ECON 4360: Empirical Finance
Spring 2014
Solutions 02

Solutions

1. [15 Points] Suppose a representative investor has utility given by $u(c) = -e^{-\alpha c}$ where $\alpha = 0.2$. Imagine that there are 20 different equally likely "states of the world" that might be possible tomorrow. The possible states are $s = 11, 12, 13, ..., 30$. The investor wants to eat 8 units of the consumption good today and $s/2$ units of the consumption good tomorrow, and he discounts the future with time-preference parameter $\beta = 0.99$. Create an m-file script that answers the following (report to two decimal places, please):

   a. What is the price today of a risky asset A that pays $x_{t+1} = 30 + s/2$ in state s? What is the expected gross return for asset A?

   Solution 1 (1.a) $p^A = 28.69$; $R^B = 1.40$

   b. What is the price today of a risky asset B that pays $x_{t+1} = 50 - s/2$ in state s? What is the expected gross return for asset B?

   Solution 2 (1.b) $p^B = 30.63$; $R^B = 1.30$

   c. What is the risk-free rate (gross)?

   Solution 3 (1.c) $R^f = 1.35$

   d. Explain the difference between the returns for assets A, B, and the risk-free asset.

   Solution 4 (1.d) *The payoff of A is positively correlated with consumption, so its expected return is higher than the risk-free rate. The payoff of B is negatively correlated with consumption, so its expected return is lower than the riskfree rate.*

2. [10 Points] Suppose a representative investor has utility given by $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ where $\gamma = 3$. Our investor still lives in a two-period world, discounting the future (tomorrow) by $\beta = 0.99$. Our investor has an endowment of \$25 (for today only) and is thinking about buying shares of a stock priced at \$15 per share with the following possible payoffs: a 10% chance of \$32, a 10% chance of \$22, a 30% chance of \$10, a 40% chance of \$23, and a 10% chance of \$48. Assume it is possible to buy fractional shares.

   (a) Determine the number of shares the investor will want to purchase (three decimals, please).

   Hint Find the FOC for the utility maximization problem and then use the fzero function in MATLAB to numerically solve for the zeros of that equation. Careful: there may be more than one zero! You'll need to find the "global optimum", so you will want to check for more than one solution by trying different "starting points". You need to write "function files" in MATLAB to solve this problem - consult the Help file on "fzero". There are also two (very simple) example .m files uploaded on the Collab site.

   Solution 5 *The global optimum is at* 0.821. *Note that you have to set up the consumer's utility maximization problem up and solve for the FOC's. See the Matlab files.*

3. [15 Points] Download annual consumption data from 1929-2010 - use the series: Real Personal Consumption Expenditures (PCECCA) from the FRED's Economic Data website:

   http://research.stlouisfed.org/.

   Using this data, write a MATLAB program that finds the mean-variance frontier (EQ 1.18 in Cochrane (2005)). The idea is that in order to graph Figure 1.1 in the text, you just need to find the y-intercept and slope. Assume that consumers have power utility with $\gamma = 3.5$ and $\beta = 0.99$.

a. What is your estimate of the y-intercept and slope for the mean-variance frontier (four decimals, please).

Solution 6 *Y-Intercept for MV Frontier* $= 1.1202$; *Slope of MV Frontier* $= 0.1134$.

b. Does your estimated placement of the mean-variance frontier make sense? Discuss in terms of the risk-free rate and the equity premium. Hint: What do you know about the historical volatility of the S&P 500?

Solution 7 *If the volatility of the S&P is about* 20% *per year, then the equity premium is:*

$$\frac{E\left(R^{mkt}\right) - R^{f}}{\sigma\left(R^{mkt}\right)} \leq \frac{\sigma(m)}{E(m)},$$

which implies

$$E\left(R^{mkt}\right) - R^{f} \leq \sigma\left(R^{mkt}\right)\frac{\sigma(m)}{E(m)} = 0.1134 * 0.20 = 2.27\%$$

The equity premium is that stocks should only earn 2.2% *more than bonds, but they actually earn much more. A Risk-Free Rate of* 11.1% *per year is too high. (But note we use aggregate consumption data, not per-capita consumption. This means the growth rate in consumption is biased high,* $E(m)$ *is biased low, and the risk-free rate is biased high.)*

4. [10 Points] Tomorrow, there are three possible states determined by the future price of AAPL stock as follows.

