## **QUALITY DATA ANALYSIS**

18/06/2021

#### **General recommendations:**

- write the solutions in CLEAR and READABLE way on paper and show (qualitatively) all the relevant plots;
- avoid (if not required) theoretical introductions or explanations covered during the course;
- always state the assumptions and report all relevant steps/discussion/formulas/expression to present and motivate your solution;
- when using hypothesis tests provide the numerical value of the test statistic and the test conclusion in terms of p-value.
- For exams in presence: to access the software on the provided laptops, go on browser → Favourites → Managed favourites → Virtual Desktop and enter your Polimi credentials.
- Exam duration: 2h 10min

#### Exercise 1 (15 points)

The energy consumption of a thermal treatment facility is known to follow a time series model  $x_t = \beta_0 + \beta_1 x_{t-1} + \varepsilon$ , with  $\varepsilon \sim N(0, \sigma_\varepsilon = 0.65)$ . Based on historical process data, the unknown coefficients estimates are available  $\widehat{\beta_0} = b_0$  and  $\widehat{\beta_1} = b_1$  and one can assume that  $E(b_0) = 9.5$ ,  $V(b_0) = 6.5$ ,  $E(b_1) = 0.64$ ,  $V(b_1) = 0.0196$ . Every month, 30 consecutive measurements (in KWh) are collected. The data collected in June and July 2020 are reported in Table 1<sup>1</sup>.

Table 1

Month	Order	Measured value	Order	Measured value
	1	26,01	16	26,47
	2	26,72	17	26,12
	3	25,24	18	26,64
	4	25,63	19	27,02
	5	25,66	20	27,32
	6	24,87	21	27,19
	7	24,68	22	26,15
June	8	24,92	23	26,28
	9	26,72	24	26,84
	10	27,76	25	26,71
	11	26,54	26	26,89
	12	27,75	27	26,87
	13	27,56	28	26,35
	14	27,02	29	26,23
	15	26,97	30	25,59
	1.5	20,37	30	23,33
Month	Order	Measured value	Order	Measured value
Month				
Month	Order	Measured value	Order	Measured value
Month	Order 1	Measured value 26,5	Order 16	Measured value 26
Month	Order 1 2	Measured value 26,5 23,74	Order 16 17	Measured value 26 24,64
Month	Order	Measured value 26,5 23,74 25,83	Order 16 17 18	Measured value 26 24,64 26,64
Month	Order  1 2 3 4	Measured value 26,5 23,74 25,83 25,47	Order 16 17 18 19	Measured value  26  24,64  26,64  25,14
Month	Order  1 2 3 4 5	Measured value 26,5 23,74 25,83 25,47 25,72	Order 16 17 18 19 20	Measured value  26  24,64  26,64  25,14  25,68
Month	Order 1 2 3 4 5	Measured value 26,5 23,74 25,83 25,47 25,72 25,6	Order  16  17  18  19  20  21	Measured value  26  24,64  26,64  25,14  25,68  24,6
	Order  1 2 3 4 5 6 7	Measured value 26,5 23,74 25,83 25,47 25,72 25,6 25,38	Order  16 17 18 19 20 21	Measured value  26  24,64  26,64  25,14  25,68  24,6  26,09
	Order  1 2 3 4 5 6 7 8	Measured value 26,5 23,74 25,83 25,47 25,72 25,6 25,38 25,67	Order  16 17 18 19 20 21 22 23	Measured value  26 24,64 26,64 25,14 25,68 24,6 26,09 25,03
	Order  1 2 3 4 5 6 7 8 9	Measured value  26,5  23,74  25,83  25,47  25,72  25,6  25,38  25,67  25,16	Order  16 17 18 19 20 21 22 23	Measured value  26 24,64 26,64 25,14 25,68 24,6 26,09 25,03 25,78
	Order  1 2 3 4 5 6 7 8 9 10	Measured value  26,5  23,74  25,83  25,47  25,72  25,6  25,38  25,67  25,16  26,13	Order  16 17 18 19 20 21 22 23 24	Measured value  26  24,64  25,14  25,68  24,6  26,09  25,03  25,78  25,25
	Order  1 2 3 4 5 6 7 8 9 10	Measured value 26,5 23,74 25,83 25,47 25,72 25,6 25,38 25,67 25,16 26,13 24,44	Order  16 17 18 19 20 21 22 23 24 25 26	Measured value  26  24,64  25,14  25,68  24,6  26,09  25,03  25,78  25,25  25,93
	Order  1 2 3 4 5 6 7 8 9 10 11	Measured value  26,5  23,74  25,83  25,47  25,72  25,6  25,38  25,67  25,16  26,13  24,44  26,36	Order  16 17 18 19 20 21 22 23 24 25 26 27	Measured value  26  24,64  25,14  25,68  24,6  26,09  25,78  25,78  25,25  25,93  24,84

<sup>&</sup>lt;sup>1</sup> An implicit use of conditional distribution is done, considering that the estimate relies on having observed the previous datapoint.

