ECON 4360: Empirical Finance
Spring 2014
Homework 01: Solutions

Solutions

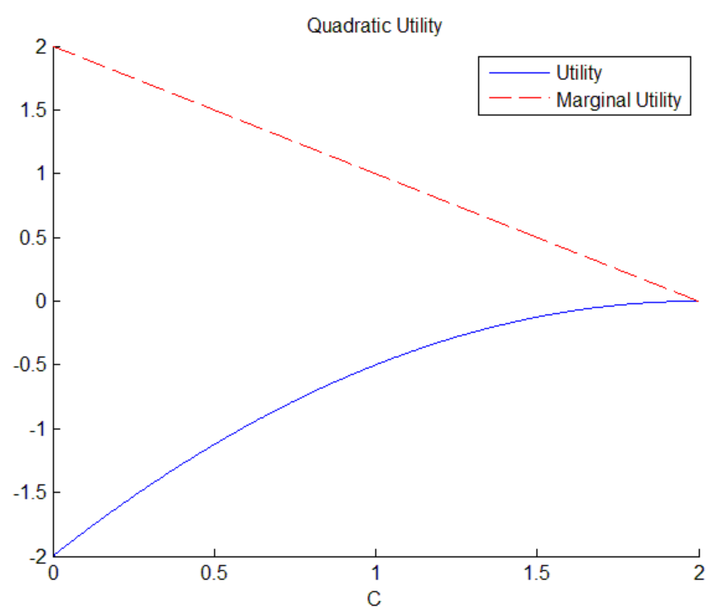
Practice with Utility Functions

1. We'll mostly use power utility in this class $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and you've already become somewhat familiar with it from our practice in class. This homework is going to focus on another common utility function: quadratic utility. Quadratic utility $u(c) = -\frac{1}{2}(c^* - c)^2$ is sometimes used because it has the convenient property that marginal utility is linear.

   (a) What are the limits over which this is a vaguely sensible utility function? (I.e., what are $c_{\min}$ and $c_{\max}$ for $c \in (c_{\min}, c_{\max})$?) Why?

   - $c_{\min} = 0$ and $c_{\max} = 2$. Utility functions make sense for the range where consumption is positive (negative consumption is physically impossible) and marginal utility is also positive (more consumption should make you happier).
   - *Why would we use quadratic utility? Because marginal utility is linear - this is really useful when you're doing asset pricing. One issue is that marginal utility really isn't linear... But how non-linear is it really? Especially in a region near where you're using the model. Quadratic utility makes sense between $c = 0$ and $c = c^*$, so it can be used as a local approximation for that region.*

   (b) Plot utility and marginal utility on the same graph **over the range which makes sense** for $c^* = 2$, being sure to clearly label your curves, axes, etc.



   (c) What is the formula for the coefficient of relative risk aversion for this utility function? Evaluate this formula for $c^* = 2$ at the points $c = 0, 1, 2, 4$ and **explain** what is going on at each of those points.

- The Arrow-Pratt measure of relative risk aversion is

$$\begin{aligned}\gamma &= -\frac{cu''(c)}{u'(c)} \\ &= -\frac{c(-1)}{(c^* - c)} \\ &= \frac{c}{c^* - c}\end{aligned}$$

so

$$\begin{aligned}\gamma(0) &= 0 \\ \gamma(1) &= 1 \\ \gamma(2) &= \infty \\ \gamma(4) &= -2\end{aligned}$$

- Note that relative risk aversion depends on c - i.e., it's not constant. We have increasing relative risk aversion (over the range that makes sense) here, so investors with higher consumption (equivalently, wealth) will be more risk averse to relative wealth bets than those with lower consumption (wealth).
- Does zero risk aversion make sense here? Sure it does. γ refers to risk aversion with respect to *relative* wealth bets... You would be risk-averse at all about making relative wealth bets here, since you have nothing to lose (or gain)! (Note that the value is zero here, even though utility is never/nowhere linear.)
- So relative risk aversion is zero at zero wealth, and increases with consumption, reaching infinity at $c = c^*$. Why? Because marginal utility is zero here... This means you wouldn't want any more consumption (even if it were given to you), so there is no relative wealth bet that you would want to take. So we get infinite risk aversion.
- Beyond $c = c^*$, marginal utility is negative (you would throw away consumption). Note that $c > c^*$ isn't part of the range that makes sense for this utility function.
- *Why do you need to know these things? Quadratic utility isn't bad as a local approximation, but the global properties of quadratic utility aren't very plausible. It's important to keep these in mind to diagnose the ways that models that use quadratic utility can sometimes go wrong.*

Get to Know the Data

2. The file data.txt contains CRSP data (yearly, from 1926-2012) for the return on the value-weighted portfolio of all US stocks (NYSE, AMEX, and NASDAQ), the 90-day T-bill return, Dividend Growth, and the D/P ratio. Know the units you are working with: returns and growth rates here are net (i.e., a 10 percent return is 0.10) and D/P is a fraction (e.g., 10 percent is 0.10). (Be careful in the future that you know if you are using net returns, gross returns, log returns, or something else. And be careful with growth rates.) Load the data file into MATLAB.

