LINEAR REGRESSION: Homework #3

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Problem 1 (2.2)

No, this conclusion does not imply that *X* and *Y* have no linear association. This result only tells us that *X* and *Y* are negative correlated, which means when *X* grows, value of *Y* decreases.

Problem 2 (2.23)

(a)

```
> anova(lm.pr19)
Analysis of Variance Table
Response: da$V1
           Df Sum Sq Mean Sq F value   Pr(>F)
da$V2       1  3.588  3.5878  9.2402 0.002917 **
Residuals 118 45.818  0.3883
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

	DF	SS	MS	F
Regression	1	3.588	3.5878	9.239763
Error	118	45.818	0.3883	
Total	119	49.406		

(b)

$$E(MSR) = \sigma^2 + \beta_1^2 \sum (X_i - \bar{X})$$

So $s^2 + \hat{\beta}_1^2 \sum (X_i - \bar{X})$ is estimated by *MSR*

$$E(MSE) = \sigma^2$$

So s^2 is estimated by *MSE*

When $\hat{\beta}_1 = 0$, *MSR* and *MSE* estimate the same quantity.

(c)

$$H_0 : \ \beta_1 = 0$$
$$H_1 : \ \beta_1 \neq 1$$
$$F^* = \frac{MSR}{MSE} = 9.239763$$

if $F^* \leq F(0.99, 1, 118)$, then conclude H_0

else $F^* > F(0.99, 1, 118)$, reject H_0

While $F(0.99, 1, 118) = 6.855 \ < \ F^*$, so we can reject H_0 and conclude $\beta_1 \neq 0$

(d)

SSR = 3.588 is the absolute magnitude of the reduction in the variation of Y when X is introduced into the regression model.

The relative reduction is $\frac{3.588}{49.406} = 0.07262276$. This is the same as coefficient of determination.

(e)

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{45.818}{49.406} = 0.0726$$
$$r = \sqrt{R^2} = \sqrt{0.0726} = +0.2695$$

(f)

I think R^2 has the more clear-cut operational interpretation. Because R^2 equals to Explained variation divided by Total variation, which represents the percentage of variation can be explained by our linear model.

Problem 3 (2.26)

(a)

```
> anova(model22)
Analysis of Variance Table
Response: d22$V1
          Df Sum Sq Mean Sq F value    Pr(>F)
d22$V2     1 5297.5  5297.5  506.51 2.159e-12 ***
Residuals 14  146.4    10.5
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

	DF	SS	MS	F
Regression	1	5297.5	5297.5	506.51
Error	14	146.4	10.5	
Total	15	5443.9		

(b)

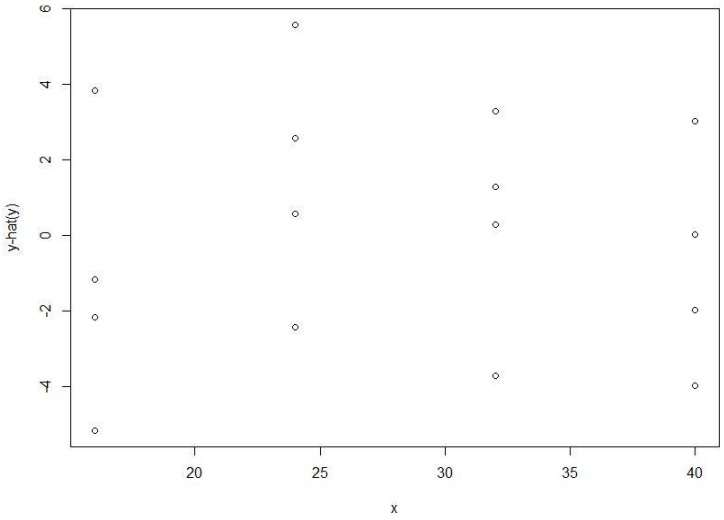
$$H_0 : \ \beta_1 = 0$$
$$H_1 : \ \beta_1 \neq 1$$
$$F^* = \frac{MSR}{MSE} = 506.51$$

