

VXB Vectors

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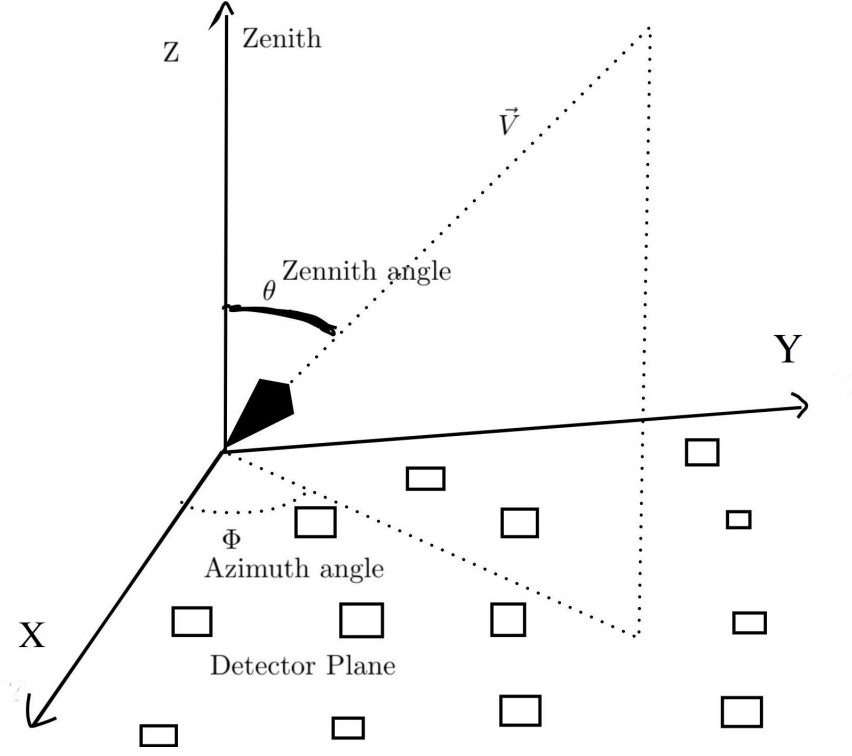


Figure 1: Detector to shower plane

To start the coordinate transformation from the detector plane to the shower plane, the unit vector of the velocity of the incoming shower is to be described in the detector plane co-ordinates noting the zenith angle θ with respect to the positive z-axis and the azimuth angle ϕ taken with respect to the positive x-axis. The $-\mathbf{v}$ is then set to be the new zenith via the following transformation where $|\mathbf{v}|$ is the length of the velocity vector:

$$\hat{\mathbf{z}}' = -\mathbf{v} = \frac{-1}{|\mathbf{v}|} \begin{pmatrix} v \sin \theta \cos \phi \\ v \sin \theta \sin \phi \\ v \cos \theta \end{pmatrix} \quad (1)$$

The magnetic field unit \mathbf{B} vector can be written out explicitly as:

$$\hat{\mathbf{B}} = \frac{1}{|\mathbf{B}|} \begin{pmatrix} B \sin \theta_B \cos \phi_B \\ B \sin \theta_B \sin \phi_B \\ B \cos \theta_B \end{pmatrix}, \quad (2)$$

To generate a second axis that is perpendicular to both the $\hat{\mathbf{v}}$ and and magnetic field $\hat{\mathbf{B}}$, we get the cross product of both unit vectors.

$$\hat{\mathbf{x}}' = \hat{\mathbf{z}}' \times \hat{\mathbf{B}} = \det \begin{pmatrix} i & j & k \\ -\sin \theta \cos \phi & -\sin \theta \sin \phi & -\cos \theta \\ \sin \theta_B \cos \phi_B & \sin \theta_B \sin \phi_B & \cos \theta_B \end{pmatrix} \quad (3)$$

$$\hat{\mathbf{x}}' = \begin{pmatrix} -\sin \theta \sin \phi \cos \theta_B + \sin \theta_B \sin \phi_B \cos \theta \\ \sin \theta \cos \phi \cos \theta_B - \sin \theta_B \sin \phi_B \cos \theta \\ \sin \theta \sin \phi \sin \theta_B \sin \phi_B - \sin \theta \sin \theta_B \sin \phi_B \cos \theta \end{pmatrix} \quad (4)$$

