Backtracking | Set 4 (Subset Sum)

Subset sum problem is to find subset of elements that are selected from a given set whose sum adds up to a given number K. We are considering the set contains non-negative values. It is assumed that the input set is unique (no duplicates are presented).

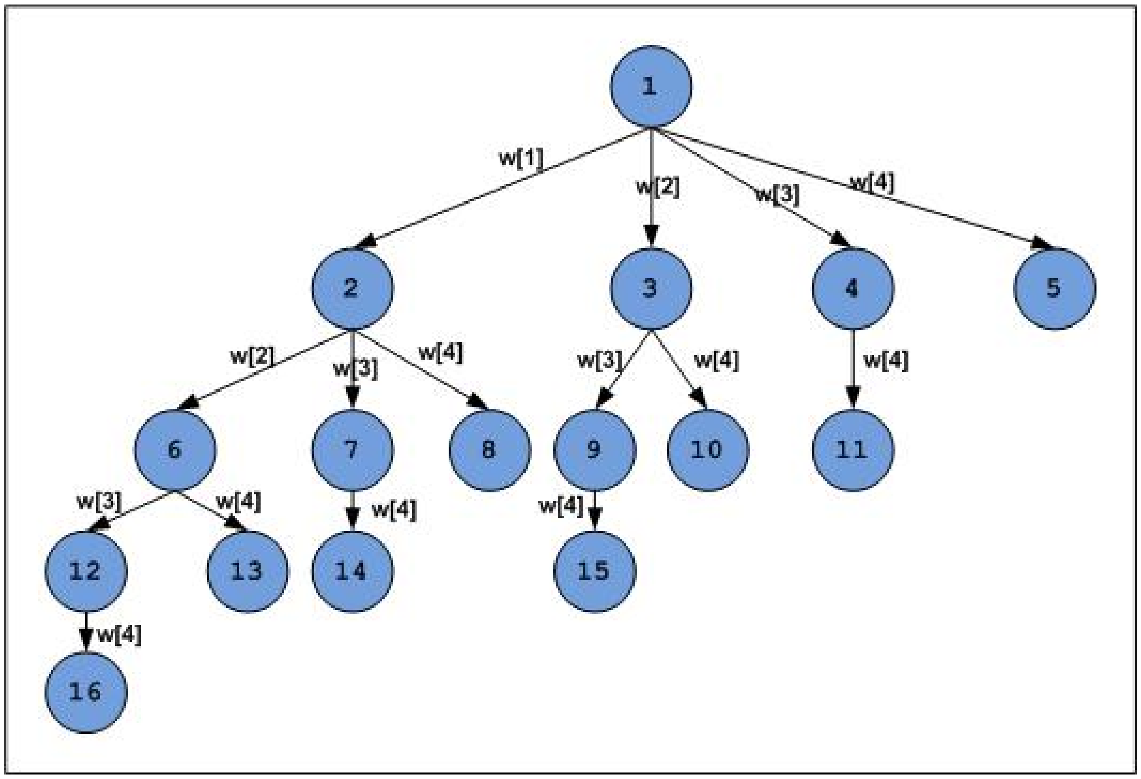
Exhaustive Search Algorithm for Subset Sum

One way to find subsets that sum to K is to consider all possible subsets. A power set contains all those subsets generated from a given set. The size of such a power set is 2^N.

Backtracking Algorithm for Subset Sum

Using exhaustive search we consider all subsets irrespective of whether they satisfy given constraints or not. Backtracking can be used to make a systematic consideration of the elements to be selected.

Assume given set of 4 elements, say **w[1] … w[4]**. Tree diagrams can be used to design backtracking algorithms. The following tree diagram depicts approach of generating variable sized tuple.



In the above tree, a node represents function call and a branch represents candidate element. The root node contains 4 children. In other words, root considers every element of the set as different branch. The next level sub-trees correspond to the subsets that includes the parent node. The branches at each level represent tuple element to be considered. For example, if we are at level 1, tuple_vector[1] can take any value of four branches generated. If we are at level 2 of left most node, tuple_vector[2] can take any value of three branches generated, and so on…

For example the left most child of root generates all those subsets that include w[1]. Similarly the second child of root generates all those subsets that includes w[2] and excludes w[1].