   (a) Find the means and standard deviations for Stocks, Bonds, and Excess Returns (the stock return - T-bill returns). Report, e.g., a 20 percent return or standard deviation as "20", not 0.20. Use the "fprintf" command to display your results in the following format (2 decimal places):

   Mean Stock Return = xx.xx, with Standard Deviation = xx.xx
   Mean Bond Return = xx.xx, with Standard Deviation = xx.xx
   Mean Excess Return = xx.xx, with Standard Deviation = xx.xx

   - Mean Stock Return = 11.59, with Standard Deviation = 20.31
   - Mean Bond Return = 3.95, with Standard Deviation = 3.44
   - Mean Excess Return = 7.64, with Standard Deviation = 20.63

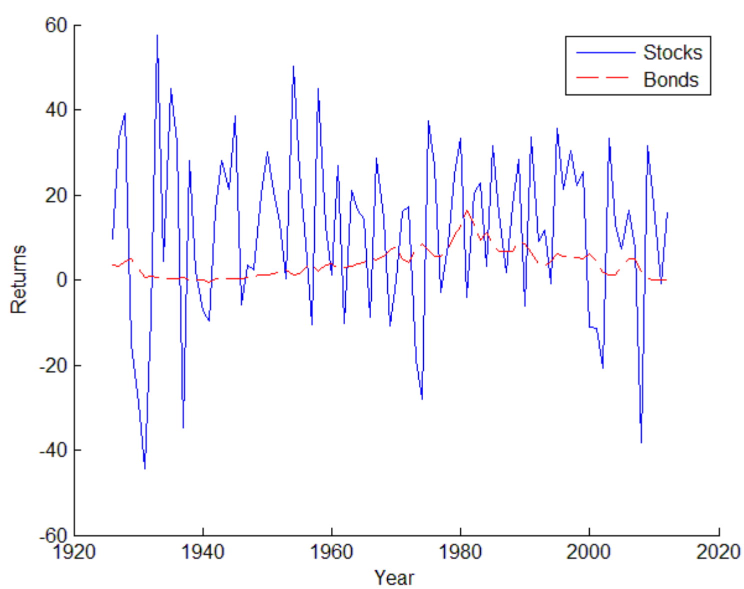
(b) Get a sense of the size of these numbers (note: they are not adjusted for inflation). How would the means of each of these change, once you adjust for inflation? (Ball-park is fine.)

- Real returns for stocks and bonds that subtract out inflation would be about 3 percent over over the sample. The excess return wouldn't change - since inflation would be subtracted from both stocks and bonds, you wouldn't have to worry about real / nominal issues here.
- *We use excess returns a lot - they are easy to work with (no real / nominal issues as above), and they isolate risk premiums from savings/investment questions. (This is essentially the return to borrowing a dollar (as a bond) and investing in stocks.)*
- *Note how much higher stock returns are compared with bonds returns... over many years, this really adds up!*
- *Note also how high the standard deviation of stock returns is - it means that, in a typical year, stocks are plus or minus* 20 *percent of the mean. Two standard deviations of* ± 40 *percent are not rare. Stocks are really volatile!*

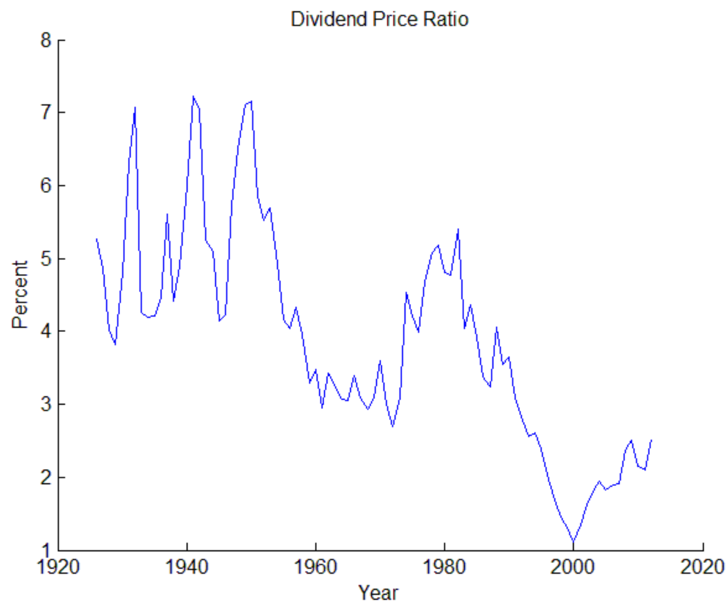
(c) The excess return is interesting - it tells you what you can get by borrowing a dollar and investing in the stock market. Is this an easy pay-day? Explain.

