Quantum State Preparation of W-State

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Introduction

- Quantum State preparation, an important starting point for a lot of algorithms, is a broad topic and we confine ourselves to the W-State preparation. The W-State holds significance due it's robustness in keeping entanglement.[1]
- Hypothesis - As the depth of circuit and the complexity of gates rise then the fidelity would get worse and runtime would grow.
- 4 circuits ranging from 3 to 8 qubits were analysed for the W-state preparation. To measure fidelity, noise was introduced in 1-qubit gates and in the 2-qubit CNOT gate.

Entanglement and the W State

Quantum computers have an advantage over classical computers by utilizing two properties: **superposition** and **entanglement**. While there are many quantum states which have some degree of entanglement, two well known entangled multipartite states have garnered much attention in the quantum computing community: the **GHZ state** (for 3-qubits):

$$\frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

and the **W state** (for 3-qubits):

$$\frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$

One major advantage of the W and GHZ states is that they can be prepared on multipartite systems of an arbitrary number of qubits. While the W state is not as entangled as the GHZ state by many measures[1] it is more robust, and will not collapse after a single measurement, which can be useful for many applications including quantum communication and cryptography[2].
In order to best prepare circuits for these applications, the W state must be efficiently and accurately applied.[8]

Noise Sources and Fidelity Measurement

- The state fidelity F for density matrix input states ρ_1, ρ_2 is given by[3]

$$F(\rho_1, \rho_2) = Tr[\sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}}]^2$$

- A custom noise model was implemented for the simulations. While this is not a real noisy device, it gives a better trend analysis for the purpose of this project. For this project, 1-qubit X, Y, or Z errors were applied with probability p_1 and 2-qubit CNOT errors were applied with probability p_2. [4][5]

References

[1] Briegel, H. J., and Raussendorf, R. (2001). Persistent entanglement in arrays of interacting particles. Physical Review Letters, 86(5), 910.

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[3] Qiskit Development Team. (2022). State Fidelity. qiskit.org. Retrieved December 15, 2022, from https://qiskit.org/documentation/stubs/qiskit.quantum_info.state_fidelity.html

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[5] QuTech. (n.d.) Error models. Quantum-inspire.com. Retrieved December 15, 2022, from https://www.quantum-inspire.com/kbase/cqasm-error-models/

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Simulating W State Preparation

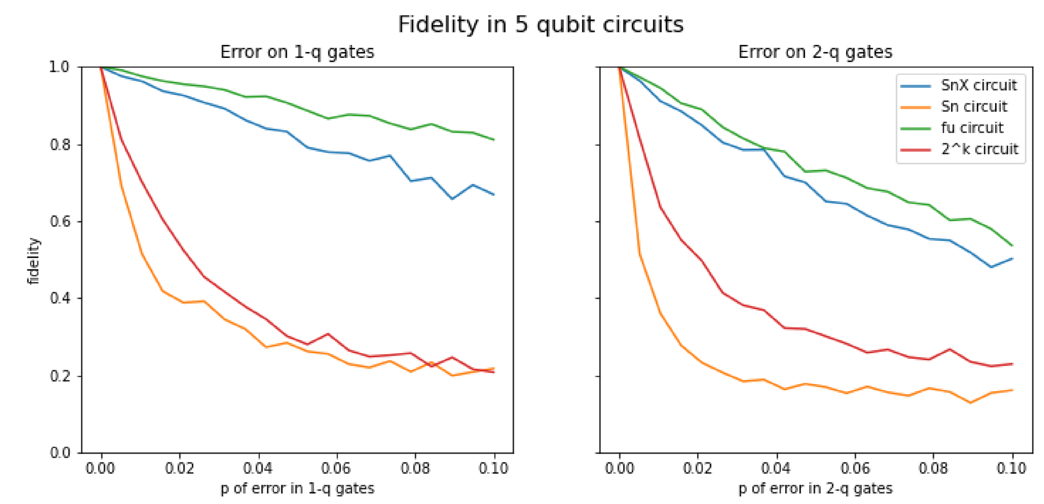


Fig. 1: **A**The fidelity as a function of 1-qubit gate error **B** Fidelity as a function of 2-qubit gate error

- We compare the fidelity of the 4 different circuits with simulating an 5 qubit W-state preparation. S_nX and FU follow a similar linear decrease in fidelity while the S_n and 2^k circuit had a exponential decrease in fidelity.

