

PROBABILITY: Homework #1

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Problem 1

(a) denote H for heads and T for tails.

$$S = \{(H, H, H), (H, H, T), (H, T, H), (H, T, T), \\ (T, H, H), (T, H, T), (T, T, H), (T, T, T)\}$$

(b)

$$A = \{(H, H, T), (H, T, H), (T, H, H), (H, H, H)\} \\ B = \{(H, H, T), (H, H, H)\} \\ C = \{(H, H, T), (H, T, T), (T, H, T), (T, T, T)\}$$

(c)

$$1) A^c = \{(H, T, T), (T, H, T), (T, T, H), (T, T, T)\} \\ 2) A \cap B = \{(H, H, T), (H, H, H)\} = B \\ 3) A \cup C = \{(H, H, T)\}$$

Problem 2

(a) denote H for heads and T for tails.

$$\text{number of sample space is } \binom{52}{5}$$

$$\text{number of choices of suits is } \binom{4}{1}$$

$$P = \frac{\#of suits}{\#of sample space} \\ = \frac{\binom{4}{1}}{\binom{52}{5}} \\ = \frac{1}{649740}$$

(b)

list possible situations as (A2345)...(10JQKA), we know there are 10 choices of value and 4 choices of suit.

$$P = \frac{10 * 4}{\binom{52}{5}} \\ = \frac{1}{64974}$$

(c)

there are 13 choices of value

there are 4 choices of suit

$$P = \frac{13 * 4}{\binom{52}{5}}$$

$$= \frac{1}{49980}$$

(d)

since the 5 cards have same suits, they should not have same value

there are $\binom{13}{5}$ choices of value

there are 4 choices of suit

$$P = \frac{\binom{13}{5} * 4}{\binom{52}{5}}$$

$$= \frac{33}{16660}$$

(e)

After chose value for the 3 cards ,there are only 12 choices for the other 2 cards.

There are 13*12*11 choices of value for no repeated value of the other 2 cards.

In this situation there are 4*4*4 choices of suit.

Besides there are 13*12 choices of value for a pair.

in this situation there are 4*4*3 choices of suit.

$$P = \frac{(13 * 12 * 11) * (4 * 4 * 4) + (13 * 12) * (4 * 4 * 3)}{\binom{52}{5}}$$

$$= \frac{188}{4165}$$

(f)

there are 13*12 choices of value.

there are $\frac{4}{2} * \frac{4}{2}$ choices of suit.

$$P = \frac{(13 * 12) * \frac{4}{2} * \frac{4}{2}}{\binom{52}{5}}$$

$$= \frac{9}{4165}$$

Problem 3

(a)

there are 16 choices for women president. And there are 48 choices for president

$$\text{so } Pr(E) = \frac{16}{48} = \frac{1}{3}$$

there are 32 choices for men vice president. And there are 48 choices for vice president

$$\text{so } Pr(F) = \frac{32}{48} = \frac{2}{3}$$

there are $16 * 15 + 32 * 31$ choices for president of same sex

there are $48 * 47$ choices for president

$$\begin{aligned} \text{so } Pr(G) &= \frac{16 * 15 + 32 * 31}{48 * 47} \\ &= \frac{77}{141} \end{aligned}$$

(b)

$E \cap F$ represent president is woman and vice president is man

there are $16 * 32$ choices for this situation

$E \cup F$ represent president is woman or vice president is man

there are $16 * 32 + 16 * 15 + 32 * 31$ choices for this situation

$E \cap F \cap G$ does not make sense

there are 0 choices for this situation

$$Pr(E \cap F) = \frac{16 * 32}{48 * 47} = \frac{32}{141}$$

$$Pr(E \cup F) = \frac{16 * 32 + 16 * 15 + 32 * 31}{48 * 47} = \frac{109}{141}$$

$$Pr(E \cap F \cap G) = 0$$

(c)

$$Pr(G|E \cup F) = \frac{Pr(G \cap (E \cup F))}{Pr(E \cup F)}$$

$G \cap (E \cup F)$ represent two president are of same sex = G

$$\text{therefore } Pr(G|E \cup F) = \frac{Pr(G)}{Pr(E \cup F)} = \frac{\frac{77}{141}}{\frac{109}{141}} = \frac{77}{109}$$