As we go down along depth of tree we add elements so far, and if the added sum is satisfying explicit constraints, we will continue to generate child nodes further. Whenever the constraints are not met, we stop further generation of sub-trees of that node, and backtrack to previous node to explore the nodes not yet explored. In many scenarios, it saves considerable amount of processing time.

The tree should trigger a clue to implement the backtracking algorithm (try yourself). It prints all those subsets whose sum add up to given number. We need to explore the nodes along the breadth and depth of the tree. Generating nodes along breadth is controlled by loop and nodes along the depth are generated using recursion (post order traversal). Pseudo code given below,

```
if(subset is satisfying the constraint)
    print the subset
    exclude the current element and consider next element
else
    generate the nodes of present level along breadth of tree and
    recur for next levels
```

Following is C implementation of subset sum using variable size tuple vector. Note that the following program explores all possibilities similar to exhaustive search. It is to demonstrate how backtracking can be used. See next code to verify, how we can optimize the backtracking solution.

```
#include <stdio.h>
#include <stdlib.h>

#define ARRAYSIZE(a) (sizeof(a))/(sizeof(a[0]))

static int total_nodes;
// prints subset found
void printSubset(int A[], int size)
{
    for(int i = 0; i < size; i++)
    {
        printf("%*d", 5, A[i]);
    }

    printf("\n");
}

// inputs
// s            - set vector
// t            - tuplet vector
// s_size       - set size
// t_size       - tuplet size so far
// sum          - sum so far
// ite          - nodes count
// target_sum   - sum to be found
void subset_sum(int s[], int t[],
                int s_size, int t_size,
                int sum, int ite,
                int const target_sum)
{
    total_nodes++;
    if( target_sum == sum )
    {
        // We found subset
        printSubset(t, t_size);
        // Exclude previously added item and consider next candidate
        subset_sum(s, t, s_size, t_size-1, sum - s[ite], ite + 1, target_sum);
        return;
    }
    else
    {
        // generate nodes along the breadth
        for( int i = ite; i < s_size; i++ )
        {
            t[t_size] = s[i];
            // consider next level node (along depth)
            subset_sum(s, t, s_size, t_size + 1, sum + s[i], i + 1, target_sum);
        }
    }
}

// Wrapper to print subsets that sum to target_sum
// input is weights vector and target_sum
void generateSubsets(int s[], int size, int target_sum)
{
    int *tuplet_vector = (int *)malloc(size * sizeof(int));
    subset_sum(s, tuplet_vector, size, 0, 0, 0, target_sum);

    free(tuplet_vector);
}

int main()
{
    int weights[] = {10, 7, 5, 18, 12, 20, 15};
    int size = ARRAYSIZE(weights);

    generateSubsets(weights, size, 35);
    printf("Nodes generated %d\n", total_nodes);
    return 0;
}
```

The power of backtracking appears when we combine explicit and implicit constraints, and we stop generating nodes when these checks fail. We can improve the above algorithm by strengthening the constraint checks and presorting the data. By sorting the initial array, we need not to consider rest of the array, once the sum so far is greater than target number. We can backtrack and check other possibilities.