	State 1	State 2	State 3
AAPL Stock	350	375	400

A share of AAPL trades today for a price of \$360.00. An AAPL call option with a strike price of \$364.00 trades today for \$17.62, an AAPL put option with a strike price of \$386.00 trades today for \$15.40.

a. Fill in the following table for tomorrow's payoffs:

	State 1	State 2	State 3
AAPL Stock, $P = \$360$	350	375	400
AAPL Call, $K = 364$, $P = \$17.62$	**0**	**11**	**36**
AAPL Put, $K = 386$, $P = \$15.40$	**36**	**11**	**0**

b. Is it possible to infer what the state prices are from the prices of the stock and the options that you observe in the market? *Yes, we can infer the state prices.*

c. Why? *Since we have three states and three securities with linearly independent payoffs, the market is complete and state prices exist. The payoff matrix is nonsingular, so we can invert it.*

d. If the answer to b) is yes, determine the value of the state prices (to four decimal places):

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = \begin{bmatrix} \mathbf{0.3974} \\ \mathbf{0.0994} \\ \mathbf{0.4591} \end{bmatrix}$$

e. If the state prices exist and are unique, determine the price of an AAPL put option that has a strike price $K = 383$. *The price of an AAPL put option that has a strike price* $K = 383$ *is* \$13.91

f. If it is possible to determine the (gross) riskfree rate in this economy, determine what it is. *The (gross) risk-free rate is* $R^f = 1.0461$.

g. What is the relationship between the set of state prices and the price of a risk-less bond? *The price of a* \$1 *par risk-less bond is simply the sum of all the state prices.*

5. [15 Points] One year from now, there are four possible states determined by the future price of a barrel of oil as follows:

	State 1	State 2	State 3	State 4
Price of Oil, per Barrel	70	75	80	85

A barrel of oil trades today for a price of \$73.75. A call option for a barrel of oil with a strike price of \$73 trades today for \$5.00, a call with strike price \$79 trades today for \$1.50, a put with strike price \$81 trades today for \$4.00, and a put with strike price \$80 trades for \$3.27.

a. Fill in the following table for tomorrow's payoffs:

	State 1	State 2	State 3	State 4
Price of Oil, per Barrel, $P = \$73.75$	70	75	80	85
Call, $K = 73$, $P = \$5.00$	**0**	**2**	**7**	**12**
Call, $K = 79$, $P = \$1.50$	**0**	**0**	**1**	**6**
Put, $K = 81$, $P = \$4.00$	**11**	**6**	**1**	**0**
Put, $K = 80$, $P = \$3.27$	**10**	**5**	**0**	**0**

b. Use the four options to determine the value of the state prices (four decimal places):

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0.3120} \\ \mathbf{0.0300} \\ \mathbf{0.3880} \\ \mathbf{0.1853} \end{bmatrix}$$

c. Using the state prices you found for part b), what do they tell you the price of a barrel of oil should be? Is there an arbitrage opportunity? *The state prices from part b) tell us that the price of a barrel of oil should be* \$70.88. *Yes, there is an arbitrage opportunity - the market price of oil is too high, relative to the four options.*

d. How can you form a replicating portfolio for the barrel of oil using the four options? Describe the position (long / short) for each asset and the quantity you would buy (Recall that "shorting" is mathematically equivalent to buying negative amounts of an asset):

	Long or Short?	Quantity
Call, $K = 73$, $P = \$5.00$	***short***	**14.17**
Call, $K = 79$, $P = \$1.50$	***long***	**42.5**
Put, $K = 81$, $P = \$4.00$	***long***	**136.67**
Put, $K = 80$, $P = \$3.27$	***short***	**143.33**

We can form a replicating portfolio for the barrel of oil as follows. Let's express the problem using linear algebra to find the replicating portfolio

$$\begin{aligned} nC &= Z \\ nCC^{-1} &= ZC^{-1} \\ n &= ZC^{-1} \end{aligned}$$

We then get that

$$n = \begin{bmatrix} -14.17 & 42.5 & 136.67 & -143.33 \end{bmatrix}.$$

e. Given the replicating portfolio you found in d), how can you exploit the arbitrage? (I.e., Do you go long or short the barrel of oil? Do you go long or short the replicating portfolio?) What is the arbitrage profit per option?

