ADVANCED DATA ANALYSIS

HW1

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Problem 1

(a)

The reject region given is $S \geq 16$, which contradicts with the two-sided alternative hypothesis. So I do in 2 ways as follow.

1)

Assume $H_a : \eta > 0$

Because any one observation is equally likely to be above or below the population median η, the number of $X_i \geq \eta = 0$ will have a binomial distribution with mean = 0.5.

$$\begin{aligned} 1 - \alpha &= Pr(S \geq 16 | H_0) \\ &= \sum_{i=16}^{25} \binom{25}{i} \times (\frac{1}{2})^{25} \\ &= 0.11476 \\ \alpha &= 0.88524 \end{aligned}$$

Therefore, the level of the test is 0.88524.

2)

Assume reject region is $S \geq 16$ **and** $S \leq 9$

Because any one observation is equally likely to be above or below the population median η, the number of $X_i \geq \eta = 0$ will have a binomial distribution with mean = 0.5.

$$\begin{aligned}1-\alpha &= Pr(S \geq 16 \text{ and } S \leq 9|H_0) \\ &= 2 \times \sum_{i=16}^{25} \binom{25}{i} \times (\frac{1}{2})^{25} \\ &= 0.22952 \\ \alpha &= 0.770477\end{aligned}$$

Therefore, the level of the test is 0.770477.

(b)

$$\begin{aligned}Pr(X_i > \eta_0) &= Pr(X_i > 0) \\ &= 1 - Pr(\frac{X_i - 0.5}{1} \leq -0.5) \\ &= 1 - Pr(Z \leq -0.5) \\ \text{where } Z \text{ follows } N(0,1). \\ &= 0.6915 \\ \text{So } S \text{ follows } Bin(25, 0.6915).\end{aligned}$$

$$\begin{aligned}\text{power} &= Pr(\text{reject } H_0|H_1) \\ &= Pr(S \geq 16|H_1) \\ &= \sum_{i=16}^{25} \binom{25}{i} \times (0.6915)^i \times (1-0.6915)^{25-i} \\ &= 0.78355\end{aligned}$$

Therefore, the power of the test is 0.78355.

Problem 2

(a)

In [16]:
```
pretest = c(30,28,31,26,20,30,34,15,28,20,
            30,29,31,29,34,20,26,25,31,29)
posttest = c(20,30,32,30,16,25,31,18,33,25,
             32,22,34,32,32,27,28,29,32,32)
diff = pretest-posttest
knitr::kable(cbind(pretest,posttest,diff))
print("mean of pretest-posttest is")
mean(diff)
print("standard deviation of pretest-posttest is")
sd(diff)
```

```
| pretest| posttest| diff|
|-------:|--------:|----:|
|      30|       20|   10|
|      28|       30|   -2|
|      31|       32|   -1|
|      26|       30|   -4|
|      20|       16|    4|
|      30|       25|    5|
|      34|       31|    3|
|      15|       18|   -3|
|      28|       33|   -5|
|      20|       25|   -5|
|      30|       32|   -2|
|      29|       22|    7|
|      31|       34|   -3|
|      29|       32|   -3|
|      34|       32|    2|
|      20|       27|   -7|
|      26|       28|   -2|
|      25|       29|   -4|
|      31|       32|   -1|
|      29|       32|   -3|
```

```
[1] "mean of pretest-posttest is"
```

-0.7

```
[1] "standard deviation of pretest-posttest is"
```

4.43787526211646

test statistis is defined as

$$\begin{aligned} t^* &= \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \\ &= \frac{-0.7 - 0}{4.4379/\sqrt{20}} \\ &= -0.7054 \end{aligned}$$

$$\text{while } t_{n-1}(\alpha/2) = 2.093 > |t^*| = 0.7054$$

$$\begin{aligned} \text{p-value} &= Pr(t > |t^*|) \\ &= 0.48912 \end{aligned}$$

Therefore, we fail to reject H_0.