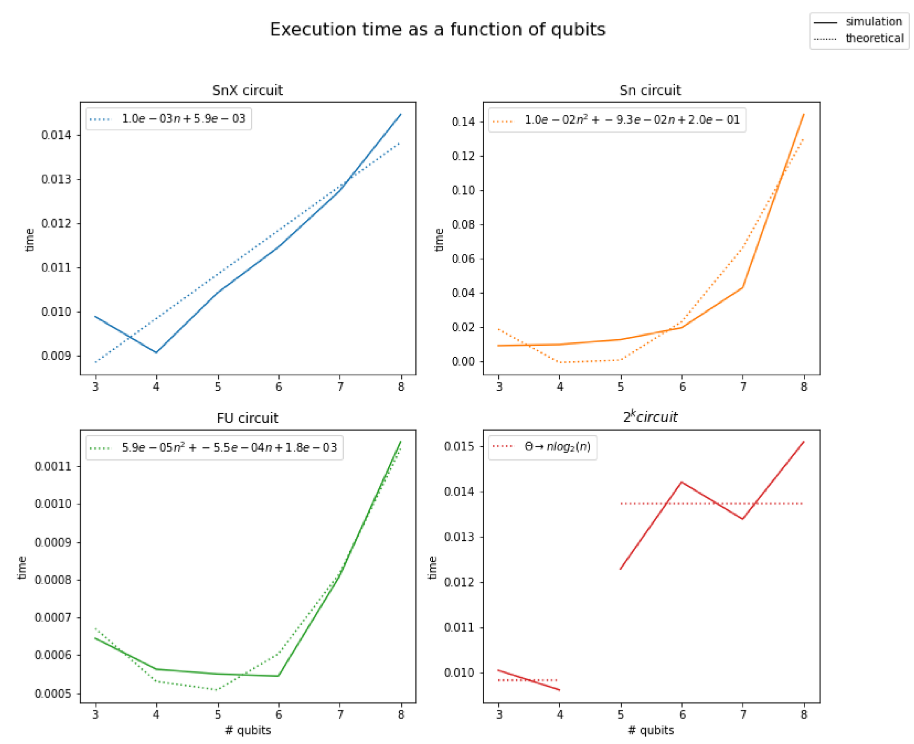


Fig. 2: Execution time vs the number of qubits plotted with the theoretical results

We find that:

- The FU circuit grows quadratically and the time implementation is 2 orders of magnitude lower than any other circuit.
- The depth of the S_n circuit is linear but the time dependence is worse than the rest of the circuits because of the complexity of the Sn controlled gates. The 2^k circuit follows a stairway trend for both of our simulation for 3 to 8 qubits.

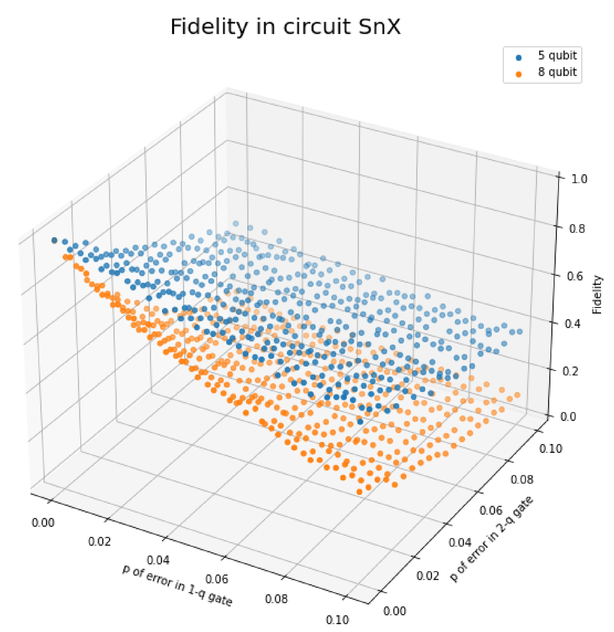
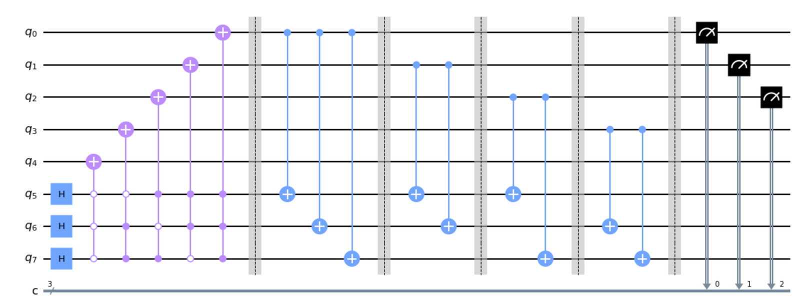


