PROBABILITY: Homework #1

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Problem 1

(a) denote H for heads and T for tails.

$$\begin{aligned} S = \{&(H,H,H),(H,H,T),(H,T,H),(H,T,T)\\ &(T,H,H),(T,H,T),(T,T,H),(T,T,T)\} \end{aligned}$$

(b)

$$\begin{aligned} A &= \{(H,H,T),(H,T,H),(T,H,H),(H,H,H)\}\\ B &= \{(H,H,T),(H,H,H)\}\\ C &= \{(H,H,T),(H,T,T),(T,H,T),(T,T,T)\} \end{aligned}$$

(c)

1) $A^c = \{(H,T,T),(T,H,T),(T,T,H),(T,T,T)\}$
2) $A \cap B = \{(H,H,T),(H,H,H)\} = B$
3) $A \cup C = \{(H,H,T),(H,T,H),(T,H,H),(H,H,H),(H,T,T),(T,H,T),(T,T,T)\}$

Problem 2

(a) denote H for heads and T for tails.

$$\begin{aligned} &\text{number of sample space is } \binom{52}{5}\\ &\text{number of choices of suits is } \binom{4}{1}\\ &P = \frac{\#of suits}{\#of sample\ space}\\ &= \frac{\binom{4}{1}}{\binom{52}{5}}\\ &= \frac{1}{649740} \end{aligned}$$

(b)

list possible situations as (A2345)...(10JQKA),we know there are 10 choices of value and 4 choices of suit. But we should exclude Royal Flush

$$\begin{aligned} P &= \frac{10*4-4}{\binom{52}{5}}\\ &\approx 1.39 \times 10^{-5} \end{aligned}$$

(c)

there are 13 choices of value

as for the other card, there are 48 choices

$$P = \frac{13 * 48}{\begin{pmatrix} 52 \\ 5 \end{pmatrix}}$$

$$= \frac{1}{4165}$$

(d)

since the 5 cards have same suits, they should not have same value

there are $\begin{pmatrix} 13 \\ 5 \end{pmatrix}$ choices of value

there are 4 choices of suit

What's more, we should exclude Royal Flush and Straight Flush

$$P = \frac{\begin{pmatrix} 13 \\ 5 \end{pmatrix} * 4 - 4 - 36}{\begin{pmatrix} 52 \\ 5 \end{pmatrix}}$$

$$= \frac{1277}{649740} \approx 1.97 \times 10^{-3}$$

(e)

Choose 3 same cards first:There are 13 choices of value and $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ choices of suit

As for the other two cards:There are $\begin{pmatrix} 52-4 \\ 2 \end{pmatrix}$ choices

$$P = \frac{13 * 4 * \begin{pmatrix} 52-4 \\ 2 \end{pmatrix}}{\begin{pmatrix} 52 \\ 5 \end{pmatrix}}$$

$$= \frac{94}{4165}$$

(f)

there are $\begin{pmatrix} 13 \\ 2 \end{pmatrix}$ choices of value.

there are $\begin{pmatrix} 4 \\ 2 \end{pmatrix} * \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ choices of value.

as for the last cards, there are only 52-4-4 choices.

$$P = \frac{\begin{pmatrix} 13 \\ 2 \end{pmatrix} * \begin{pmatrix} 4 \\ 2 \end{pmatrix} * \begin{pmatrix} 4 \\ 2 \end{pmatrix} * (52 - 4 - 4)}{\begin{pmatrix} 52 \\ 5 \end{pmatrix}}$$

$$= \frac{198}{4165}$$

Problem 3

(a)

there are 16 choices for women president. And there are 48 choices for president

$so\ Pr(E) = \frac{16}{48} = \frac{1}{3}$

there are 32 choices for men vice president. And there are 48 choices for vice president

$so\ Pr(F) = \frac{32}{48} = \frac{2}{3}$

there are$16 * 15 + 32 * 31$ choices for president of same sex

there are 48*47 choices for president

$so\ Pr(G) = \frac{16 * 15 + 32 * 31}{48 * 47}$

$= \frac{77}{141}$

(b)

$E \cap F$ represent president is woman and vice president is man

there are 16*32 choices for this situation

$E \cup F$ represent president is woman or vice president is man

there are 16*32+16*15+32*31 choices for this situation

$E \cap F \cap G$ does not make sense

there are 0 choices for this situation

$Pr(E \cap F) = \frac{16 * 32}{48 * 47} = \frac{32}{141}$

$Pr(E \cup F) = \frac{16 * 32 + 16 * 15 + 32 * 31}{48 * 47} = \frac{109}{141}$

$Pr(E \cap F \cap G) = 0$

(c)

$$Pr(G|E\cup F)=\frac{Pr(G\cap(E\cup F))}{Pr(E\cup F)}$$

$G\cap(E\cup F)$represent two president are of same sex $=G$

$$therefore\ Pr(G|E\cup F)=\frac{Pr(G)}{Pr(E\cup F)}=\frac{\frac{77}{141}}{\frac{109}{141}}=\frac{77}{109}$$