- No! You can essentially borrow bonds and invest in stocks and get about a 7 percent return on average. But the standard deviation of this long-short strategy is almost 20 percent!
- *Note the Sharpe ratio - the mean to standard deviation of excess returns - is equal to* 0.37.

(d) Make one plot that displays both the stock return and T-bill return (being sure that you can tell which one is which). Display the year on the x-axis and percent return (e.g., 40 percent as 40) on the y-axis.



(e) Make another plot that displays the dividend-price ratio. Display the year on the x-axis and percent (e.g., 40 percent as 40) on the y-axis.



(f) From your plots in (d) and (e), do stocks look serially correlated over time? Bonds? D/P? (Eye-balling is fine here.)

- The would seem serially correlated if an above average return in one year meant an above average (for positive) or below average (for negative) return in the next.
- Stocks looks uncorrelated, but bonds and the dividend yield both look highly correlated over time (positively).
- You should note that the stock returns line up with major booms and crashes that you know about. The bond returns rise in the 70s with inflation, and then fall. The dividend yield (price/dividend upside down) should also line up with major booms and busts. Note also that the dividend yield gives you a sense of the big historical rises and falls in asset market valuations.

(g) Run the following regressions for stocks, bonds, and excess returns:

$$R_{t+1} = a + bR_t + \varepsilon_t \text{ for } t = 1, 2, ..., T-1$$

Report the slope coefficient, its t-statistic, and the regression R^2 in the following format:

Stocks: b = xx.xx, t(b) = xx.xx, R2 = xx.xx
Bonds: b = xx.xx, t(b) = xx.xx, R2 = xx.xx
Excess: b = xx.xx, t(b) = xx.xx, R2 = xx.xx

(You can use the 'regstats' function in MATLAB here - see the help file for the syntax.)

- Stocks: b = 0.01, t(b) = 0.06, R2 = 0.00
- Bonds: b = 0.92, t(b) = 20.42, R2 = 0.83
- Excess: b = 0.01, t(b) = 0.11, R2 = 0.00

(h) What do you learn about return predictability in (g)? Does it support your answer for (c)?

- Stock returns are essentially uncorrelated (and unpredictable) over time. We see that the T-bill return has a large autocorrelation coefficient (0.92), so we see that these returns are predictable over time. The risk premium component of the excess return, then, is essentially unpredictable over time. This accords with the classic random-walk view of the world.

- Yes, it supports the fact that there are no easy pay days! We can't predict when the excess return is likely to be high, and when it's likely to be low! Otherwise, if excess returns were predictable, you could borrow and invest in stocks, making a costless profit. If interest rates are predictable, you can save more or less - but this is a weak corrective force. This is why we usually focus on excess returns - to focus on the part of returns that does not even contain the small predictable component coming from bonds.

(i) Now regress the excess return on the D/P ratio (this is a forecasting regression):

$$R^e_{t+1} = a + b\,(D/P)_t + \varepsilon_t \text{ for } t = 1, 2, ..., T-1$$

Report the slope coefficient, its t-statistic, and the regression R^2 in the following format:

Excess: b = xx.xx, t(b) = xx.xx, R2 = xx.xx

- Excess: b = 3.68, t(b) = 2.54, R2 = 0.07

(j) Provide an interpretation for the b coefficient and the R^2 of the regression.

- The coefficient is big here (with a 2.5 t-stat and a 0.07 R^2, which is "big" for finance). A coefficient of 1 would mean that prices don't move - a one percent more dividend yield means a one percent more return. A coefficient of 3.68 means that prices move the "wrong way", reinforcing the dividend yield.

(k) For the regression you were working with in (i), also report $E(R^e)$ and $\sigma\left[E_t\left(R^e_{t+1}\right)\right]$ (this latter is $\sigma\left[E_t\left(R^m_{t+1} - R^f_{t+1}\right)\right] = \sigma\left(a + b\left(D/P_t\right)\right)$) in the following format:

E(R) = xx.xx and sigma(E(R)) = xx.xx

- E(R) = 7.66 and sigma(E(R)) = 5.54

(l) What do $E(R^e)$ and $\sigma\left[E_t\left(R^e_{t+1}\right)\right]$ together tell you about the equity premium?

- You can see that the standard deviation of expected returns is as about as big as the expected return itself! I.e., the equity premium varies over time - it doesn't just sit at 7.5 percent - is jumps around over time by about its average magnitude... jumping roughly between 0 and 12.

(m) What can (i), (j), and (k) tell you about the "significance" of this regression?

- The slope coefficient b in the regression for part (i) is statistically significant (a t-stat above 2.0 is the number for statistical significance here), but "economic significance" is more important. The variation in expected returns is a better measure of the latter here, and in this regression from parts (k) and (l), that number is huge - so the regression really isn't economically significant.
- *(The slope coefficient is statistically significant, but the variation in expected returns is not small relative to the unconditional average return.)*