Similarly, assume the array is presorted and we found one subset. We can generate next node excluding the present node only when inclusion of next node satisfies the constraints. Given below is optimized implementation (it prunes the subtree if it is not satisfying contraints).

```
#include <stdio.h>
#include <stdlib.h>

#define ARRAYSIZE(a) (sizeof(a))/(sizeof(a[0]))

static int total_nodes;

// prints subset found
void printSubset(int A[], int size)
{
    for(int i = 0; i < size; i++)
    {
        printf("%*d", 5, A[i]);
    }

    printf("\n");
}

// qsort compare function
int comparator(const void *pLhs, const void *pRhs)
{
    int *lhs = (int *)pLhs;
    int *rhs = (int *)pRhs;

    return *lhs > *rhs;
}

// inputs
// s            - set vector
// t            - tuplet vector
// s_size       - set size
// t_size       - tuplet size so far
// sum          - sum so far
// ite          - nodes count
// target_sum   - sum to be found
void subset_sum(int s[], int t[],
                int s_size, int t_size,
                int sum, int ite,
                int const target_sum)
{
    total_nodes++;

    if( target_sum == sum )
    {
        // We found sum
        printSubset(t, t_size);

        // constraint check
        if( ite + 1 < s_size && sum - s[ite] + s[ite+1] <= target_sum )
        {
            // Exclude previous added item and consider next candidate
            subset_sum(s, t, s_size, t_size-1, sum - s[ite], ite + 1, target_sum);
        }
        return;
    }
    else
    {
        // constraint check
        if( ite < s_size && sum + s[ite] <= target_sum )
        {
            // generate nodes along the breadth
            for( int i = ite; i < s_size; i++ )
            {
                t[t_size] = s[i];

                if( sum + s[i] <= target_sum )
                {
                    // consider next level node (along depth)
                    subset_sum(s, t, s_size, t_size + 1, sum + s[i], i + 1, target_sum);
                }
            }
        }
    }
}

// Wrapper that prints subsets that sum to target_sum
void generateSubsets(int s[], int size, int target_sum)
{
    int *tuplet_vector = (int *)malloc(size * sizeof(int));

    int total = 0;

    // sort the set
    qsort(s, size, sizeof(int), &comparator);

    for( int i = 0; i < size; i++ )
    {
        total += s[i];
    }

    if( s[0] <= target_sum && total >= target_sum )
    {

        subset_sum(s, tuplet_vector, size, 0, 0, 0, target_sum);
    }

    free(tuplet_vector);
}

int main()
{
    int weights[] = {15, 22, 14, 26, 32, 9, 16, 8};
    int target = 53;

    int size = ARRAYSIZE(weights);

    generateSubsets(weights, size, target);

    printf("Nodes generated %d\n", total_nodes);

    return 0;
}
```

As another approach, we can generate the tree in fixed size tuple analogs to binary pattern. We will kill the sub-trees when the constraints are not satisfied.

– – – **Venki**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

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Puneet · 3 months ago

public class SubsetSum {

public static void main(String[] args) {

int[] a = new int[]{1, 1, 1, 1, 1, 1, 2, 3, 1, 1, 4, 5, 2, 1};

boolean[] b = new boolean[a.length];

subset(a, b, 0, 5);

}

private static void subset(int[] a, boolean[] b, int index, int sum) {

if (sum == 0) {

for (int i=0; i < a.length; i++) {

if (b[i]) System.out.print(a[i] + " ");

}

see more

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Ankit Bhodia • 3 months ago

https://ideone.com/Rdta1P

I think once we print the subset, we can directly backtrack rather than recursing after the print. I commented the recursive call after print and I think it works fine. Do let me know if I am wrong.

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Aditya Goel • 3 months ago

Can someone check why this code is not working - http://ideone.com/7lj8XT

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ell • 4 months ago

if the set include negative integers , cant it be implemented with backtrack algorithm ?

2 • Reply • Share ›

Gaurav Gupta • 10 months ago

http://ideone.com/9lXga4

using vectors and backtracking. Generates Subsets as well

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RACHIT SAXENA • a year ago

can one find all possible subsets of a big array say no. of elements =10^5 in less tha o(n^2)???

