

PROBABILITY: Homework #1

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Problem 1

(a) denote H for heads and T for tails.

$$S = \{(H, H, H), (H, H, T), (H, T, H), (H, T, T), \\ (T, H, H), (T, H, T), (T, T, H), (T, T, T)\}$$

(b)

$$A = \{(H, H, T), (H, T, H), (T, H, H), (H, H, H)\} \\ B = \{(H, H, T), (H, H, H)\} \\ C = \{(H, H, T), (H, T, T), (T, H, T), (T, T, T)\}$$

(c)

- 1) $A^c = \{(H, T, T), (T, H, T), (T, T, H), (T, T, T)\}$
- 2) $A \cap B = \{(H, H, T), (H, H, H)\} = B$
- 3) $A \cup C = \{(H, H, T), (H, T, H), (T, H, H), (H, H, H), (H, T, T), (T, H, T), (T, T, T)\}$

Problem 2

(a) denote H for heads and T for tails.

$$\text{number of sample space is } \binom{52}{5}$$

$$\text{number of choices of suits is } \binom{4}{1}$$

$$P = \frac{\#of suits}{\#of sample space} \\ = \frac{\binom{4}{1}}{\binom{52}{5}} \\ = \frac{1}{649740}$$

(b)

list possible situations as (A2345)...(10JQKA), we know there are 10 choices of value and 4 choices of suit. But we should exclude Royal Flush

$$P = \frac{10 * 4 - 4}{\binom{52}{5}} \\ \approx 1.39 \times 10^{-5}$$

(c)

there are 13 choices of value
 as for the other card, there are 48 choices

$$P = \frac{13 * 48}{\binom{52}{5}} \\ = \frac{1}{4165}$$

(d)

since the 5 cards have same suits, they should not have same value

there are $\binom{13}{5}$ choices of value

there are 4 choices of suit

What's more, we should exclude Royal Flush and Straight Flush

$$P = \frac{\binom{13}{5} * 4 - 4 - 36}{\binom{52}{5}} \\ = \frac{1277}{649740} \approx 1.97 \times 10^{-3}$$

(e)

Choose 3 same cards first: There are 13 choices of value and $\binom{4}{3}$ choices of suit

As for the other two cards: There are $\binom{52-4}{2}$ choices

$$P = \frac{13 * 4 * \binom{52-4}{2}}{\binom{52}{5}} \\ = \frac{94}{4165}$$

(f)

there are $\binom{13}{2}$ choices of value.

there are $\binom{4}{2} * \binom{4}{2}$ choices of value.

as for the last cards, there are only 52-4-4 choices.

$$P = \frac{\binom{13}{2} * \binom{4}{2} * \binom{4}{2} * (52 - 4 - 4)}{\binom{52}{5}}$$

$$= \frac{198}{4165}$$

Problem 3

(a)

there are 16 choices for women president. And there are 48 choices for president

$$\text{so } Pr(E) = \frac{16}{48} = \frac{1}{3}$$

there are 32 choices for men vice president. And there are 48 choices for vice president

$$\text{so } Pr(F) = \frac{32}{48} = \frac{2}{3}$$

there are $16 * 15 + 32 * 31$ choices for president of same sex

there are $48 * 47$ choices for president

$$\text{so } Pr(G) = \frac{16 * 15 + 32 * 31}{48 * 47}$$

$$= \frac{77}{141}$$

(b)

$E \cap F$ represent president is woman and vice president is man

there are $16 * 32$ choices for this situation

$E \cup F$ represent president is woman or vice president is man

there are $16 * 32 + 16 * 15 + 32 * 31$ choices for this situation

$E \cap F \cap G$ does not make sense

there are 0 choices for this situation

$$Pr(E \cap F) = \frac{16 * 32}{48 * 47} = \frac{32}{141}$$

$$Pr(E \cup F) = \frac{16 * 32 + 16 * 15 + 32 * 31}{48 * 47} = \frac{109}{141}$$

$$Pr(E \cap F \cap G) = 0$$

(c)

$$Pr(G|E \cup F) = \frac{Pr(G \cap (E \cup F))}{Pr(E \cup F)}$$

$G \cap (E \cup F)$ represent two president are of same sex = G

$$\text{therefore } Pr(G|E \cup F) = \frac{Pr(G)}{Pr(E \cup F)} = \frac{\frac{77}{141}}{\frac{109}{141}} = \frac{77}{109}$$

Problem 4

consider the event as inserting four adjacent aces into 48 shuffled cards

of order of adjacent aces are $4*3*2*1$

of choices of inserting the four adjacent are $49!$

in sample space is $52!$

$$\text{therefore } P = \frac{4 * 3 * 2 * 49!}{52!} \approx 1.81 \times 10^{-4}$$

