ECON 4360: Empirical Finance

Spring 2014

Solutions 04

Solutions

GMM Exercises: The Consumption-Based Model

1. [20 Points] Use the data on the Collab website that we used in class for the following exercises.

   (a) Find the mean and the correlation matrix of consumption growth (column 2) and the three excess returns (columns 3-5). Is consumption growth positively correlated with the returns? Which ones have greater correlations?

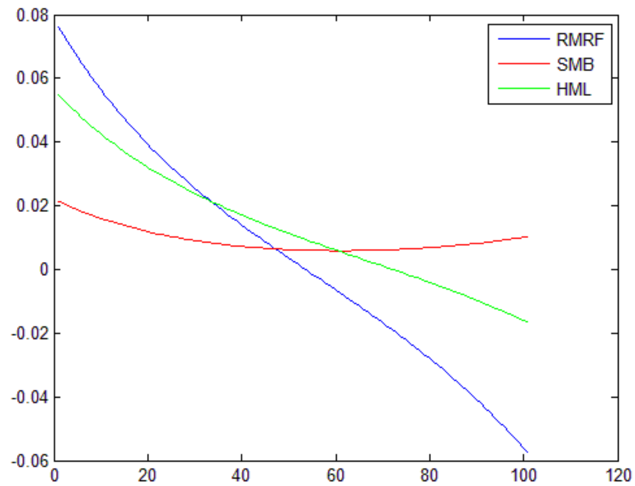
   Solution 1 *See below.*

$\mu(Cg)$	1.0218
$\mu\left(R^{mRf}\right)$	0.0763
$\mu\left(R^{SMB}\right)$	0.0213
$\mu\left(R^{HML}\right)$	0.0548

	Cg	$RMRF$	SMB	HML
Cg	1	0.3167	0.1245	0.1642
$RMRF$		1	0.2230	-0.0682
SMB			1	-0.2651
HML				1

   (b) Plot $E\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}R^e_{t+1}\right]$ for a fixed $\beta = 1$ and for γ from 0 to 100 for each $R^e = R^m - R^f$, SMB, and HML on one plot (use a different color or symbol for each line). Two of the lines will look "ok", though they will differ on γ. One line will look "wrong". What's wrong?

Solution 2 *SMB looks "wrong" - no value of gamma alone will work here.*



2. [30 Points] Use the data on the Collab website that we used in class for the following exercises.

   (a) Use the data for consumption growth (column 1), excess returns for the $R^m - R^f$ (column 3), and the risk-free rate (column 6) to estimate the discount factor β and the risk aversion coefficient γ in the consumption-based model using GMM. Assume i.i.d. moment conditions. Report $\widehat{\beta}_{GMM}$ and $\widehat{\gamma}_{GMM}$ and their standard errors.

   Solution 3 $\widehat{\beta}_{GMM} = 2.1393$ $se = 0.7158$ *and* $\widehat{\gamma}_{GMM} = 52.5236$ $se = 26.8689$

   (b) Use the data for consumption growth (column 1), excess returns for the $R^m - R^f$ (column 3), the risk-free rate (column 6), SMB (column 4), and HML (column 5) to estimate the discount factor β and the risk aversion coefficient γ in the consumption-based model using GMM. Use the HAC structure (use lag length of $q = 5$) and iterate your estimations until convergence. (I.e., go beyond just first-stage and second-stage GMM). Report $\widehat{\beta}_{GMM}$ and $\widehat{\gamma}_{GMM}$ and their standard errors.

   Solution 4 $\widehat{\beta}_{GMM} = 2.3947$ $se = 0.6288$ *and* $\widehat{\gamma}_{GMM} = 66.5951$ $se = 17.9362$

3. [50 Points] Use the NEW data file "qdata.m" for the following exercises. Note that this data is quarterly data (not annual data, as in questions 1 and 2) - the first column is real, per-capita consumption growth, the second column is SMB, the third column is HML, and fourth column is $R^m - R^f$, and the fifth column is R^f.