Problem 4

consider the event as inserting four adjacent aces into 48 shuffled cards

# of order of adjacent aces are 4*3*2*1

# of choices of inserting the four adjacent are 49!

in sample space is 52!

$$therefore\ P=\frac{4*3*2*49!}{52!}\approx 1.81\times 10^{-4}$$

Problem 5

(a)

$$\#\text{ of the sample space }=\binom{60}{30}$$

there are$2\times\binom{60-5}{30}$choices for this situation.

$$so\ P=\frac{2\times\binom{55}{30}}{\binom{60}{30}}$$

$$=\frac{117}{2242}$$

(b)

$$\#\text{ of the sample space }=\binom{60}{30}$$

there are$2*\binom{5}{4}*\binom{60-5}{30-4}$choices for this situation.

$$so\ P=\frac{2\times\binom{5}{4}*\binom{60-5}{30-4}}{\binom{60}{30}}$$

$$=\frac{675}{2242}$$

(c)

$$\# \text{ of the sample space} = \binom{60}{30}$$

there are $2 \times \binom{60-5}{30-1}$ choices for this situation.

$$so\ P = \frac{2 \times \binom{60-5}{30-1}}{\binom{60}{30}}$$

$$= \frac{135}{2242}$$

Problem 6

(a)

this event could be either first up then down or first down then up

$so\ P = p * (1-p) + (1-p) * p$

$= 2p(1-p)$

(b)

there should be 1 day down and 2 days up

$so\ P = \binom{3}{1} * p^2 * (1-p)$

$= 3p^2(1-p)$

(c)

denote E for price increasing on the first day,then $Pr(E) = p$

denote F for after three days price increased by 1,then $Pr(F) = 3p^2(1-p)$

the probability that the first day price goes up and three days later price still increased by 1 should be

$Pr(E \cap F) = p * \binom{2}{1} * p * (1-p)$

$= 2p^2(1-p)$

$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$

$= \frac{2p^2(1-p)}{3p^2(1-p)}$

$= \frac{2}{3}$

Problem 7

under strategy (a),the answer could be correct when either husband or wife gives the correct answer.

$so\ P = \frac{1}{2} * p + \frac{1}{2} * p = p$

under strategy (b),the answer could be correct in the following situations:.

1)husband correct wife correct and either of their answer to be given.

2)husband correct wife wrong and husband's answer is given.

3)wife correct husband wrong and wife's answer is given.

$so\ P = p * p + \frac{1}{2} * p * (1-p) + \frac{1}{2} * p * (1-p) = p$

therefore,the two strategies have same efficiency.

Problem 8

(1)

$$Pr(agree) = p * p + (1-p) * (1-p) = 2p^2 - 2p + 1$$

from problem 7 we know $Pr(correct) = p$

$$Pr(correct|agree) = \frac{Pr(correct \cap agree)}{Pr(agree)} = \frac{p * p}{2p^2 - 2p + 1} = \frac{9}{13}$$

(2)

$$Pr(disagree) = p * (1-p) + (1-p) * p = 2p(1-p)$$

$$Pr(correct \cap disagree) = \frac{1}{2} * p * (1-p) + \frac{1}{2}(1-p) * p = p(1-p)$$

$$Pr(correct|disagree) = \frac{Pr(correct \cap disagree)}{Pr(disagree)} = \frac{p(1-p)}{2p(1-p)} = \frac{1}{2}$$

Problem 9

we will calculate the chance that there is no head

$Pr_{(no\ head)} = (1-p)^n$.

in contrast, the probability that at least one head is

$Pr_{(at\ least\ one\ head)} = 1 - (1-p)^n$

$1 - (1-p)^n \geq 0.5 \Leftrightarrow (1-p)^n \leq 0.5$

$1 - p \in [0, 1];\ so\ n * log(1-p) \leq log(0.5)$

$\Leftrightarrow n \geq \frac{log(\frac{1}{2})}{log(1-p)}$

and p could not equal to 1

Problem 10

denote the event a silver coin is found as E

$$Pr(E) = \frac{1}{3} * \frac{1}{2} + \frac{1}{3} * 1 = \frac{1}{2}$$

denote the event two silver coins is found as F

$$Pr(F) = \frac{1}{3}$$

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)}$$

$$= \frac{Pr(F)}{Pr(E)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Problem 11