Pos. Today Long/Short?		Cash Flow Today	Payoffs State 1	Payoffs State 2	Payoffs State 3	Payoffs State 4
	Barrel of Oil	**+73.75**	**−70**	**−75**	**−80**	**−85**
	Replicating Portfolio	**−70.88**	**+70**	**+75**	**+80**	**+85**
—	**TOTAL**	**2.87**	**0**	**0**	**0**	**0**

You exploit the arbitrage buy shorting the barrel of oil and longing the replicating portfolio. The arbitrage profit you make per transaction is \$2.87.

6. [5 Points] A mutual fund manager must decide which of two stocks, Y or Z, to combine with stock X. The portfolio will hold 50 percent in X and 50 percent in either Y or Z. You know the following information about the stocks:

	Expected Return	Standard Deviation	Correlation with X
X	12%	16%	1.0
Y	8%	7%	0.40
Z	8%	5%	0.85

The manager concludes that Z has the lower risk, so it is the obvious choice. Do you agree or disagree? Explain why. *Disagree. The portfolio expected return is* 10% *in either case, however the standard deviation is lower in a portfolio that combines X with Y (*9.93%*) as opposed to a portfolio that combines X with Z (*10.21%*). Stock Y is a better choice because though the individual stock has a larger standard deviation, it has a lower correlation with X, which helps to reduce the portfolio risk.*

7. [15 Points] You are trying to decide the best way to divide your money among 3 stocks, X, Y, and Z. You have already estimated the following information

	Expected Return	Standard Deviation
X	7%	16%
Y	9%	9%
Z	8%	11%

and

Correlations
$\rho_{XY} = -0.8$
$\rho_{XZ} = 0.15$
$\rho_{YZ} = 0.22$

Use Matlab to answer the following questions:

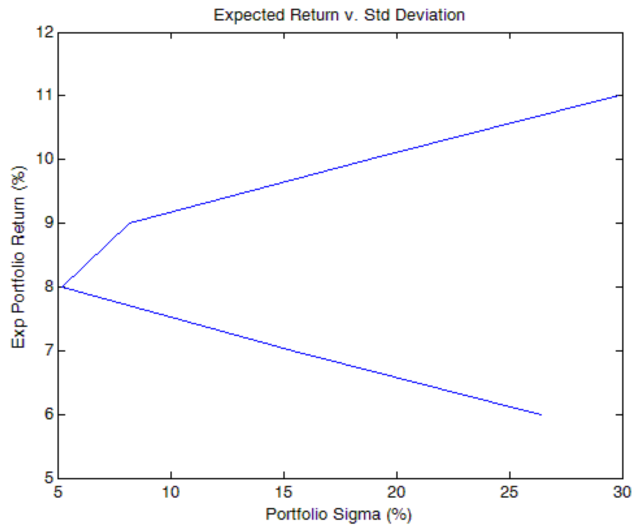
a. Assume that short sales are allowed. Find the weights with which will give you the lowest overall portfolio standard deviation. Report your results in the table below.

w_X	w_Y	w_Z	$E(R^p)$	σ^p
0.3872	0.7298	−0.1170	8.34%	3.5%

b. Again, assume that short sales are allowed. Find the optimal weights which minimize portfolio variance while giving expected returns of 6%, 7%, 8%, 9%, 10%, and 11%. Report your results in the table below.

w_X	w_Y	w_Z	$E(R^p)$	σ^p
1.0705	−0.9295	0.8590	6.0%	26.4%
0.7788	−0.2212	0.4423	7.0%	15.4%
0.4871	0.4871	0.0257	8.0%	5.2%
0.1954	1.1954	−0.3909	9.0%	8.1%
−0.0963	1.9037	−0.8075	10.0%	18.8%
−0.3880	2.6120	−1.2241	11.0%	29.9%

c. Graph your answers from part b) and include your graph as a separate page.



d. Now assume that you are restricted from making short sales. Find the optimal weights which minimize portfolio variance while giving expected returns of 7%, 8%, and 9%. Report your results in the table below.