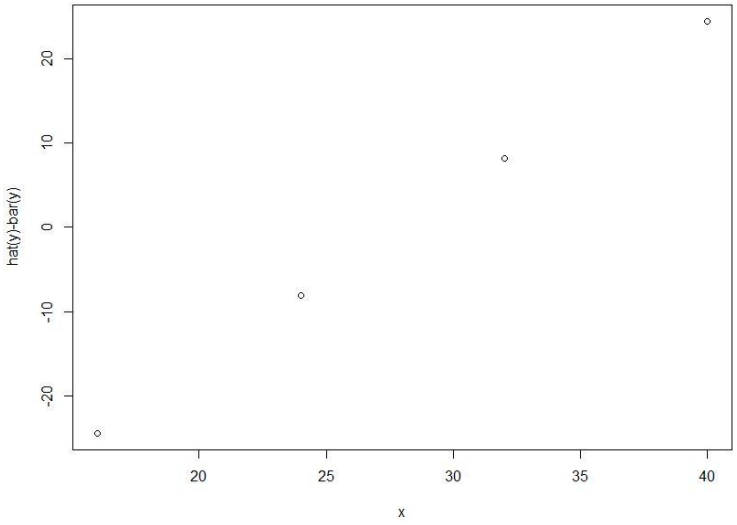
if $F^* \leq F(0.99, 1, 14)$, then conclude H_0

else $F^* > F(0.99, 1, 118)$, reject H_0

While $F(0.99, 1, 14) = 8.861593 \ < \ F^*$, so we can reject H_0 and conclude $\beta_1 \neq 0$

(c)





From the graph we can say that SSR appear to be the larger component of SST. While $R^2 = \frac{SSR}{SST}$ so R^2 is large.

Problem 4 (2.56)

(a)

$$\begin{aligned}\bar{Y} = \hat{\beta}_0 + \hat{\beta}_1\bar{X} &= 5 + 3 \times 8 = 29\\ E(MSE) &= \sigma^2\\ &= 0.6^2 = 0.36\\ E(MSR) &= \sigma^2 + \hat{\beta}_1^2 \sum (X_i - \bar{X})^2\\ &= 0.36 + 9 \times \sum (X_i - 8)^2\\ &= 1026.36\end{aligned}$$

(b)

It would be worse to observe $X = 6, 7, 8, 9, 10$. When n becomes larger *MSR* will become larger which means it will bring more variation to our model. What's more, these observations do not change the range of predictive variables.

But when we want to estimate the mean response for $X = 8$, adding these observations to our model will help us improve our preciseness. Because these observation points lie near $X = 8$, which provides more information around $X = 8$.

Problem 5 (2.61)

$$\begin{aligned}\frac{SSR}{SST} &= \frac{b_1^2 \sum (X_i - \bar{X})^2}{\sum (Y_i - \bar{Y})^2}\\ &= \frac{\left(\frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}\right)^2 \sum (X_i - \bar{X})^2}{\sum (Y_i - \bar{Y})^2}\\ &= \frac{\left(\sum (X_i - \bar{X})(Y_i - \bar{Y})\right)^2}{\sum (X_i - \bar{X})^2 \sum (Y_i - \bar{Y})^2}\end{aligned}$$

As for this fraction, the denominator and numerator each has same expression for X and Y, which means X and Y have symmetric expressions. Therefore, the ratio is the same whether $X(or Y_1)$ is regressed on $Y(or Y_2)$ or $Y(or Y_2)$ is regressed on $X(or Y_1)$.

Problem 6 (2.66)

(a)

```
> e=rnorm(5, mean = 0, sd = 5)
[1] -4.9633070  6.4244096 -0.4761277 -2.8104307 -0.4428723
> X=c(4,8,12,16,20)
> Y=20+4*X+e
> Y
[1] 31.03669 58.42441 67.52387 81.18957 99.55713


> model66 <- lm(Y~X)
> summary(model66)
Call:
lm(formula = Y ~ X)
Residuals:
       1        2        3        4        5
-4.54844  6.85868 -0.02246 -2.33737  0.04959
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  19.6045     5.1806   3.784  0.03235 *
X             3.9952     0.3905  10.231  0.00199 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

So $b_0 = 19.6045$ and $b_1 = 3.9952$
When $X_h = 10,\ Y_h = 19.6045 + 3.9952 \times 10 = 59.55603$
The 95 percent confidence interval is

$$\hat{Y_h} \pm t(1-\alpha/2,\ n-2)s_{\{\hat{Y}_h\}}$$

which is

$$59.5565 \pm 3.182446 \times 2.343011$$
$$[\ 52.09952,\ 67.01254\ ]$$

(b)

