

**ECON 4360: Empirical Finance**  
**Spring 2014**  
**Homework 02**  
**Due Monday 24 February 2014 by 1400**

## Instructions

Please write your final answers/results **on this homework page** in the spaces provided. Show all work in the pages that follow - this includes any derivations you might need, .m file scripts, output results, etc. Final answers without supporting work will receive no credit.

You may work in groups no larger than three for this assignment. If you choose to work in a group, please turn in only ONE copy of your group's assignment - just be sure to include the names of everyone in your group.

## Questions

1. [15 Points] Suppose a representative investor has utility given by  $u(c) = -e^{-\alpha c}$  where  $\alpha = 0.2$ . Imagine that there are 20 different equally likely "states of the world" that might be possible tomorrow. The possible states are  $s = 11, 12, 13, \dots, 30$ . The investor wants to eat 8 units of the consumption good today and  $s/2$  units of the consumption good tomorrow, and he discounts the future with time-preference parameter  $\beta = 0.99$ . Create an m-file script that answers the following (report to two decimal places, please):
  - a. What is the price today of a risky asset  $A$  that pays  $x_{t+1} = 30 + s/2$  in state  $s$ ? What is the expected gross return for asset  $A$ ?
  - b. What is the price today of a risky asset  $B$  that pays  $x_{t+1} = 50 - s/2$  in state  $s$ ? What is the expected gross return for asset  $B$ ?
  - c. What is the risk-free rate (gross)?
  - d. Explain the difference between the returns for assets  $A$ ,  $B$ , and the risk-free asset.

2. [10 Points] Suppose a representative investor has utility given by  $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$  where  $\gamma = 3$ . Our investor still lives in a two-period world, discounting the future (tomorrow) by  $\beta = 0.99$ . Our investor has an endowment of \$25 (for today only) and is thinking about buying shares of a stock priced at \$15 per share with the following possible payoffs: a 10% chance of \$32, a 10% chance of \$22, a 30% chance of \$10, a 40% chance of \$23, and a 10% chance of \$48. Assume it is possible to buy fractional shares.

(a) Determine the number of shares the investor will want to purchase (three decimals, please).

**Hint** Find the FOC for the utility maximization problem and then use the `fzero` function in MATLAB to numerically solve for the zeros of that equation. Careful: there may be more than one zero! You'll need to find the "global optimum", so you will want to check for more than one solution by trying different "starting points". You need to write "function files" in MATLAB to solve this problem - consult the Help file on "fzero". There are also two (very simple) example .m files uploaded on the Collab site.

3. [15 Points] Download annual consumption data from 1929-2010 - use the series: Real Personal Consumption Expenditures (PCECCA) from the FRED's Economic Data website:

<http://research.stlouisfed.org/>.

Using this data, write a MATLAB program that finds the mean-variance frontier (EQ 1.18 in Cochrane (2005)). The idea is that in order to graph Figure 1.1 in the text, you just need to find the y-intercept and slope. Assume that consumers have power utility with  $\gamma = 3.5$  and  $\beta = 0.99$ .

a. What is your estimate of the y-intercept and slope for the mean-variance frontier (four decimals, please).

b. Does your estimated placement of the mean-variance frontier make sense? Discuss in terms of the risk-free rate and the equity premium. Hint: What do you know about the historical volatility of the S&P 500?

4. [10 Points] Tomorrow, there are three possible states determined by the future price of AAPL stock as follows.

|            | State 1 | State 2 | State 3 |
|------------|---------|---------|---------|
| AAPL Stock | 350     | 375     | 400     |

A share of AAPL trades today for a price of \$360.00. An AAPL call option with a strike price of \$364.00 trades today for \$17.62, an AAPL put option with a strike price of \$386.00 trades today for \$15.40.

- a. Fill in the following table for tomorrow's payoffs:

|                                      | State 1 | State 2 | State 3 |
|--------------------------------------|---------|---------|---------|
| AAPL Stock, $P = \$360$              | 350     | 375     | 400     |
| AAPL Call, $K = 364$ , $P = \$17.62$ |         |         |         |
| AAPL Put, $K = 386$ , $P = \$15.40$  |         |         |         |

- b. Is it possible to infer what the state prices are from the prices of the stock and the options that you observe in the market?

- c. Why?

- d. If the answer to b) is yes, determine the value of the state prices (to four decimal places):

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

- e. If the state prices exist and are unique, determine the price of an AAPL put option that has a strike price  $K = 383$ .

- f. If it is possible to determine the (gross) riskfree rate in this economy, determine what it is.

- g. What is the relationship between the set of state prices and the price of a risk-less bond?

5. [15 Points] One year from now, there are four possible states determined by the future price of a barrel of oil as follows:

|                          | State 1 | State 2 | State 3 | State 4 |
|--------------------------|---------|---------|---------|---------|
| Price of Oil, per Barrel | 70      | 75      | 80      | 85      |

A barrel of oil trades today for a price of \$73.75. A call option for a barrel of oil with a strike price of \$73 trades today for \$5.00, a call with strike price \$79 trades today for \$1.50, a put with strike price \$81 trades today for \$4.00, and a put with strike price \$80 trades for \$3.27.

- a. Fill in the following table for tomorrow's payoffs:

|   | State 1 | State 2 | State 3 | State 4 |
|---|---------|---------|---------|---------|
| Price of Oil, per Barrel, $P = \$73.75$ | 70      | 75      | 80      | 85      |
| Call, $K = 73$ , $P = \$5.00$           |         |         |         |         |
| Call, $K = 79$ , $P = \$1.50$           |         |         |         |         |
| Put, $K = 81$ , $P = \$4.00$            |         |         |         |         |
| Put, $K = 80$ , $P = \$3.27$            |         |         |         |         |

- b. Use the four options to determine the value of the state prices (four decimal places):

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{bmatrix}$$

- c. Using the state prices you found for part b), what do they tell you the price of a barrel of oil should be? Is there an arbitrage opportunity?