- 1) Determine if the energy consumption data in Table 1 are in-control or not by using a special cause control chart (use  $ARL_0 = 371$ ).
- 2) The head of the quality department aims to replace the special cause control chart with a control chart designed to keep under control the estimated coefficients  $b_0$  and  $b_1$ . To this aim, the model coefficients are estimated every month, i.e., the i-th values of  $b_0$  and  $b_1$  in the i-th month are estimated by fitting the model to the measurements acquired in the i-th month only. Design two control charts suitable to monitor the mean of the estimated model coefficients (assuming an overall  $ARL_0 = 371$ ).
- 3) Fit two models for June's and July's data, respectively, and estimate their model coefficients.
- 4) Based on control charts designed in point 2) and using the model coefficients estimated in point 3), is the process in these two months in-control or not? Discuss and compare the pros and cons of these control charts compared with the control chart used in point 1).

#### Exercise 2 (15 points)

The mean of piston ring diameters is monitored by means of an  $\bar{X}$  control chart. To this aim, a sample of piston rings is collected every 3 hours, and the  $\bar{X}$  control chart is designed assuming  $ARL_0 = 200$ .

- 1) Estimate the curves of the average time to signal (ATS) as a function of the mean shift  $\delta$  expressed in standard deviation units with a sample size n = 5 and n = 10, respectively (show the two curves for  $\delta \in [0, 4]$  and report the ATS values for  $\delta = 1$  and  $\delta = 2$ ).
- 2) Estimate the curves of the ATS as a function of the sample size n for two values of the shift,  $\delta = 1$  and  $\delta = 2$ . (show the two curves for  $n \in [2\ 20]$ ).
- 3) The head of the quality assurance department is interested in optimizing the process monitoring solution for two mean shifts of major interest, i.e.,  $\delta = 1$  and  $\delta = 2$ . Assuming the cost of a process in its in-control state as reference baseline, when the process mean shifts to an out-of-control state, an additional cost is due and equal to C1 = 1 for each hour spent in the out-of-control state when  $\delta = 1$  or equal to C1 = 6 for each hour spent in the out-of-control state when  $\delta = 2$ . The inspection cost is C2 = 0.8 for each measurement of the piston ring diameters. Compute the optimal value of the sample size, n, minimizing the overall expected costs when  $\delta = 1$  and when  $\delta = 2$ , respectively. Discuss the results.

### Exercise 3 (3 points)

A piston ring diameter is normally distributed  $D \sim N(\mu_0, \sigma^2)$  with  $\mu_0 = 20\,$  mm and  $\sigma = 1\,$  mm. The lower and upper specification limits are  $LSL = 18\,$  mm and  $USL = 22\,$  mm, respectively. The process produces 1 piston ring per minute and each conforming item costs  $0.1\epsilon$  while each nonconforming item costs  $2\epsilon$ .

- 1) What are the expected costs per hour when the process is in its in-control state?
- 2) What are the expected costs per hour when the process moves out-of-control with a new mean  $\mu_1 = \mu_0 + \delta \sigma$  with  $\delta = 1$ ?

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#### Multichance students can skip:

Exercise 1 point 4;

Exercise 2 point 1 and

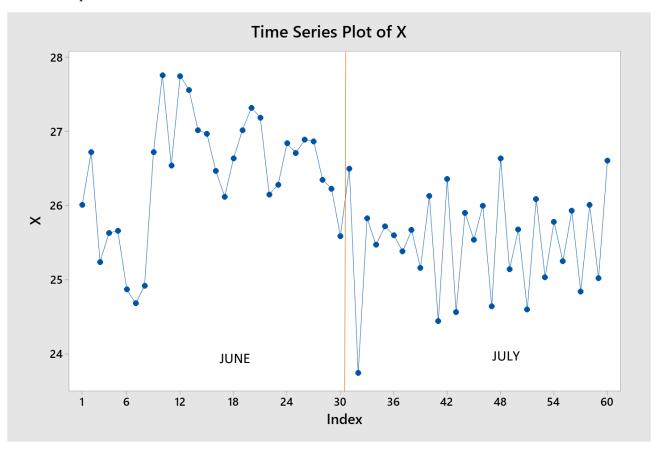
Exercice 3 point 2.

## **Solutions**

## Exercise 1

1)

Time series plot:



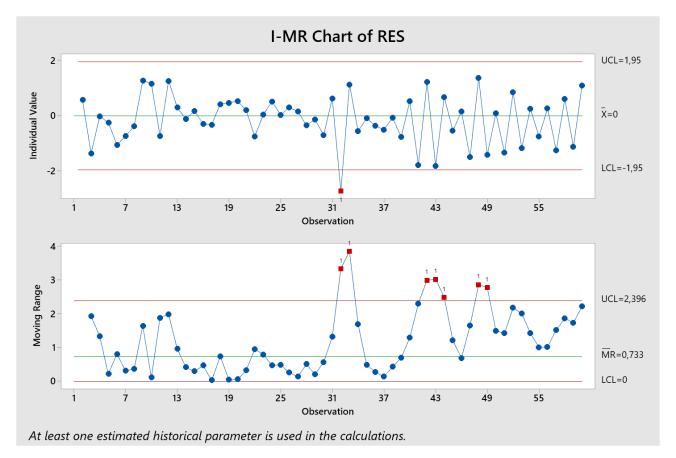
Measured data in June and July exhibit to different patterns, possibly related to a change in the autocorrelation structure.