   (a) Find the correlation matrix of consumption growth, the three excess returns, and the risk-free rate. Is consumption growth positively correlated with the returns? Which ones have greater correlations?

Solution 5 *See below.*

	Cg	SMB	HML	$R^m - R^f$	R^f
Cg	1	0.0750	0.0432	**0.1560**	0.1273
$RMRF$		1	-0.1145	0.4072	-0.1593
SMB			1	-0.2966	0.0428
HML				1	-0.0103
R^f					1

(b) Find the correlation matrix of **the lead of** consumption growth, the three excess returns, and the risk-free rate. Is consumption growth positively correlated with the returns? Which ones have greater correlations?

Solution 6 *See below.*

	Cg	SMB	HML	$R^m - R^f$	R^f
Cg	1	0.0983	0.0126	**0.2638**	0.0357
$RMRF$		1	-0.1162	0.4070	-0.1627
SMB			1	-0.2984	0.0352
HML				1	-0.0120
R^f					1

*Note that the **lead of consumption growth** appears to be more correlated with the excess market return here.*

(c) Now, given what we're learned from parts (a) and (b), assume that the SDF takes the form

$$m_{t,t+1} = \beta \left(\frac{c_{t+2}}{c_{t+1}} \right)^{-\gamma}$$

so that we're now working with the lead of consumption growth instead of contemporaneous consumption growth. Use the data for consumption growth, $R^m - R^f$, and R^f to estimate the discount factor β and the risk aversion coefficient γ in the consumption-based model with this new discount factor using GMM. Assume i.i.d. moment conditions. Report $\widehat{\beta}_{GMM}$ and $\widehat{\gamma}_{GMM}$ and their standard errors.

Solution 7 $\widehat{\beta}_{GMM} = 1.4449$ *se* $= 0.1367$ *and* $\widehat{\gamma}_{GMM} = 115.6506$ *se* $= 37.0352$.

(d) From part (c), do you see an equity premium puzzle and/or a risk-free rate puzzle? Can you statistically reject the model using this data? Explain.

Solution 8 *These results confirm the equity premium puzzle. The model "fits" the data; but it only does so with values of γ and β that are too high to pass the common-sense test. What does a $\beta > 1$ mean?? Such a strange value is needed to keep the riskfree rate down to a reasonable level*

- but this means people would prefer to consume tomorrow rather than today! (Which I hope you see makes no sense.) I hope you see that it is not even possible to statistically reject or not reject the model because there are no overidentifying restrictions to test - yes, this was a bit of a trick question!

(e) Now estimate this same consumption-based model with $m_{t,t+1} = \beta\left(c_{t+2}/c_{t+1}\right)^{-\gamma}$ using all four return series: $R^m - R^f$, R^f, SMB, and HML. Use 2-step GMM with HAC errors with $q = 5$. Report $\widehat{\beta}_{GMM}$ and $\widehat{\gamma}_{GMM}$ and their standard errors.

Solution 9 $\widehat{\beta}_{GMM} = 1.4633$ *se* $= 0.1631$ *and* $\widehat{\gamma}_{GMM} = 136.4939$ *se* $= 35.0144$.

(f) Test the overidentifying restrictions you used in part (e). Can you reject the model based on this information? Should you reject the model?

Solution 10 *The calculated* $J-stat$ *is* 2.99*, so we cannot reject the model at the* 5 *percent significance level (see the Matlab code for calculations). Note that the overidentifying restrictions are testing whether or not the moment condidtions are close to zero for the coefficients we estimated. We know that the moments are identically zero when only the excess market return and the risk-free rate are used - so the overidentifying restrictions are really looking at whether or not* SMB *and* HML *are priced consistently with these oter two. We know from FF 1992 and FF 1993 that mean excess returns for* SMB *and* HML *are difficult to explain using market betas; but here, we are not using market betas, we are really using consumption. Even though the mean return on* HML *is bigger than* SMB*, the former's covariance with consumption growth is lower than the latter's - this is inconsistent, but not strong enough in the data for the model to be rejected. Even though we cannot reject the model statistically, we can (and should) certainly reject the model on economic grounds.*