Measuring Chaos

Andrew Wilson, Supervised by Dr. Anthony Quas

University of Victoria | Mathematics & Statistics

This research was supported by the Jamie Cassels Undergraduate Research Award

What is Chaos?

Roughly speaking, we say that a system is chaotic if it is sensitive to initial conditions. That is, in a deterministic system, if small changes are made to an input, we expect to see vastly different outputs.

A 'system' can roughly be thought of as a set of points and a map that moves those points around.

For example: air molecules in the atmosphere and the wind, or the position of the end of a double pendulum under the influence of gravity.

Modelling Chaos: Dynamical Systems

In order to study chaos, we need models of the systems we would like to investigate. How can we abstract some of the previous examples in a way that can be studied mathematically?

Definition: A Dynamical System is a pair (X, T) where X is a set of points, and T is function from X to X.

Formalizing our first example from before, define

$$X = \{\text{air particles in the atmosphere}\}$$

and T to map an air particle to its position after being acted on by the wind for one second.

The double pendulum system happens to be a well studied dynamical system. So much so, that the map T has been dubbed 'The Standard Map'. (Include Formula?). Below are some illustrations of the orbits of points under T.

Figure 2: Caption

Caption

Stable and Unstable Manifolds

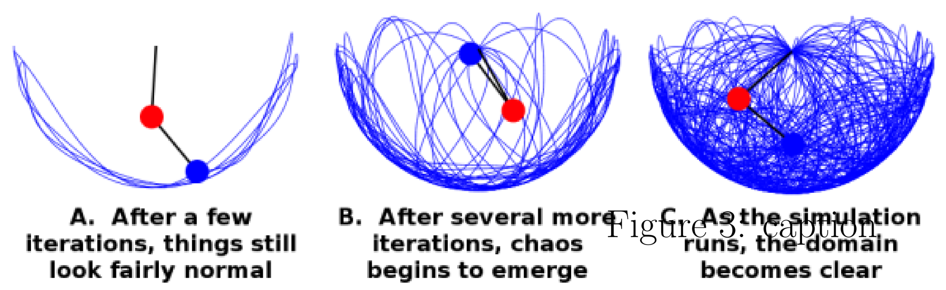

Figure 3: Caption

Figure 1

textblock

textblock

textblock

Some Examples of Manifolds

Manifolds along the orbit of a periodic point.

Manifolds along chaotic point

textblock

Computing Manifolds(?)/

textblock

References

large minimum degree, 2015, v3, arXiv:1503.08191.