```
ln[2]:= y1[d_{-}, x_{-}, r1_{-}] := \frac{d}{r1} - \frac{1-r1}{r1} x;
     y2[d_{-}, x_{-}, r2_{-}] := \frac{d}{1-r^{2}} - \frac{r2}{1-r^{2}}x;
     x1[d_{-}, y_{-}, r1_{-}] := \frac{d}{1 - r1} - \frac{r1}{1 - r1} y;
     x2[d_{-}, y_{-}, r2_{-}] := \frac{d}{r^{2}} - \frac{1-r2}{r^{2}}y;
ln[6]:= p[x, s] := PDF[BetaDistribution[2, 2], x/s]:
\ln[7] = \text{pi2}[d_{,s_{,r_{2}}}] := \text{Evaluate@FullSimplify} \left[ \int_{\theta}^{x_{2}[d,\theta,r_{2}]} \int_{\theta}^{y_{2}[d,x,r_{2}]} p[x,s] * p[y,s] \, dy \, dx \right];
      piS[d_{-}, s_{-}, r1_{-}, r2_{-}] := Evaluate@FullSimplify \left[ \int_{0}^{d} \int_{0}^{y_{2}[d, x, r2]} p[x, s] * p[y, s] dy dx + \int_{0}^{x_{1}[d, 0, r1]} \int_{0}^{y_{1}[d, x, r1]} p[x, s] * p[y, s] dy dx \right]; 
In[9]:= results = ParallelTable[{α, {
                  With [\{q = \alpha, s = 5, d = 1/3\}, NSolve[\{D[pi2[d, s, r], r] + (q-1) * D[piS[d, s, r, r], r] == 0, r > d/s, r < 1/2, r \in Reals\}, r]],
                     With [\{q = \alpha, s = 5, d = 1/3\}, NSolve[\{D[pi2[d, s, r2], r2] + (q-1) * D[piS[d, s, r1, r2], r2] == 0, r2 > d/s, r2 < 1/2, r2 \in Reals\}, r2]]
              } // Timing}, \{\alpha, \{105/100, 11/10, 3/2, 2, 25/10, 3, 4, 6, 8, 10\}\}\};
      time = Total[Map[Function[r, r[[2]][[1]]], results]];
      Grid[Prepend[Map[Function[r, Flatten[{r[[1]], r[[2]][[2]][[1]], r[[2]][[2]], r[[2]][[1]]}]], results],
                                                                                                                \{"\alpha", "results r^*", "results r^E", "CPU time"\}], Frame <math>\rightarrow All]
```

## StringForm["Total CPU time was ``s", time]

|a| = \$Assumptions = a > 1 & 0 < r1 < 1 / 2 & 0 < r2 < 1 / 2 & 0 < d < s & d < r2 \* s.

Out[11]=	α	results r*	results r <sup>E</sup>	CPU time
	2 <u>1</u>	r → 0.49079	r2 → 0.495285	0.053592
	11	$r \rightarrow \textbf{0.482387}$	r2 → 0.49079	0.027963
	3 2	r → 0.43342	r2 → 0.460895	0.053261
	2	$r \rightarrow 0.395437$	r2 → 0.43342	0.030082
	<u>5</u> 2	$r \rightarrow 0.369224$	r2 → 0.412386	0.054996
	3	$r \rightarrow 0.349411$	$r2 \rightarrow 0.395437$	0.053777
	4	$r \rightarrow 0.320523$	$r2 \rightarrow \textbf{0.369224}$	0.027517
	6	$r \rightarrow 0.283816$	r2 → 0.333602	0.026635
	8	$r \rightarrow 0.26014$	$r2 \rightarrow 0.309417$	0.018948
	10	r → 0.24296	$r2 \rightarrow \textbf{0.291344}$	0.019273

Out[12]= Total CPU time was 0.366044`s