STAT GU4261/GR5261 Statistical Methods in Finance

Spring 2018

Homeowork 4 Suggested Solution

Due date: 22 Feb 2017 (Thu)

(1) Denote A to be the corresponding matrix in the question.

(a) $\det(A-\lambda I)=0$ gives $(1-\lambda)^2-\rho^2=0$. Hence, the eigenvalues are $1\pm\rho$. The corresponding eigenvectors for $1+\rho$ and $1-\rho$ are $(1/\sqrt{2},1/\sqrt{2})$ and $(1/\sqrt{2},-1/\sqrt{2})$ respectively. The proportion explained by the first principal component is $\max\{1+\rho,1-\rho\}/2$.

(b) $\det(A-\lambda I)=0$ gives $(1-\lambda)^3-(1-\lambda)\rho^2=0$. Hence, the eigenvalues are 1 and $1\pm\rho$. The corresponding eigenvectors for 1, $1+\rho$ and $1-\rho$ are $(0,0,1)$, $(1/\sqrt{2},1/\sqrt{2},0)$ and $(1/\sqrt{2},-1/\sqrt{2},0)$ respectively. The proportion explained by the first principal component is $\max\{1,1+\rho,1-\rho\}/3=\max\{1+\rho,1-\rho\}/3$.

Note that eigenvectors are only unique up to a scalar multiple, i.e. if v is an eigenvector, then kv is also an eigenvector for any k. But it is a common practice to pick the one with a unit norm, i.e. this vector has a length of 1.

(2) (a) Note that returns are scale-free and are directly comparable, there is no need of further standardization. But it is fine if you have scaled the input in this question.

(i) 57.2%.

```
Importance of components:
                           PC1    PC2    PC3
Standard deviation     0.0862 0.0581 0.0468
Proportion of Variance 0.5719 0.2595 0.1686
Cumulative Proportion  0.5719 0.8314 1.0000
```

(ii) Each column represents one loading vector.

```
         PC1    PC2    PC3
ge     0.594 -0.142 -0.792
ibm    0.634 -0.523  0.569
mobil 0.495  0.840  0.221
```

(iii) Each row represents principal components of each observation. Note that the question does not state clearly whether the principal components are to be calculated based on the raw data or the centered data, so both solutions are accepted.

- Using the centered data (output given by R functions **prcomp** or **princomp**):

```
           PC1     PC2     PC3
[1,] -0.0735  0.0169 -0.0245
[2,] -0.1008 -0.0578  0.0296
```

- Using the raw data:

```
           PC1     PC2     PC3
[1,] -0.0518  0.0211 -0.0275
[2,] -0.0792 -0.0536  0.0265
```

(b) Any reasonable comments describing differences in the estimates. (eg. GE and IBM have more negative coefficients on SMB, meaning that they were becoming more like a big stock.)

(i)

	ge	ibm	mobil
(Intercept)	0.322	0.226	0.390
Mkt.RF[ind]	1.133	0.807	1.146
SMB[ind]	-0.334	-0.172	-0.586
HML[ind]	0.107	-0.158	0.315

(ii)

	ge	ibm	mobil
(Intercept)	0.33936	-0.00054	0.14640
Mkt.RF[ind]	1.12173	0.74824	0.86945
SMB[ind]	-0.46179	-0.61333	-0.15081
HML[ind]	-0.16594	-0.61853	0.50187

(3) (a) $\frac{1000-P}{P} = 10\% + 0.5(17\% - 10\%) = 13.5\% \implies P = 881.$

(b) $\frac{1000-P}{P} = 10\% + 0.7(17\% - 10\%) = 14.9\% \implies P = 870.$