#

Challenge Problem

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1. Create data

```
Socio_tab <- c(rep("Low",4), rep("Medium",4), rep("High",4))
Boyscout_tab <- rep(c(rep("Yes",2),rep("No",2)),3)
delinquency_tab <- rep(c("Yes","No"),6)
frequency <- c(10,40,40,160,18,132,18,132,8,192,2,48)
Socioeconomic <- factor(c(rep(Socio_tab[1:12],frequency[1:12])))
Boyscout <- factor(c(rep(Boyscout_tab[1:12],frequency[1:12])))
delinquency <- factor(c(rep(delinquency_tab[1:12],frequency[1:12])))
```

Now we get 3 variables named ***Socioeconomic***, ***Boyscout*** and ***delinquency***. They all have **800** observations. *Socioeconomic* has 3 levels("Low", "Medium" and "High") while *Boyscout* and *delinquency* both have two levels("Yes" and "No").

2. exploratory data analysis

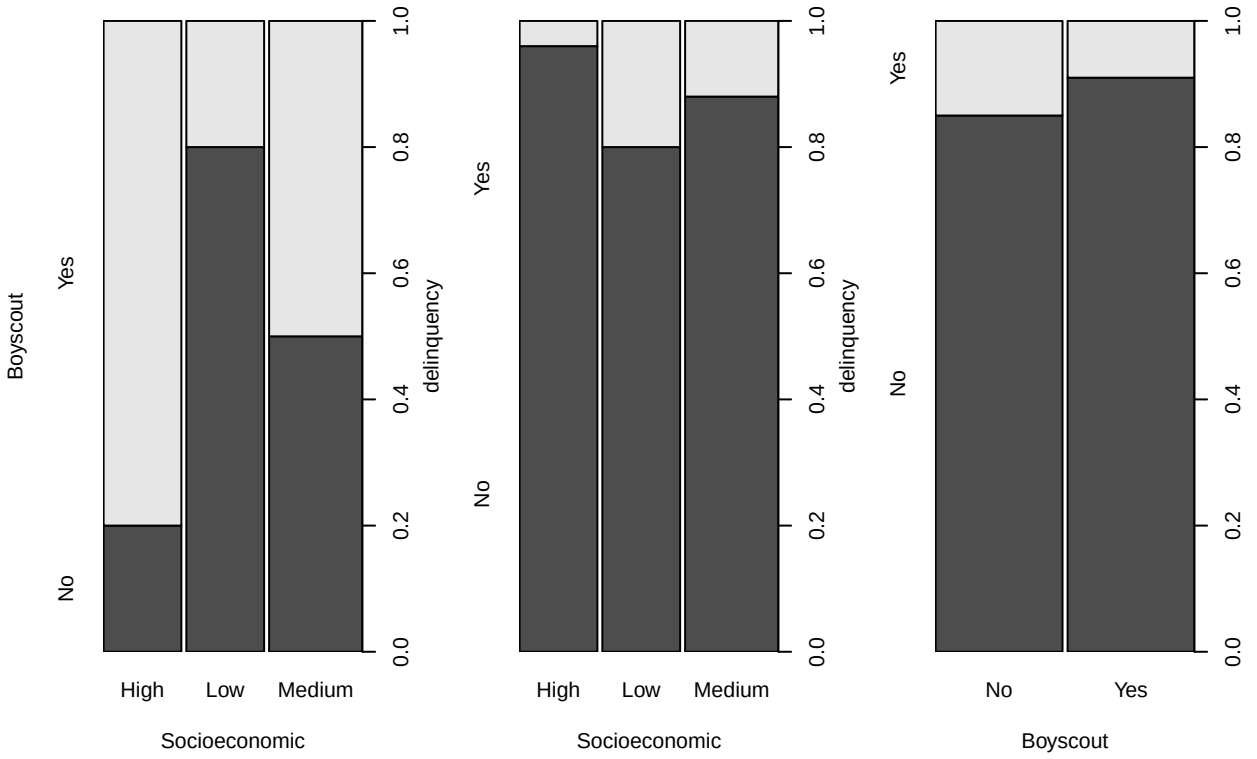
Graphical descriptive statistics

First let's draw pair-wise comparisons of the three variables.

The below three plots are pair-wise comparisons of *"Socioeconomic-Boyscout"*, *"Socioeconomic-delinquency"* and *"Boyscout-delinquency"*. In each plot, the dark bars reprensent "No" for y-axis while the bright bars represent "Yes" for y-axis. And the width of each bar in each plot stands for the number of corresponding group. We can easily find that the 3 levels of Socioeconomic have similar number of observations.

We can draw conclusion from below plots: (1). For the comparision of *"Socioeconomic-Boyscout"*, response of Boyscout influenced significantly by the levels of Socioeconomic. (2). For the other comparisons, different levels do not show significant differences.

```
options(repr.plot.width=8, repr.plot.height=3)
par(mfrow=c(1,3))
plot(Socioeconomic,Boyscout,xlab="Socioeconomic",ylab="Boyscout")
plot(Socioeconomic,delinquency,xlab="Socioeconomic",ylab="delinquency")
plot(Boyscout,delinquency,xlab="Boyscout",ylab="delinquency")
```



Numerical descriptive statistics

Denote x_1 : Socioeconomic status; x_2 : Boy scout; x_3 : delinquency.

Now let's use logistic regression to determine the relationship between the 3 variables.