The third axis is then a cross product of $\hat{\mathbf{z}}$ and $\hat{\mathbf{x}}$, this creates a 3rd mutually perpendicular axis to both $\hat{\mathbf{z}} \times \hat{\mathbf{x}}$ i.e. $-\hat{\mathbf{v}} \times -\hat{\mathbf{v}} \times \hat{\mathbf{B}}$.

$$\hat{\mathbf{y}}' = \begin{pmatrix} (\sin \theta \cos \phi \cos \theta_B - \sin \theta_B \sin \phi_B \cos \theta) \cos \theta - (\sin \theta \sin \phi \sin \phi_B \sin \phi_B - \sin \theta \sin \theta_B \sin \phi_B \cos \phi) \sin \theta \sin \phi \\ -(-\sin \theta \sin \phi \cos \theta_B + \sin \theta_B \sin \phi_B \cos \theta) \cos \theta + (\sin \theta \sin \phi \sin \theta_B \sin \phi_B - \sin \theta \sin \theta_B \sin \phi_B \cos \phi) \sin \theta \cos \phi \\ (-\sin \theta \sin \phi \cos \theta_B + \sin \theta_B \sin \phi_B \cos \theta) \sin \theta \sin \phi - (\sin \theta \cos \phi \cos \theta_B - \sin \theta_B \sin \phi_B \cos \theta) \sin \theta \cos \phi \end{pmatrix} \quad (5)$$

In an orientation that will give rise to the relationship $\hat{\mathbf{z}}' \times \hat{\mathbf{y}}' = -\hat{\mathbf{x}}'$ and $\hat{\mathbf{x}}' \times \hat{\mathbf{y}}' = \hat{\mathbf{z}}'$ and can be tested for orthogonality via the dot product = 0 test.

For the Aires and ZHAires code, the magnetic field unit vector \mathbf{B} is given to have the following components:

$$\hat{\mathbf{B}} = \frac{1}{|\mathbf{B}|} \begin{pmatrix} B \cos \theta_B \\ 0 \\ -B \sin \theta_B \end{pmatrix} \quad (6)$$

This gives;

$$\hat{\mathbf{x}}' = \begin{pmatrix} \sin \phi \sin \theta \sin \theta_B \\ -\sin \phi * \sin \theta_B \cos \theta - \cos \phi \cos \theta_B \\ \sin \phi \sin \theta \cos \theta_B \end{pmatrix} \quad (7)$$

$$\hat{\mathbf{y}}' = \begin{pmatrix} (-\sin \phi \sin \theta_B \cos \theta - \cos \phi \cos \theta_B) \cos \phi - \sin^2 \phi \sin^2 \theta \cos \theta_B \\ \sin^2 \phi \sin \theta \cos \theta \cos \theta_B - \sin \phi \sin \theta \sin \theta_B \cos \phi \\ -(-\sin \phi \sin \theta_B \cos \theta - \cos \phi \cos \theta_B) \sin \phi \cos \theta + \sin^2 \phi \sin^2 \theta \sin \theta_B \end{pmatrix} \quad (8)$$

With the orientation of the magnetic field taking down i.e. pointing to the center of the earth, as positive, θ_B is the magnetic field inclination angle.

$$T_{\text{det to shower plane co-or transformation}} = \begin{bmatrix} \hat{\mathbf{x}}'_1 & \hat{\mathbf{x}}'_2 & \hat{\mathbf{x}}'_3 \\ \hat{\mathbf{y}}'_1 & \hat{\mathbf{y}}'_2 & \hat{\mathbf{y}}'_3 \\ \hat{\mathbf{z}}'_1 & \hat{\mathbf{z}}'_2 & \hat{\mathbf{z}}'_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$