas → GeeksforGeeks · 4 years ago

oh, thanks very much, got it!

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@GeeksforGeeks i don't n know what is this long...

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o manish

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