$\text{logit}(\pi(x_1|x_2, x_3)) = \beta_0 + \beta_1 x_2 + \beta_2 x_3$

Socioeconomic vs **Boyscout** + **delinquency**

Since Socioeconomic is a ordinal categorical variable with 3 hierarchies: "Low", "Medium" and "High". We now use **multinomial logistic regression model**.

From the below above, we get the fitted model as:

$$\log\left(\frac{\hat{\pi}_{\text{High}}}{\hat{\pi}_{\text{Low}}}\right) = -1.1164 - 0.8038x_2 + 0.3526x_3 \log\left(\frac{\hat{\pi}_{\text{Low}}}{\hat{\pi}_{\text{Medium}}}\right) = 0.2403 - 0.8038x_2 + 0.3526x_3$$

```
library(MASS)
polr.cred<-polr(Socioeconomic~Boyscout+delinquency)
summary(polr.cred)$coefficients
```

```
##
## Re-fitting to get Hessian
```

```
##                     Value Std. Error    t value
## BoyscoutYes    -0.8037657 0.13459568  -5.971705
## delinquencyYes  0.3526039 0.19354302   1.821838
## High|Low       -1.1163846 0.10467675 -10.665068
## Low|Medium      0.2403070 0.09648995   2.490487
```

```
Re-fitting to get Hessian
```

Value

Std. Error

t value

BoyscoutYes

-0.8037657

0.13459568

-5.971705

delinquencyYes

0.3526039

0.19354302

1.821838

High|Low

-1.1163846

0.10467675

-10.665068

Low|Medium

0.2403070

0.09648995

2.490487

Use can **use χ^2 test** (**Likelihood Ratio Test**) to check the significance of our model.

```
drop1(polr.cred,test="Chi")
```

```
## Single term deletions
##
## Model:
## Socioeconomic ~ Boyscout + delinquency
##             Df    AIC    LRT  Pr(>Chi)
## <none>         1717.9
## Boyscout     1 1752.1 36.252 1.734e-09 ***
## delinquency  1 1719.2  3.347   0.06732 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Df

AIC

LRT

Pr(>Chi)

<none>

NA

1717.873

NA

NA

Boyscout

1

1752.126

36.252338

1.733520e-09

delinquency

1

1719.220

3.347117

6.732282e-02

Notice that the p-value for coefficients of Boyscout is very small, say less than 0.01. While p-value for coefficients of delinquency is greater than 0.05. Therefore under $\alpha = 0.05$ and "H_0: $\beta_2 = 0$ vs H_A: otherwise", we fail to reject H_0 and conclude that $\beta_2 = 0$.

use AIC for model selection

```
step(polr.cred,direction = "backward")$anova
```

```
## Start:  AIC=1717.87
## Socioeconomic ~ Boyscout + delinquency
##
##               Df    AIC
## <none>           1717.9
## - delinquency  1 1719.2
## - Boyscout     1 1752.1
```

```
##   Step Df Deviance Resid. Df Resid. Dev      AIC
## 1      NA       NA       796   1709.873 1717.873
```

```
Start:  AIC=1717.87
Socioeconomic ~ Boyscout + delinquency

              Df    AIC
<none>           1717.9
- delinquency  1 1719.2
- Boyscout     1 1752.1
```

Step

Df

Deviance

Resid. Df

Resid. Dev

AIC

NA

NA

796

1709.873

1717.873

In the above output, when we delete delinquency in the model, the AIC is 1719.2 which is very similar to the full model. According to Likelihood Ratio Test and AIC, we can remove delinquency and refit the model.

refit model

```
polr.fit<-polr(Socioeconomic~Boyscout)
summary(polr.fit)$coefficients
```

```
##
## Re-fitting to get Hessian
```

```
##                   Value Std. Error    t value
## BoyscoutYes -0.8243628 0.13416508  -6.144392
## High|Low    -1.1707291 0.10052016 -11.646710
## Low|Medium   0.1812920 0.09089892   1.994435
```

Re-fitting to get Hessian

Value

Std. Error

t value

BoyscoutYes

-0.8243628

0.13416508

-6.144392

High|Low

-1.1707291

0.10052016

-11.646710

Low|Medium

0.1812920

0.09089892

1.994435

From the below above, we get the fitted model as:

$$\log\left(\frac{\hat{\pi}_{\text{High}}}{\hat{\pi}_{\text{Low}}}\right) = -1.1707291 - 0.8243628x_2 \tag{1}$$

$$\log\left(\frac{\hat{\pi}_{\text{Low}}}{\hat{\pi}_{\text{Medium}}}\right) = 0.1812920 - 0.8243628x_2 \tag{2}$$

Summary

In this report, we first draw pair-wise comparison of the three variables. And in view of graph we find that only Boy scout and Socioeconomic Status have some correlation. The number of levels of Socioeconomic differ at different levels of Boy scout. Then we conduct Numerical descriptive statistics. Our main model

is **Multinomial Logistic Regression Model**. Set response variable as Socioeconomic and other two as predictor variables. In the first fitted model, we conduct χ^2 test to determine

H_0: $\beta_2 = 0$ vs H_A: otherwise

where β_2 is the coefficient of delinquency. Then we get the p-value is 6.732282e-02 and under $\alpha = 0.05$, we fail to reject H_0 and conclude that $\beta_2 = 0$. Therefore we delete the variable **delinquency** and refit the model.

And get our final model as below:

$$\text{logit}(\pi(x_1|x_2)) = \beta_0 + \beta_1 x_2$$

where $\pi()$ is computed in (1) and (2) and response variable is Socioeconomic and predictor(x_2) is Boy scout.