• Reply • Share ›

sujeet singh • a year ago

void print_subarray_sum(int *arr,int arr_size,int given_sum)

{

int sum =0;

shorting_array(arr,arr_size);

for(int i =0;i<arr_size;i++) for(int="" j="i;j&lt;arr_size;j++)" {="" sum="sum+*(arr+j);" if(sum="=given_sum)" {="" print_array(arr+i,j-i+1);="" return;="" }="" else="" if(sum="">
given_sum)

{

sum =0;break;

}

}

}

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AlienOnEarth • a year ago

Another solution without backtracking (using Algorithm of string combination)

#include<stdio.h>

#include<stdlib.h>

#include<stdbool.h>

void print(int arr[], int n)

{

int i;

printf("subset: ");

for(i=0;i<n;i++) {="" printf("%d,="" ",arr[i]);="" }="" printf("\n");="" }="" void="" printallsubsets(int="" arr[],="" int="" n,="" int="" output[],="" int="" start,="" int="" index,="" int="" sum,="" int="" k)="" {="" int="" i="0;" base="" case="" 1:="" sum="" subset="" not="" found="" if(k="">sum) see more

1 • Reply • Share ›

disqus_0z6aYV2hDC • a year ago

Isn't the following wrong;

if(target_sum == sum)

{

// We found subset

printSubset(t, t_size);

// Exclude previously added item and consider next candidate

subset_sum(s, t, s_size, t_size-1, sum - s[ite], ite + 1, target_sum);

return;

}

We called subset_sum with i = ite+1, and in the loop when sum == target_sum, we are calling subset_sum with again i = ite+1. We should have called it with i = ite. Also, the sum passed is sum - s[ite]. It should be sum-s[ite -1]

1 • Reply • Share ›

prashant • a year ago

int fun(int arr[],int low,int high,int s)

{

if(s==0)

return 1;

if(s<0)

return 0;

if(low==high)

{

if(arr[low]==s)

{

cout<<arr[low]<<" ";="" return="" 1;="" }="" else="" return="" 0;="" }="" if(arr[low]="">s) cout<<arr[low]<<" ";=" return=" 1;=" }=" else=" return=" 0;=" }=" if(arr[low]=" >s)

return fun(arr,low+1,high,s);

for(int k=low;k<high;k++) {="" int="" p="fun(arr,k+1,high,s-arr[k]);" if(p)="" {="" cout<<arr[k] <<"=" ";=" return=" 1;=" }=" }=" return=" 0;=" }=">

1 ⌃ | ⌄ • Reply • Share ›

prashant jha • a year ago

/*

int fun(int arr[],int low,int high,int s)

{

if(s==0)

return 1;

if(s<0)

return 0;

if(low==high)

{

if(arr[low]==s)

{

see more

⌃ | ⌄ • Reply • Share ›

Ajp • a year ago

```
private static void subSetSum(Integer[] arr, Integer[] tArr, int ts,
int sum, int ite, int num) {
if (num == sum) {
printSubset(tArr, ts);
} else if (sum < num) {
for (int i = ite; i < arr.length; i++) {
tArr[ts] = arr[i];
subSetSum(arr, tArr, ts + 1, sum + arr[i], i, num);
}
} else if (sum > num) {
// Nothing do
}
}
```

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C# Rules • a year ago

/// Logic is sort numbers and use this logic. Below method assumes that number are already sorted.

```
public void FindSubset(int sum, int tSum, int[] allInts,ArrayList posibbleset,int currentIndex)
{
if (sum == tSum) {PrintResult(posibbleset);}
else if (sum > tSum){
for (int i = currentIndex; i < allInts.Length; i++)
{
ArrayList ps = (ArrayList) posibbleset.Clone();
if (tSum + allInts[i] <= sum)
{
ps.Add(allInts[i]);
FindSubset(sum, tSum + allInts[i], allInts, ps, (i + 1));
}
else break;
}
}
```

see more

• Reply • Share ›

Vijay Kumar • a year ago

```
public static void select(int i,int n,int k,String output) {

if(k==0 && n==0){

count++;

System.out.println(output);

return;

}

if(i<1 || n<0 || k==0){return;}

select(i-1,n,k,output); // not choosing i

select(i-1,n-i,k-1,output+i +" ");

}
```