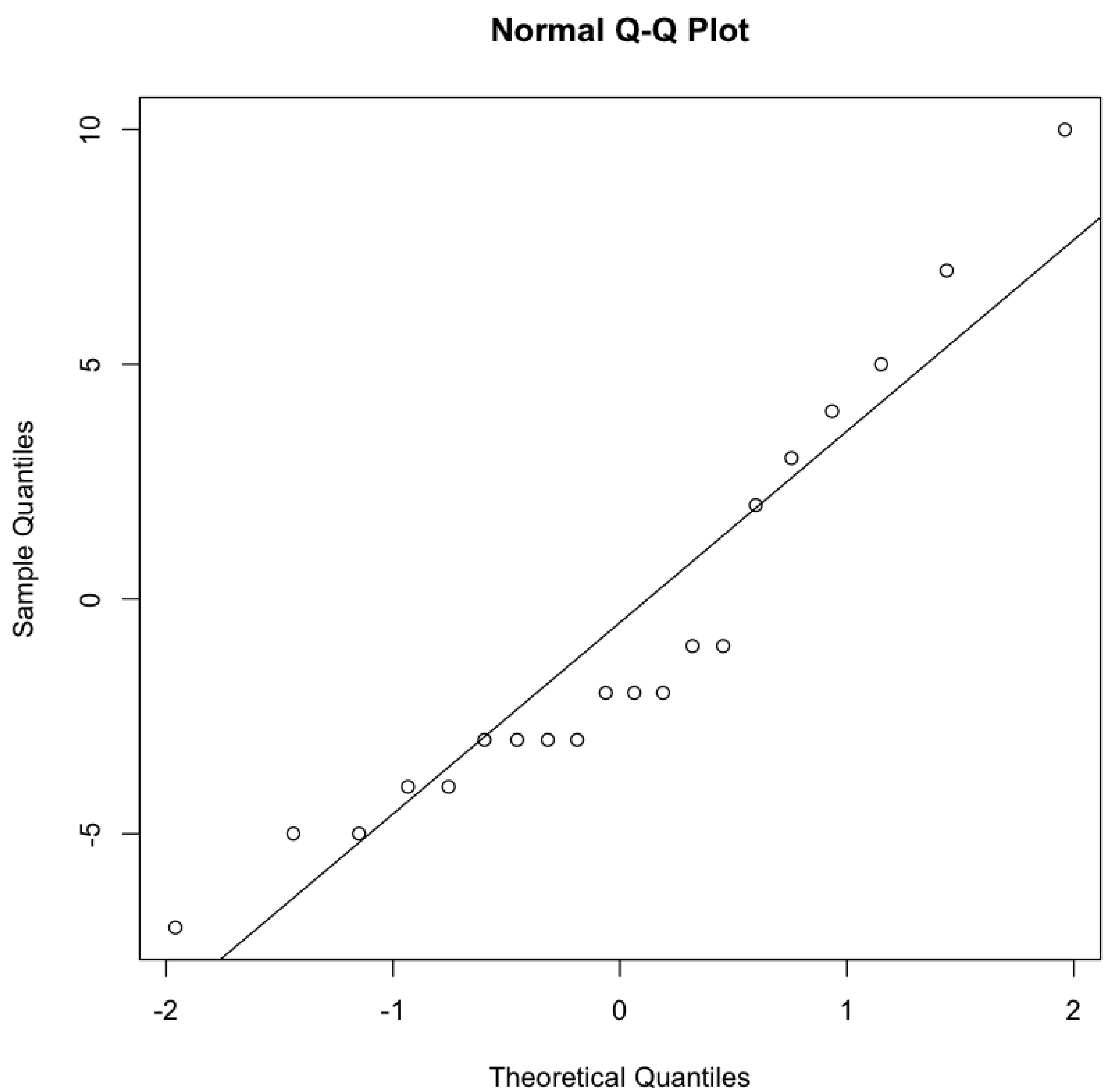
In [32]:
```
t.test(diff, mu=0)
```

```
        One Sample t-test

data:  diff
t = -0.7054, df = 19, p-value = 0.4891
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -2.77699  1.37699
sample estimates:
mean of x
     -0.7
```

We need to assume that pretest-posttest follows normal distribution.