- d. How can you form a replicating portfolio for the barrel of oil using the four options? Describe the position (long / short) for each asset and the quantity you would buy (Recall that "shorting" is mathematically equivalent to buying negative amounts of an asset):

|                               | Long or Short? | Quantity |
|-------------------------------|----------------|----------|
| Call, $K = 73$ , $P = \$5.00$ |                |          |
| Call, $K = 79$ , $P = \$1.50$ |                |          |
| Put, $K = 81$ , $P = \$4.00$  |                |          |
| Put, $K = 80$ , $P = \$3.27$  |                |          |

- e. Given the replicating portfolio you found in d), how can you exploit the arbitrage? (I.e., Do you go long or short the barrel of oil? Do you go long or short the replicating portfolio?) What is the arbitrage profit per option?

| Pos. Today  |                       | Cash Flow | Payoffs | Payoffs | Payoffs | Payoffs |
|-------------|-----------------------|-----------|---------|---------|---------|---------|
| Long/Short? |                       | Today     | State 1 | State 2 | State 3 | State 4 |
|             | Barrel of Oil         |           |         |         |         |         |
|             | Replicating Portfolio |           |         |         |         |         |
| —           | <b>TOTAL</b>          |           |         |         |         |         |

6. [5 Points] A mutual fund manager must decide which of two stocks,  $Y$  or  $Z$ , to combine with stock  $X$ . The portfolio will hold 50 percent in  $X$  and 50 percent in either  $Y$  or  $Z$ . You know the following information about the stocks:

|     | Expected Return | Standard Deviation | Correlation with $X$ |
|-----|-----------------|--------------------|----------------------|
| $X$ | 12%             | 16%                | 1.0                  |
| $Y$ | 8%              | 7%                 | 0.40                 |
| $Z$ | 8%              | 5%                 | 0.85                 |

The manager concludes that  $Z$  has the lower risk, so it is the obvious choice. Do you agree or disagree? Explain why.

7. [15 Points] You are trying to decide the best way to divide your money among 3 stocks,  $X$ ,  $Y$ , and  $Z$ . You have already estimated the following information

|     | Expected Return | Standard Deviation |
|-----|-----------------|--------------------|
| $X$ | 7%              | 16%                |
| $Y$ | 9%              | 9%                 |
| $Z$ | 8%              | 11%                |

and

| Correlations       |
|--------------------|
| $\rho_{XY} = -0.8$ |
| $\rho_{XZ} = 0.15$ |
| $\rho_{YZ} = 0.22$ |

Use Matlab to answer the following questions:

- a. Assume that short sales are allowed. Find the weights with which will give you the lowest overall portfolio standard deviation. Report your results in the table below.

| $w_X$ | $w_Y$ | $w_Z$ | $E(R^p)$ | $\sigma^p$ |
|-------|-------|-------|----------|------------|
|       |       |       |          |            |

- b. Again, assume that short sales are allowed. Find the optimal weights which minimize portfolio variance while giving expected returns of 6%, 7%, 8%, 9%, 10%, and 11%. Report your results in the table below.

| $w_X$ | $w_Y$ | $w_Z$ | $E(R^p)$ | $\sigma^p$ |
|-------|-------|-------|----------|------------|
|       |       |       | 6.0%     |            |
|       |       |       | 7.0%     |            |
|       |       |       | 8.0%     |            |
|       |       |       | 9.0%     |            |
|       |       |       | 10.0%    |            |
|       |       |       | 11.0%    |            |

- c. Graph your answers from part b) and include your graph as a separate page.
- d. Now assume that you are restricted from making short sales. Find the optimal weights which minimize portfolio variance while giving expected returns of 7%, 8%, and 9%. Report your results in the table below.

| $w_X$ | $w_Y$ | $w_Z$ | $E(R^p)$ | $\sigma^p$ |
|-------|-------|-------|----------|------------|
|       |       |       | 7.0%     |            |
|       |       |       | 8.0%     |            |
|       |       |       | 9.0%     |            |

- e. What conclusions can you draw about the effects of restricting short sales on the mean-variance frontier?

8. [10 Points] Now assume you only have the use of the two assets  $X$  and  $Y$  from the previous question and a risk-free asset that has a return of  $R^f = 4\%$ . Use Matlab to answer the following questions:

- a. Graph the possible risk-return combinations of  $X$  and  $Y$ . On the same graph, plot the new Efficiency Frontier when the possibility of investing in the risk-free asset is included. To do this, you'll need to first find the tangency portfolio. What are the weights,  $w_X$  and  $w_Y$ , for the tangency portfolio? What is the expected return and standard deviation of this portfolio? Answer in the table below and include your graph as a separate page.

| $w_X$ | $w_Y$ | $E(R^p)$ | $\sigma^p$ |
|-------|-------|----------|------------|
|       |       |          |            |

- b. If you wanted to invest part of your total investment budget in the tangency portfolio (you can think of this as a risky mutual fund) and part of your budget in the risk-free asset to provide an overall portfolio expected return of 12%, how could you do this? What would be the standard deviation of that portfolio?

9. [5 Points] You own two risky stocks,  $A$  and  $B$ , where  $\sigma_A = 2\sigma_B$  and  $\rho_{AB} = -1.0$ . You are extremely risk-averse, so you want to divide your wealth so that there is *no* risk in your portfolio. What must the weights  $w_A$  and  $w_B$  be?