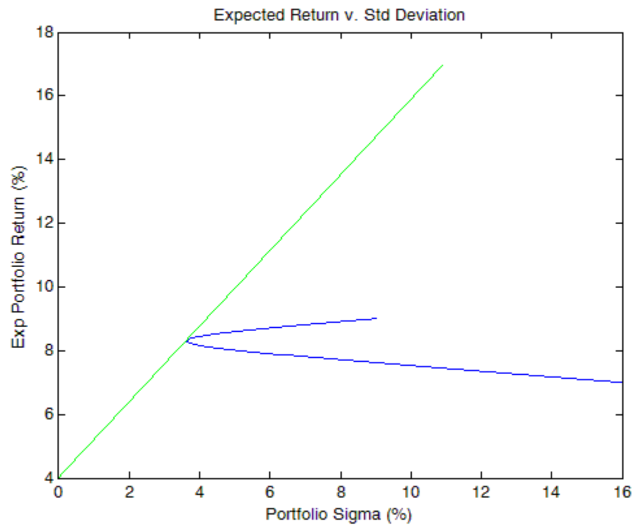
w_X	w_Y	w_Z	$E(R^p)$	σ^p
1.0	0.0	0.0	7.0%	16.0%
0.4871	0.4871	0.0257	8.0%	5.2%
0.0	1.0	0.0	9.0%	9.0%

e. What conclusions can you draw about the effects of restricting short sales on the mean-variance frontier? *Short sale restrictions shift the frontier down and to the right - i.e., for a given level of expected return, the portfolio is riskier. (You can see this by plotting the new portfolio frontier on top of the old one.)*

8. [10 Points] Now assume you only have the use of the two assets X and Y from the previous question and a risk-free asset that has a return of $R^f = 4\%$. Use Matlab to answer the following questions:

a. Graph the possible risk-return combinations of X and Y. On the same graph, plot the new Efficiency Frontier when the possibility of investing in the risk-free asset is included. To do this, you'll need to first find the tangency portfolio. What are the weights, w_X and w_Y, for the tangency portfolio? What is the expected return and standard deviation of this portfolio? Answer in the table below and include your graph as a separate page.

w_X	w_Y	$E(R^p)$	σ^p
0.3333	0.6667	8.33%	3.64%



b. If you wanted to invest part of your total investment budget in the tangency portfolio (you can think of this as a risky mutual fund) and part of your budget in the risk-free asset to provide an overall portfolio expected return of 12%, how could you do this? What would be the standard deviation of that portfolio? *To get an expected return of* 12%*, let* w_c *be the proportion of wealth invested in the risky portfolio. Then*

$$\begin{aligned} 12 &= w_c 8.33 + (1 - w_c)\,4 \\ 8 &= 4.33 w_c \\ w_c &= 1.8476 \end{aligned}$$

So we buy 1.8476 *of the risky portfolio and short* 0.8476 *of the risk-free asset. The standard deviation of the overall portfolio would be*

$$\sigma_p = \sqrt{(1.8476)^2\,(0.0364)^2} = 6.72\%$$

9. [5 Points] You own two risky stocks, A and B, where $\sigma_A = 2\sigma_B$ and $\rho_{AB} = -1.0$. You are extremely risk-averse, so you want to divide your wealth so that there is *no* risk in your portfolio. What must the weights w_A and w_B be? *Use the variance expression to find*

$$\begin{aligned} \sigma_p^2 &= w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \rho \sigma_A \sigma_B \\ 0 &= 4w_A^2\sigma_B^2 + w_B^2\sigma_B^2 - 4w_A w_B \sigma_B^2 \\ 0 &= \sigma_B^2\left(4w_A^2 - 4w_A w_B + w_B^2\right) \\ 0 &= (w_B - 2w_A)^2 \\ w_B &= 2w_A \end{aligned}$$

And since $w_A + w_B = 1$*, we get that* $w_A = 1/3$ *and* $w_B = 2/3$.