In [40]:
```
qqnorm(diff)
qqline(diff)
```



We can conclude that the difference is approximately follows normal distribution.

(b)

The $100(1-\alpha)\%$ confidence interval is

$$\bar{X} \pm t_{n-1}(\alpha/2)s/\sqrt{n}$$

which is

$$-0.7 \pm 2.093 \times 4.4379/\sqrt{20}$$
$$[-2.77699, 1.37698]$$

In [44]:
```
-0.7+ 2.093*4.4379 / sqrt(20)
-0.7- 2.093*4.4379 / sqrt(20)
```

1.37697726398858

-2.77697726398858

(c)

In [46]:
```
signdiff = diff / abs(diff)
sum(signdiff>0)
```

6

The test statistic $T^* = \sum I(X_i > 0) = 6$
and $T \sim Bin(n, 0.5)$
$|T - n/2| = |6 - 10| = 4$

$$1 - \alpha = Pr(T > T')$$
$$= \sum_{i=T'}^{20} \binom{20}{i} 0.5^{20}$$

when $T' = 7,\ Pr(T > 7) = 0.94234$ and when $T' = 6,\ Pr(T > 6) = 0.979305$
when $T' = 13,\ Pr(T < 13) = 0.94234$ and when $T' = 14,\ Pr(T < 14) = 0.979305$
Let's calculate the p-value
p-value= $2min(P(T \leq 6), P(T \geq 6)) = 0.115318$, which is less than α
Therefore, we fail to reject H_0.

(d)

In [65]:
```
library(BSDA)
SIGN.test(diff, md=0,,alternative="two.sided",conf.level=0.95)
```

```
        One-sample Sign-Test

data:  diff
s = 6, p-value = 0.1153
alternative hypothesis: true median is not equal to 0
95 percent confidence interval:
 -3.000000  1.650588
sample estimates:
median of x
         -2

Achieved and Interpolated Confidence Intervals:

                  Conf.Level L.E.pt  U.E.pt
Lower Achieved CI     0.8847     -3 -1.0000
Interpolated CI       0.9500     -3  1.6506
Upper Achieved CI     0.9586     -3  2.0000
```

So the 95% confidence interval for η is $[-3.000000, 1.650588]$

Problem 3

In [112]:
```
Active = c(9.00,9.50,9.75,10.00,13.00,9.50)
Noexe = c(11.50,12.00,9.00,11.50,13.25,13.00)
diffexe = Noexe - Active
knitr::kable(cbind(Active,Noexe,diffexe))
print("mean of Noexe - Active is")
mean(diffexe)
print("standard deviation of Noexe - Active is")
sd(diffexe)
```

```
| Active| Noexe| diffexe|
|------:|-----:|-------:|
|   9.00| 11.50|    2.50|
|   9.50| 12.00|    2.50|
|   9.75|  9.00|   -0.75|
|  10.00| 11.50|    1.50|
|  13.00| 13.25|    0.25|
|   9.50| 13.00|    3.50|
```

```
[1] "mean of Noexe - Active is"
```

1.58333333333333

```
[1] "standard deviation of Noexe - Active is"
```

1.58640053790544

one sample t-test

Denote μ as the mean of the difference between the two groups.
$H_0 : \mu = 0; \quad H_1 : \mu \neq 0$