Problem 4

consider the event as inserting four adjacent aces into 48 shuffled cards

of order of adjacent aces are $4 * 3 * 2 * 1$

of choices of inserting the four adjacent are $\binom{49}{1}$

$$\text{therefore } P = \frac{4 * 3 * 2 * 1}{49} = \frac{24}{49}$$

Problem 5

(a)

$$\# \text{ of the sample space} = \binom{60}{30}$$

$$\text{there are } \binom{60-5}{30} \text{ choices for this situation.}$$

$$\begin{aligned} \text{so } P &= \frac{\binom{55}{30}}{\binom{60}{30}} \\ &= \frac{117}{4484} \end{aligned}$$

(b)

$$\# \text{ of the sample space} = \binom{60}{30}$$

$$\text{there are } \binom{5}{4} * \binom{60-5}{30-4} \text{ choices for this situation.}$$

$$\begin{aligned} \text{so } P &= \frac{\binom{5}{4} * \binom{60-5}{30-4}}{\binom{60}{30}} \\ &= \frac{675}{4484} \end{aligned}$$

(c)

$$\# \text{ of the sample space} = \binom{60}{30}$$

$$\text{there are } \binom{60-5}{30-1} \text{ choices for this situation.}$$

$$\begin{aligned} \text{so } P &= \frac{\binom{60-5}{30-1}}{\binom{60}{30}} \\ &= \frac{135}{4484} \end{aligned}$$

Problem 6

(a)

this event could be either first up then down or first down then up

$$\begin{aligned} \text{so } P &= p * (1 - p) + (1 - p) * p \\ &= 2p(1 - p) \end{aligned}$$

(b)

there should be 1 day down and 2 days up

$$\begin{aligned} \text{so } P &= \binom{3}{1} * p^2 * (1 - p) \\ &= 3p^2(1 - p) \end{aligned}$$

(c)

denote E for price increasing on the first day, then $Pr(E) = p$

denote F for after three days price increased by 1, then $Pr(F) = 3p^2(1 - p)$

the probability that the first day price goes up and three days later price still increased by 1 should be

$$\begin{aligned} Pr(E \cap F) &= p * \binom{2}{1} * p * (1 - p) \\ &= 2p^2(1 - p) \\ Pr(E|F) &= \frac{Pr(E \cap F)}{Pr(F)} \\ &= \frac{2p^2(1 - p)}{3p^2(1 - p)} \\ &= \frac{2}{3} \end{aligned}$$

Problem 7

under strategy (a), the answer could be correct when either husband or wife gives the correct answer.

$$\text{so } P = \frac{1}{2} * p + \frac{1}{2} * p = p$$

under strategy (b), the answer could be correct in the following situations:

- 1) husband correct wife correct and either of their answer to be given.
- 2) husband correct wife wrong and husband's answer is given.
- 3) wife correct husband wrong and wife's answer is given.

$$\text{so } P = p * p + \frac{1}{2} * p * (1 - p) + \frac{1}{2} * p * (1 - p) = p$$

therefore, the two strategies have same efficiency.

Problem 8

(1)

$$Pr(\text{agree}) = p * p + (1 - p) * (1 - p) = 2p^2 - 2p + 1$$

from problem 7 we know $Pr(\text{correct}) = p$

$$Pr(\text{correct}|\text{agree}) = \frac{Pr(\text{correct} \cap \text{agree})}{Pr(\text{agree})} = \frac{p * p}{2p^2 - 2p + 1} = \frac{9}{13}$$

(2)

$$Pr(disagree) = p * (1 - p) + (1 - p) * p = 2p(1 - p)$$

$$Pr(correct \cap disagree) = \frac{1}{2} * p * (1 - p) + \frac{1}{2}(1 - p) * p = p(1 - p)$$

$$Pr(correct|disagree) = \frac{Pr(correct \cap disagree)}{Pr(disagree)} = \frac{p(1 - p)}{2p(1 - p)} = \frac{1}{2}$$

Problem 9

we will calculate the chance that there is no head

$$Pr(no\ head) = (1 - p)^n.$$

in contrast, the probability that at least one head is

$$Pr(at\ least\ one\ head) = 1 - (1 - p)^n$$

$$1 - (1 - p)^n \geq 0.5 \Leftrightarrow (1 - p)^n \leq 0.5$$

$$1 - p \in [0, 1]; \text{ so } n * \log(1 - p) \leq \log(0.5)$$

$$\Leftrightarrow n \geq \frac{\log(\frac{1}{2})}{\log(1 - p)}$$

and p could not equal to 1

Problem 10

denote the event a silver coin is found as E

$$Pr(E) = \frac{1}{3} * \frac{1}{2} + \frac{1}{3} * 1 = \frac{1}{2}$$

denote the event two silver coins ia found as F

$$Pr(F) = \frac{1}{3}$$

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)}$$

$$= \frac{Pr(F)}{\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Problem 11