(a)

let's separate this event into two parts:

a)a red ball is drawn from A; b)any ball other than red is drawn form A

$$then\ P = \frac{4}{4+3+2} * \frac{3}{3+3+4} + \frac{3+2}{4+3+2} * \frac{2}{2+3+4+1} = \frac{11}{45}$$

(b)

denote E as drawn red from A and F as drawn red from B

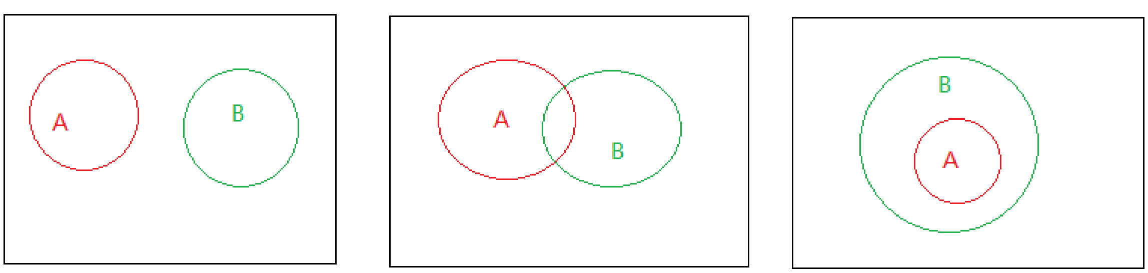
from part (a) we get $Pr(F) = \frac{11}{45}$

$$Pr(E \cap F) = \frac{4}{4+3+2} * \frac{3}{3+3+4} = \frac{2}{15}$$

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$$

$$= \frac{Pr(E \cap F)}{Pr(F)} = \frac{\frac{2}{15}}{\frac{11}{45}} = \frac{6}{11}$$

Problem 12



from the above venn diagram,we come to the following conclusion:
As for the right diagram, there is no intersection between A and B, but Pr(A)+Pr(B)=1.1>1, so there should be intersection
1) when A and B have the smallest intersection, $Pr(A \cup B) = 1$
$Pr(A \cap B) = Pr(A) + Pr(B) - Pr(A \cup B) = 0.1$
2) when B is a sub set of A, then $Pr(A \cap B)$ get the maximum value 0.4.

Problem 13

difference	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

as the graph above depicts, we get:
$$P = \frac{24}{36} = \frac{2}{3}$$

Problem 14

the number in sample space should be20^12
the situation that no box have more than one ball is the same as there are 12 boxes with one box each only and 8 empty boxes,but order matters.
so number of choices of this situation is $\begin{matrix} 20! \\ 12! \end{matrix}$

$$therefore\ P = \frac{\begin{pmatrix} 20! \\ 12! \end{pmatrix}}{20^12}$$
$$= 1.24 \times 10^{-6}$$

Problem 15

$$\text{the number in sample space should be} \binom{35}{10}$$

$$\text{number of choices of this situation is} \binom{35-2}{10-2} + \binom{35-2}{10}$$

$$= \binom{33}{8} + \binom{33}{10}$$

$$so\ P = \frac{\binom{33}{8} + \binom{33}{10}}{\binom{35}{10}}$$

$$= \frac{69}{119}$$

Problem 16

$$\text{the number in sample space should be} \binom{52}{13, 13, 13, 13}$$

$$\text{number of choices of this situation is} \binom{12}{3, 3, 3, 3} * \binom{40}{10, 10, 10, 10}$$

$$so\ P = \frac{\binom{12}{3, 3, 3, 3} * \binom{40}{10, 10, 10, 10}}{\binom{52}{13, 13, 13, 13}}$$

$$= \frac{\frac{12!}{(3!)^4} * \frac{40!}{(10!)^4}}{\frac{52!}{(13!)^4}}$$

$$= \frac{\frac{12!}{(3!)^4} * \frac{40!}{(10!)^4}}{\frac{52!}{(13!)^4}}$$

$$= \frac{148933}{4594023} \approx 0.0324$$

Problem 17

$$\text{the number in sample space should be} \binom{30*3}{10}$$

$$\text{number of choices of missing one color is } 3 \times \binom{30*3 \times 2}{10} - 2 * \binom{30}{10}$$

$$\text{number of choices of missing two color is } 3 * \binom{30}{10}$$

$$so\ P = \frac{\binom{30*2}{10} - 2 * \binom{30}{10} + 3 * \binom{30}{10}}{\binom{30*3}{10}}$$

$$= \frac{2357}{59590} \approx 0.040$$