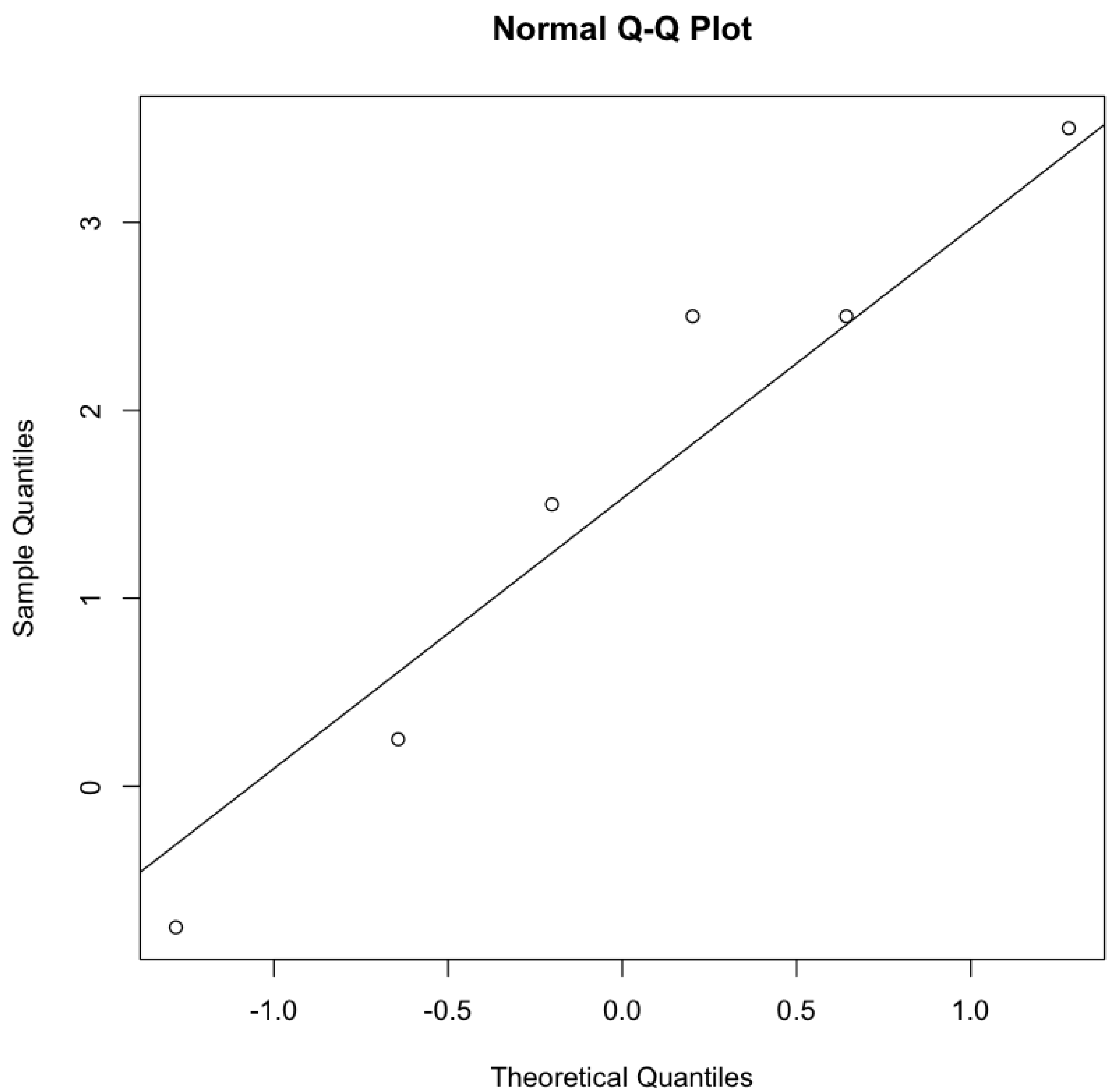
test statistis is defined as

$$\begin{aligned} t^* &= \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \\ &= \frac{1.5833 - 0}{1.5864/\sqrt{6}} \\ &= 2.44475 \\ \text{while } t_{n-1}(\alpha/2) = 2.57058 &> |t^*| = 2.44475 \\ \text{p-value} &= Pr(t > |t^*|) \\ &= 0.0583115 \end{aligned}$$

Therefore, we fail to reject H_0.