(a)

let's separate this event into two parts:

a) a red ball is drawn from A; b) any ball other than red is drawn from A

$$\text{then } P = \frac{4}{4+3+2} * \frac{3}{3+3+4} + \frac{3+2}{4+3+2} * \frac{2}{2+3+4+1} = \frac{11}{45}$$

(b)

denote E as drawn red from A and F as drawn red from B

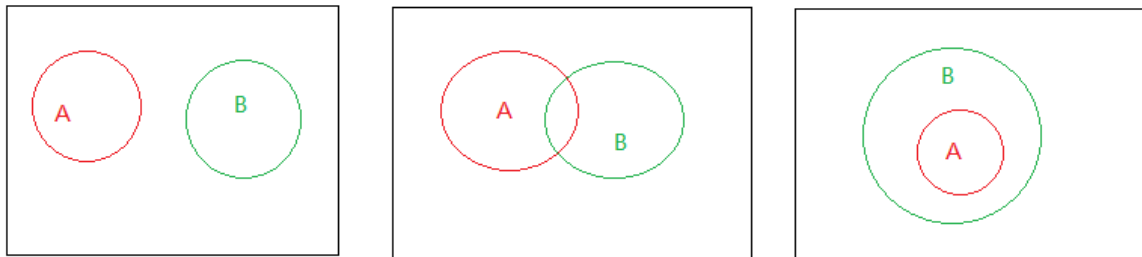
from part (a) we get $Pr(F) = \frac{11}{45}$

$$Pr(E \cap F) = \frac{4}{4+3+2} * \frac{3}{3+3+4} = \frac{2}{15}$$

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$$

$$= \frac{Pr(E \cap F)}{Pr(F)} = \frac{\frac{2}{15}}{\frac{11}{45}} = \frac{6}{11}$$

Problem 12



from the above venn diagram,we come to the following conclusion:

- 1) when A and B are independent, then $Pr(A \cap B)$ get the minimum value 0.
- 2) when B is a sub set of A, then $Pr(A \cap B)$ get the maximum value 0.7.

Problem 13

difference	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

as the graph above depicts, we get:

$$P = \frac{24}{36} = \frac{2}{3}$$

Problem 14

the number in sample space should be $\binom{12+20-1}{12} = \binom{31}{12}$

the situation that no box have more than one ball is the same as there are 12 boxes with one box each only and 8 empty boxes

so number of choices of this situation is $\binom{20}{12}$

$$\begin{aligned} \text{therefore } P &= \frac{\binom{20}{12}}{\binom{31}{12}} \\ &= \frac{646}{723695} \end{aligned}$$

Problem 15

the number in sample space should be $\binom{35}{10}$

number of choices of this situation is $\binom{35-2}{10-2} + \binom{35-2}{10}$

$$= \binom{33}{8} + \binom{33}{10}$$

$$\text{so } P = \frac{\binom{33}{8} + \binom{33}{10}}{\binom{35}{10}}$$

$$= \frac{69}{119}$$

Problem 16

the number in sample space should be $\binom{52}{13, 13, 13, 13}$

number of choices of this situation is $\binom{12}{3, 3, 3, 3} * \binom{40}{10, 10, 10, 10}$

$$\text{so } P = \frac{\binom{12}{3, 3, 3, 3} * \binom{40}{10, 10, 10, 10}}{\binom{52}{13, 13, 13, 13}}$$

$$= \frac{\frac{12!}{(3!)^4} * \frac{40!}{(10!)^4}}{\frac{52!}{(13!)^4}}$$

$$= \frac{\frac{12!}{(3!)^4} * \frac{40!}{(10!)^4}}{\frac{52!}{(13!)^4}}$$

$$= \frac{148933}{4594023} \approx 0.0324$$

Problem 17

the number in sample space should be $\binom{30 * 3}{10}$

number of choices of missing one color is $\binom{30 * 2}{10} - 2 * \binom{30}{10}$

number of choices of missing two color is $3 * \binom{30}{10}$

$$\text{so } P = \frac{\binom{30 * 2}{10} - 2 * \binom{30}{10} + 3 * \binom{30}{10}}{\binom{30 * 3}{10}}$$

$$= \frac{9734}{738289} \approx 0.0132$$