1 • Reply • Share ›

Guest · 2 years ago

```
#include <stdio.h>

#include <stdlib.h>

void sumOfSubsets(int x[],int w[],int n,int s,int k,int r,int m)

{

int i;

x[k]=1;

if(s+w[k]==m)

{

printf("SOLUTION : ");

for(i=1;i<=n;i++)
```

see more

^ | ˅ • Reply • Share ›

Guest · 2 years ago

I think this is a better solution if we are not allowed to modify the array by sorting it.

```
#include <stdio.h>
#include <stdlib.h>
void sumOfSubsets(int x[],int w[],int n,int s,int k,int r,int m)
{
int i;
x[k]=1;
if(s+w[k]==m)
{
printf("SOLUTION : ");
for(i=1;i<=n;i++)
{
if(i>k)
{
x[i]=0;
}
printf("%d ",x[i]);
```

see more

• Reply • Share ›

abhishek08aug • 2 years ago

Intelligent :D

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rahul23 • 2 years ago

@venki--
Firstly following isnt not needed i think..And it can be modified to handle duplicates if we need this...say 2,4,4 and we need to find 6...2 4 matched..now ite is at 3rd element i.e 4..wta we so....we do sum-s[ite-1],i.e exclude previous and + s[ite]..add next..agin we get 2,4.....correct me if wrng...
// it would not be needed in case unique elements....
if(ite + 1 < s_size && sum - s[ite] + s[ite+1] <= target_sum)
{
// Exclude previous added item and consider next candidate
subset_sum(s, t, s_size, t_size-1, sum - s[ite], ite + 1, target_sum);
}
return;
}

3 • Reply • Share ›

bugfinder • 2 years ago

isnt this code finding repeated same subsets which satisfies the constraint.

for example :

consider the tree : ..

1
/ \
2
/ \
6 7

suppose 6 leads to a subset ... Then
[code][/code]
if(target_sum == sum)
{
// We found sum
printSubset(t, t_size);

see more

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Balaji • 2 years ago

Thanks for explaining how to accomplish this through backtracking. Just to be more clear and to verify my belief - i'm thinking we can model this problem as the coin change problem which essentially asks in how many ways can you make change to a given sum 'N' using 'm' coins S1<S2<..<Sm, the solution to which is simply the recurrence C(N,m) = C(N,m - 1) + C(N - Sm,m)

• Reply • Share ›

kalyan • 3 years ago

if(ite + 1 < s_size && sum - s[ite] + s[ite+1] <= target_sum)
{
// Exclude previous added item and consider next candidate
subset_sum(s, t, s_size, t_size-1, sum - s[ite], ite + 1, target_sum);
}

i think this condition is redundant ?can some one tell me why is it required ...i ran it without that condition its running fine ..

• Reply • Share ›

Anil • 3 years ago

```
I feel below check is redundant and not required. Could u please explain the necessity of t

My point is

if we got subset whose sum is equal to target sum, then there is no question of being check

        if(ite + 1 < s_size && sum - s[ite] + s[ite+1] <= target_sum)
        {
            subset_sum(s, t, s_size, t_size-1, sum - s[ite], ite + 1, target_sum);
        }
```

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Anil ➦ Anil • 3 years ago

I am talking about optimized version.

Also in unoptimized version the return statement is redundant.

if(target_sum == sum)

{
// We found subset
printSubset(t, t_size);
// Exclude previously added item and consider next candidate
subset_sum(s, t, s_size, t_size-1, sum - s[ite], ite + 1, target_sum);
return; // AKT the return is not required redundant code
}

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dreamer Anil • 3 years ago

exactly my point.. this statement is not required.. the exclusion statement... it doesnt make any difference .. plus we should exclude "s[ite-1]" here.. coz it is the previously added item.. wasted hours figuring out why we are excluding s[ite] when we haven't included it yet

```
/* Paste your code here (You may delete these lines if not writing code) */
```