In order to use this test we need to assume that X follows normal distribution. Now we use qqplot to check.

In [92]:
```
qqnorm(diffexe)
qqline(diffexe)
```



According to the above qqplot, we can assume the difference follows normal distribution.

In [93]:
```
t.test(diffexe, mu=0)
```

```
	One Sample t-test

data:  diffexe
t = 2.4448, df = 5, p-value = 0.05831
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -0.08149193  3.24815860
sample estimates:
mean of x 
 1.583333 
```

Sign test

In [94]:
```
signdiffexe = diffexe / abs(diffexe)
sum(signdiffexe>0)
```

5

The test statistic $T^* = \sum I(X_i > 0) = 5$
and $T \sim Bin(n, 0.5)$
$|T - n/2| = |4 - 3| = 1$

$$1 - \alpha = Pr(T > T')$$
$$= \sum_{i=T'}^{6} \binom{6}{i} 0.5^6$$

when $T' = 2,\ Pr(T > 2) = 0.890625$ and when $T' = 1,\ Pr(T > 1) = 0.984375$
when $T' = 4,\ Pr(T < 4) = 0.890625$ and when $T' = 5,\ Pr(T < 5) = 0.984375$
Let's calculate the p-value
p-value= $2min(P(T \leq 5), P(T \geq 5)) = 0.21875$, which is greater than α
Therefore, we fail to reject H_0.

In [106]:
```
SIGN.test(diffexe, md=0,,alternative="two.sided",conf.level=0.95)
```

```
        One-sample Sign-Test

data:  diffexe
s = 5, p-value = 0.2187
alternative hypothesis: true median is not equal to 0
95 percent confidence interval:
 -0.65  3.40
sample estimates:
median of x
          2

Achieved and Interpolated Confidence Intervals:

                  Conf.Level L.E.pt U.E.pt
Lower Achieved CI     0.7812   0.25    2.5
Interpolated CI       0.9500  -0.65    3.4
Upper Achieved CI     0.9688  -0.75    3.5
```

The Sign test do not need to assume normal sample, we only need the sample to be indenpendent.

appendix

In [1]:
```
from IPython.display import display_html
display_html("""<button onclick="$('.input, .prompt, .output_stderr, .ou
tput_error').toggle();">Toggle Code</button>""", raw=True)
```

Toggle Code

In [111]:
```
# 1.a
p=0
for (i in 16:25){
    p=p+choose(25,i)*0.5^25
}
1-2*p
```

0.770477056503296

In [107]:
```
# 1.b
p=0
for (i in 16:25){
    p=p+choose(25,i)*pnorm(0.5)^(i)*(1-pnorm(0.5))^(25-i)
}
p
pnorm(0.5)
p*pnorm(0.5)^25
1-p*0.5^25
1-choose(25,16)*0.5^25
1-0.0609
pnorm(-0.5)
pnorm(0.5)
```

0.783551130937035

0.691462461274013

7.73183424118108e-05

0.999999976648357

0.939114600419998

0.9391

In [113]:
```
# 2.c
p=0
for (i in 0:14){
    p=p+choose(20,i)*0.5^20
}
p
2*p
```

0.979305267333984

1.95861053466797

In [105]:
```
# 3.t-test
mean(diffexe)/sd(diffexe)*sqrt(6)
qt(0.975,5)
(1-pt(mean(diffexe)/sd(diffexe)*sqrt(6),5))*2
```

2.44475381011115

2.57058183563631

0.0583115211038363

In [103]:
```
# 3.signtest
p=0
for (i in 5:6){
    p=p+choose(6,i)*0.5^6
}
2*p
```

0.21875