```
> for (i in 1:200){
+     e=rnorm(5, mean = 0, sd = 5)
+     X=c(4,8,12,16,20)
+     Y=20+4*X+e
+     model66 <- lm(Y~X)
+     B1[i]=model66$coefficients[2]
+ }
```

(c)

The mean and standard deviation of the 200 estimates are:

```
> mean(B1)
[1] 3.967998
> sd(B1)
[1] 0.3660616
```

As for theoretical results,

$$\sigma^2(b_1) = \frac{\sigma^2}{\sum(X_i - \bar{X})^2}$$

```
> sqrt(25/sum((X-mean(X))^2))
[1] 0.3952847
```

The mean of the 200 estimates is 3.967998, which is very close to theoretical results 4.

And the theoretical expectation of $\sigma(b_1)$ should be 0.3952847. Compared with the standard deviation 0.3660616, we find the result differs from the theoretical expectation a little, but the difference is just about 0.3

(d)

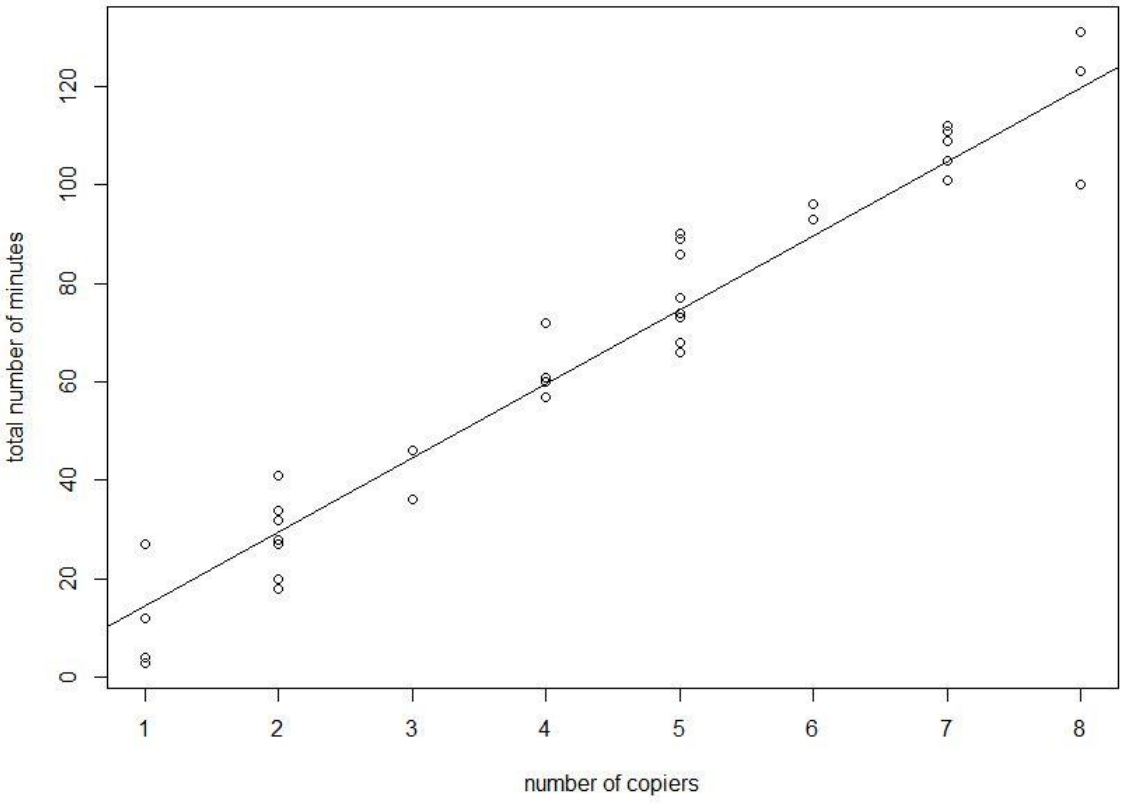
```
> for (i in 1:200){
+   e=rnorm(5, mean = 0, sd = 5)
+   X=c(4,8,12,16,20)
+   Y=20+4*X+e
+   model66 <- lm(Y~X)
+   newdata=data.frame(X=10)