Problem 5

(a)

$$\# \text{ of the sample space} = \binom{60}{30}$$

there are $2 \times \binom{60-5}{30}$ choices for this situation.

$$\begin{aligned} \text{so } P &= \frac{2 \times \binom{55}{30}}{\binom{60}{30}} \\ &= \frac{117}{2242} \end{aligned}$$

(b)

$$\# \text{ of the sample space} = \binom{60}{30}$$

there are $2 * \binom{5}{4} * \binom{60-5}{30-4}$ choices for this situation.

$$\begin{aligned} \text{so } P &= \frac{2 \times \binom{5}{4} * \binom{60-5}{30-4}}{\binom{60}{30}} \\ &= \frac{675}{2242} \end{aligned}$$

(c)

$$\begin{aligned}
& \# \text{ of the sample space} = \binom{60}{30} \\
& \text{there are } 2 \times \binom{60-5}{30-1} \text{ choices for this situation.} \\
& \text{so } P = \frac{2 \times \binom{60-5}{30-1}}{\binom{60}{30}} \\
& = \frac{135}{2242}
\end{aligned}$$

Problem 6

(a)

this event could be either first up then down or first down then up

$$\begin{aligned}
\text{so } P &= p * (1 - p) + (1 - p) * p \\
&= 2p(1 - p)
\end{aligned}$$

(b)

there should be 1 day down and 2 days up

$$\begin{aligned}
\text{so } P &= \binom{3}{1} * p^2 * (1 - p) \\
&= 3p^2(1 - p)
\end{aligned}$$

(c)

denote E for price increasing on the first day, then $Pr(E) = p$

denote F for after three days price increased by 1, then $Pr(F) = 3p^2(1 - p)$

the probability that the first day price goes up and three days later price still increased by 1 should be

$$\begin{aligned}
Pr(E \cap F) &= p * \binom{2}{1} * p * (1 - p) \\
&= 2p^2(1 - p) \\
Pr(E|F) &= \frac{Pr(E \cap F)}{Pr(F)} \\
&= \frac{2p^2(1 - p)}{3p^2(1 - p)} \\
&= \frac{2}{3}
\end{aligned}$$

Problem 7

under strategy (a), the answer could be correct when either husband or wife gives the correct answer.

$$\text{so } P = \frac{1}{2} * p + \frac{1}{2} * p = p$$

under strategy (b), the answer could be correct in the following situations:.

1) husband correct wife correct and either of their answer to be given.

2) husband correct wife wrong and husband's answer is given.

3) wife correct husband wrong and wife's answer is given.

$$\text{so } P = p * p + \frac{1}{2} * p * (1 - p) + \frac{1}{2} * p * (1 - p) = p$$

therefore, the two strategies have same efficiency.

Problem 8

(1)

$$Pr(\text{agree}) = p * p + (1 - p) * (1 - p) = 2p^2 - 2p + 1$$

from problem 7 we know $Pr(\text{correct}) = p$

$$Pr(\text{correct}|\text{agree}) = \frac{Pr(\text{correct} \cap \text{agree})}{Pr(\text{agree})} = \frac{p * p}{2p^2 - 2p + 1} = \frac{9}{13}$$

(2)

$$Pr(\text{disagree}) = p * (1 - p) + (1 - p) * p = 2p(1 - p)$$

$$Pr(\text{correct} \cap \text{disagree}) = \frac{1}{2} * p * (1 - p) + \frac{1}{2} * (1 - p) * p = p(1 - p)$$

$$Pr(\text{correct}|\text{disagree}) = \frac{Pr(\text{correct} \cap \text{disagree})}{Pr(\text{disagree})} = \frac{p(1 - p)}{2p(1 - p)} = \frac{1}{2}$$

Problem 9

we will calculate the chance that there is no head

$$Pr(\text{no head}) = (1 - p)^n.$$

in contrast, the probability that at least one head is

$$Pr(\text{at least one head}) = 1 - (1 - p)^n$$

$$1 - (1 - p)^n \geq 0.5 \Leftrightarrow (1 - p)^n \leq 0.5$$

$$1 - p \in [0, 1]; \text{ so } n * \log(1 - p) \leq \log(0.5)$$

$$\Leftrightarrow n \geq \frac{\log(\frac{1}{2})}{\log(1 - p)}$$

and p could not equal to 1

Problem 10

denote the event a silver coin is found as E

$$Pr(E) = \frac{1}{3} * \frac{1}{2} + \frac{1}{3} * 1 = \frac{1}{2}$$

denote the event two silver coins is found as F

$$Pr(F) = \frac{1}{3}$$

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)}$$

$$= \frac{Pr(F)}{Pr(E)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Problem 11