Fig. 3: Fidelity vs PoE in 1 qubit gate and PoE in 2-qubits gate for the W-state preparation in the S_nX circuit.

Circuit Algorithms and Structures for $|W_5\rangle$

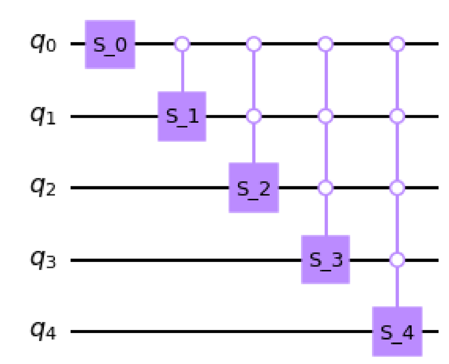
2^k circuit



- Standard gates
- More qubits needed
- k controlled gates
- Depth: $C_z(k)n$
- Probabilistic results

Fig. 4: 2k circuit preparing $|W_5\rangle$

Sn circuit

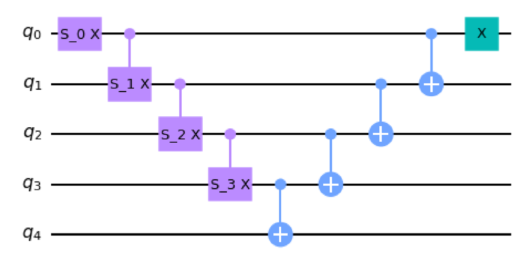


$$S_n = \frac{1}{\sqrt{n}}\begin{bmatrix}\sqrt{n-1} & 1 \\ 1 & -\sqrt{n-1}\end{bmatrix}$$

- Non-standard gates
- Multi-controlled gates
- Depth O(n)

Fig. 5: **A** Sn circuit preparing $|W_5\rangle$. **B** Matrix for the unitary splitter gate S_n where n = number of qubits.

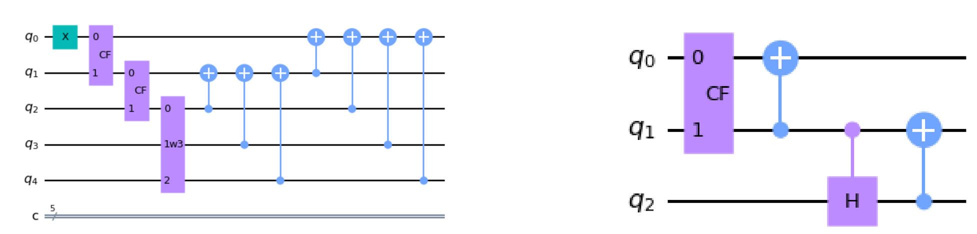
SnX circuit



- Non-standard gates
- Multi-controlled gates
- Depth O(n)

Fig. 6: Sn circuit preparing $|W_5\rangle$. Optimal construction of $|W_5\rangle$ [2].

FU circuit



$$CF = \begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & \sin\theta & 0 & -\cos\theta\end{bmatrix}$$

Fig. 7: **A** FU circuit **B** $W3$ circuit **C** CF matrix [6][7]

- Non-standard gates
- Singly controlled gates
- Depth $O(n^2)$

Conclusion

- There is a cost-benefit to each circuit. Implementing an ideal W-state requires optimizing it to the specific hardware or machine it is being run on.
- According to the simulation the scaling of the circuits behaved as expected compared to the theoretical data. To make the results even more accurate, we could have implemented additional or more specific sources of noise; however we felt that it was best to isolate variables where possible.