+   newy=predict(model66, newdata)
+   Bool66[i]<-(newy%in%predict(model66,newdata,interval="confidence"))*1
+ }
> sum(Bool66) / length(Bool66)
[1] 1
```

From the results above, we find 100% of the 200 confidence intervals include $E\{Y_h\}$. This result is consistent with theoretical expectations.

Problem 7 (2.68)

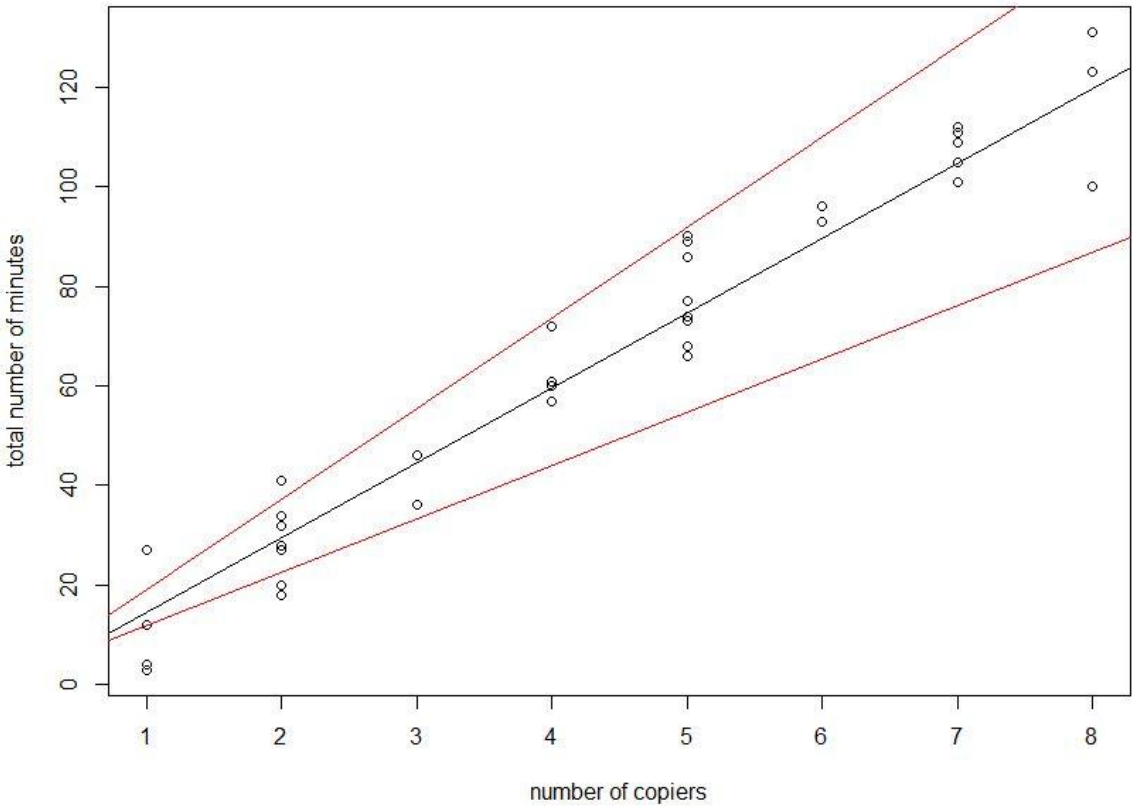
(a)

```
> plot(d20$V2[d20$V2<=8],d20$V1[d20$V2<=8],
+      xlab="number of copiers",ylab="total number of minutes")
> abline(model20$coefficients[1],model20$coefficients[2])
```



(b)

```
> F = 2*qf(0.9,2,43)
> W = sqrt(F)
> Yup=pred+W*s
> Ylow=pred-W*s
>
> abline(newdata,Yup,col='red')
> abline(newdata,Ylow,col='red')
```



The fitted regression line entirely lies between the confidence band. Except for some few points, most points also lie between the bands and near the fitted regression line. So the true regression relation has been precisely estimated.