(a)

let's separate this event into two parts:

a) a red ball is drawn from A; b) any ball other than red is drawn from A

$$\text{then } P = \frac{4}{4+3+2} * \frac{3}{3+3+4} + \frac{3+2}{4+3+2} * \frac{2}{2+3+4+1} = \frac{11}{45}$$

(b)

denote E as drawn red from A and F as drawn red from B

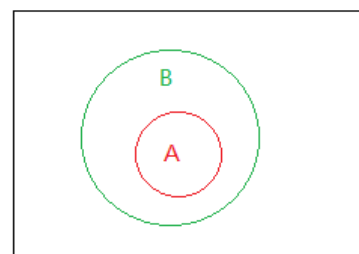
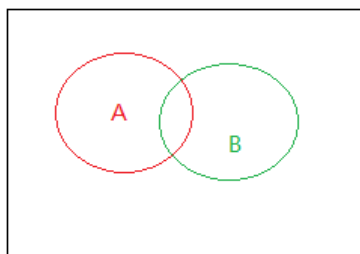
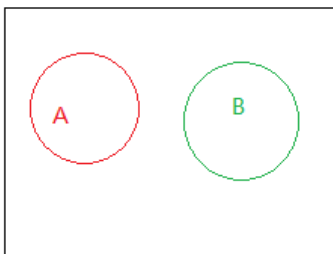
from part (a) we get $Pr(F) = \frac{11}{45}$

$$Pr(E \cap F) = \frac{4}{4+3+2} * \frac{3}{3+3+4} = \frac{2}{15}$$

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$$

$$= \frac{Pr(E \cap F)}{Pr(F)} = \frac{\frac{2}{15}}{\frac{11}{45}} = \frac{6}{11}$$

Problem 12



from the above venn diagram,we come to the following conclusion:

As for the right diagram, there is no intersection between A and B, but $\Pr(A)+\Pr(B)=1.1>1$, so there should be intersection

1) when A and B have the smallest intersection, $\Pr(A \cup B) = 1$

$$\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B) = 0.1$$

2) when B is a sub set of A, then $\Pr(A \cap B)$ get the maximum value 0.7.

Problem 13

difference	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

as the graph above depicts, we get:

$$P = \frac{24}{36} = \frac{2}{3}$$

Problem 14

the number in sample space should be 20^{12}

the situation that no box have more than one ball is the same as there are 12 boxes with one box each only and 8 empty boxes, but order matters.

so number of choices of this situation is $\frac{20!}{12!}$

$$\begin{aligned} \text{therefore } P &= \frac{\binom{20!}{12!}}{20^{12}} \\ &= 1.24 \times 10^{-6} \end{aligned}$$

Problem 15

$$\begin{aligned}
& \text{the number in sample space should be } \binom{35}{10} \\
& \text{number of choices of this situation is } \binom{35-2}{10-2} + \binom{35-2}{10} \\
& = \binom{33}{8} + \binom{33}{10} \\
& \text{so } P = \frac{\binom{33}{8} + \binom{33}{10}}{\binom{35}{10}} \\
& = \frac{69}{119}
\end{aligned}$$

Problem 16

$$\begin{aligned}
& \text{the number in sample space should be } \binom{52}{13, 13, 13, 13} \\
& \text{number of choices of this situation is } \binom{12}{3, 3, 3, 3} * \binom{40}{10, 10, 10, 10} \\
& \text{so } P = \frac{\binom{12}{3, 3, 3, 3} * \binom{40}{10, 10, 10, 10}}{\binom{52}{13, 13, 13, 13}} \\
& = \frac{\frac{12!}{(3!)^4} * \frac{40!}{(10!)^4}}{\frac{52!}{(13!)^4}} \\
& = \frac{\frac{12!}{(3!)^4} * \frac{40!}{(10!)^4}}{\frac{52!}{(13!)^4}} \\
& = \frac{148933}{4594023} \approx 0.0324
\end{aligned}$$

Problem 17

$$\begin{aligned}
& \text{the number in sample space should be } \binom{30 * 3}{10} \\
& \text{number of choices of missing one color is } 3 * \binom{30 * 3 * 2}{10} - 2 * \binom{30}{10} \\
& \text{number of choices of missing two color is } 3 * \binom{30}{10} \\
& \text{so } P = \frac{\binom{30 * 2}{10} - 2 * \binom{30}{10} + 3 * \binom{30}{10}}{\binom{30 * 3}{10}} \\
& = \frac{2357}{59590} \approx 0.040
\end{aligned}$$