

Notes on logic

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Recommend for further reading and the sources I am using


- HOW TO PROVE IT - A STRUCTURED APPROACH, by *Daniel J. Velleman*.
 - This is the primary book I am following.
- Appendix: the basis of mathematical logic in ANALYSIS I: THIRD EDITION, by *Terence Tao*.
 - This is a really well-written description of logic. Recommend, if you can get the book.

1 Statements

Definition 1. An *mathematical argument* consists of a sequence of premises and a conclusion. Let p_1, p_2, \dots, p_n be the premises and c be the conclusion. Then we say the argument is *valid* if the conclusion cannot be false when all of the premises holds true.

Abstract:

p_1
 p_2
 \dots
 p_n

 $\therefore c$


Example:

If it rains today, **then** I will wear my raincoat **and** gummies.
 I have **not** worn my raincoat today.

Therefore, it is not raining today.

Saying that an argument is valid is merely the assertion that, *if* the premises holds true, then the conclusion is as well. If any of the premises are false, then the argument does not make any claims of any conclusion. In this case the argument will stay valid. Similarly, if any of the premises turns out to be false in reality, then the argument is still valid. Since it's merely an assertion that is made *when* the premises are true.

For the sake of proving that an argument is valid, we merely need to know the whether each premise is true or false, and in the cases where all of the premises are true check if the conclusion is true as well.

An premise or conclusion can have multiple parts, each of which can be true or false. "I will wear my raincoat and gummies" has four scenarios we have to consider depending on whether "I will wear my raincoat" is true or false, and "I will wear my gummies" is true or false. (Can you see why?)

"I will wear my raincoat and gummies" is a smaller part of the more larger statement "If it rains today, then I will wear my raincoat and gummies." This says that if "it rains today" is true, then "I will wear my raincoat and gummies" is true.

The next premise says that "I have not put on my raincoat today". What does that mean, if the previous premise holds true? If I have not worn my raincoat today, then it cannot be that it rains today. For it if rained today, then I must have both worn my raincoat and gummies on, i.e. I must have worn my raincoat on today. Therefore, it is not raining today. Hence the conclusion.