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Mayautobot • 3 years ago

```
 //this program prints all subsets in the array that sum to the given sum

#include<iostream>

using namespace std;

void Tour(int arr[],int size,int reSum)
{
        int *stack=new int[size];
        int sp=-1;
        int sum=0;
        int arrp;

        for(int i=0;i<size;i++)
        {
                sp=-1;
                stack[++sp]=i;
                sum+=arr[stack[sp]];
```

see more

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spark9 • 3 years ago

your code will not work properly when the set will contain 0.

like
[1,2,0] target sum = 3.

```
/* Paste your code here (You may delete these lines if not writing code) */
```

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kartik ➦ spark9 • 3 years ago
@spark9: I tried the code for your input. It seems to be working. It produced two subsets as output:

1 2 0
1 2

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spark9 ➦ kartik • 3 years ago
Did you try with the first version of the code (the one without sorting) ?

```
/* Paste your code here (You may delete these lines if not writing code) */
```

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sachin • 3 years ago
in the first code,
what if sum becomes greater than the target sum,then in that case also,it should exclude current element and consider next?

Reply • Share ›

Venki • 3 years ago
@Arvind, thanks for pointing this. You have provided a set with duplicated entries. It was assumed that the elements are unique.

With the duplicated entries, during branch generation upon step back by excluding the current element it is assumed that the current element is excluded to consider next elements. In case of duplicate elements it is not true.

I have updated the problem description.

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Arvind • 3 years ago
The code doesn;t work Example:

For
int weights[] = {10, 7, 5, 18, 12, 12, 15};

Output
18 7 10
18 7 12
18 5 12
18 5 12
Nodes generated 55

Which is not right

^ | ⌄ • Reply • Share ›

Arvind ➦ Arvind • 3 years ago

```
 if( target_sum == sum )
{
    // We found sum
    printSubset(t, t_size);

    // constraint check
    if( ite + 1 < s_size && sum - s[ite] + s[ite+1] <= target_sum )
    {
        // Exclude previous added item and consider next candidate
        subset_sum(s, t, s_size, t_size-1, sum - s[ite], ite + 1, target_sum);
    }
    return;
}
This is not right. why do we do  Exclude previous added item and consider next candi

Output for {10, 7, 5, 18, 12, 12, 15};
   10    7   18 (sum = 35)
   10    7   12 (sum = 29)
    5   18   12 (sum = 35)
    5   18   12
Nodes generated 124
```

^ | ⌄ • Reply • Share ›

Arvind • 3 years ago

The code doesn't work

^ | ⌄ • Reply • Share ›

kamlesh • 4 years ago

what is the time complexity of Backtracking Algorithm for Subset Sum

^ | ⌄ • Reply • Share ›

Sambasiva • 4 years ago

http://effprog.blogspot.com/20...

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Venki Sambasiva • 4 years ago

@Sambasiva, thanks for the comment. We are missing your comments on GFG :). Welcome back.

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Sambasiva Venki • 4 years ago

Hi Venki,

Thank you. Really nice posts.

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rahul • 4 years ago

@Venki isn't the complexity exponential can u write exact complexity , also

Reply • Share ›

Venki rahul • 4 years ago

@rahul, I hope Karthik clarified your query.

Adding further, backtracking explores all solution vectors in a systematic manner. It will stop generating subtrees (pruning) when implicit constraints are not satisfied. So that significant amount of time will be saved. However, in the worst case we need to explore all possibilities. Try to generate the worst case test data for above subset sum problem :) .

Where as in brute force search, we will check each and every option. To understand the fact, try the above two algorithms with the following input

```
 int weights[] = {2, 7, 5, 1, 3};
int target = 7;
```

There are two solutions for the above problem. Set {2, 7} and set {7}. The first program generates 28 nodes (including unnecessary leaf node calls) and second program generates 12 nodes.

Usually the complexity of backtracking problems will be O(P(n) * n!) or O(P(n) * 2^n) where P(n) is polynomial in 'n' that depends on the way nodes are generated.

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kartik rahul • 4 years ago

@rahul: Time complexity is exponential in worst case. Same is the case with all other Backtracking algos like Rat in a Mzae, Knight Tour, N Queen.. etc

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  forgot to see that part ;)

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