The model in known,  $x_t = 9.5 + 0.64x_{t-1} + \varepsilon$ . The corresponding model residuals for June's and July's data are:

	Х	AR	FITS	RES
	26,01			
	26,72	26,01	26,1464	0,5736
	25,24	26,72	26,6008	-1,3608
	25,63	25,24	25,6536	-0,0236
	25,66	25,63	25,9032	-0,2432
	24,87	25,66	25,9224	-1,0524
June	24,68	24,87	25,4168	-0,7368
Julie	24,92	24,68	25,2952	-0,3752
	26,72	24,92	25,4488	1,2712
	27,76	26,72	26,6008	1,1592
	26,54	27,76	27,2664	-0,7264
	27,75	26,54	26,4856	1,2644
	27,56	27,75	27,26	0,3
	27,02	27,56	27,1384	-0,1184

26,47 26,97 26,7608 -0,2908 26,12 26,47 26,4408 -0,3208 26,64 26,12 26,2168 0,4322 27,02 26,64 26,5496 0,4704 27,32 27,02 26,64 26,5496 0,5272 27,19 27,32 26,9848 0,2052 26,15 27,19 26,9016 -0,7516 26,28 26,15 26,236 0,044 26,84 26,28 26,3192 0,5208 26,71 26,84 26,6776 0,0324 26,87 26,89 26,7096 0,1604 26,87 26,89 26,7096 0,1604 26,35 26,87 26,6968 -0,3468 26,23 26,35 26,364 -0,134 25,59 26,23 26,2872 -0,6972 23,74 26,5 25,59 25,8776 0,6224 25,47 25,83 26,0312 -0,5612 25,57 25,38 25,7432 -0,0732 25,66 25,72 25,9608 -0,3608 25,38 25,6 25,884 -0,504 25,67 25,38 25,7432 -0,0732 25,16 25,67 25,9888 -0,7688 26,13 25,16 25,6024 0,5276 24,44 26,13 26,232 -1,7832 26,36 24,44 25,1416 1,2184 24,56 26,36 26,3704 -1,8104 25,59 24,56 25,2184 0,6816 25,54 25,9 26,076 -0,536 25,14 26,64 26,5496 -1,4096 25,68 25,14 25,5896 0,0904 24,64 26 26,14 -1,5 26,64 24,64 25,2696 1,3704 25,14 26,64 26,5496 -1,4096 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 -1,4096 25,68 25,54 25,992 -1,2552 26,09 24,6 25,244 0,846 25,03 26,09 26,1976 -1,1676 25,78 25,03 25,5192 0,2608 25,25 25,78 25,9992 -0,7492 25,93 25,25 25,66 0,27 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,3976 0,6124 25,02 26,01 26,1464 -1,1264					
26,12 26,47 26,4408 -0,3208 26,64 26,12 26,2168 0,4232 27,02 26,64 26,5496 0,4704 27,32 27,02 26,67928 0,5272 27,19 27,32 26,9848 0,2052 26,15 27,19 26,9016 -0,7516 26,28 26,15 26,236 0,044 26,84 26,28 26,3192 0,5208 26,71 26,84 26,6776 0,0324 26,89 26,71 26,5944 0,2956 26,87 26,89 26,71 26,5944 0,2956 26,87 26,89 26,7096 0,1604 26,35 26,87 26,6968 -0,3468 26,23 26,35 26,364 -0,134 25,59 26,23 26,2872 -0,6972 25,53 23,74 24,6936 1,1364 25,47 25,83 26,0312 -0,5612 25,72 25,47 25,808 -0,3608 25,38 25,6 25,72 25,9608 -0,3608 25,38 25,6 25,72 25,9608 -0,3608 25,38 25,6 25,884 -0,504 25,67 25,38 25,7432 -0,0732 25,16 25,67 25,9288 -0,7688 26,33 25,16 25,6024 0,5276 24,44 26,13 26,232 -1,7832 26,36 24,44 25,1416 1,2184 24,56 26,36 26,370 -1,8104 25,59 24,56 25,74 25,8966 0,1544 24,64 26 26,370 -1,8104 25,59 24,56 25,2184 0,6816 25,68 25,14 25,896 0,0904 24,6 25,68 25,9352 -1,3352 26,09 24,6 25,244 0,846 25,03 26,09 26,1976 -1,1676 25,78 25,03 25,5192 0,2608 25,25 25,78 25,992 -1,2552 26,01 24,84 25,9376 0,6124 25,02 26,01 26,1464 -1,15		26,97	27,02	26,7928	0,1772
26,64 26,12 26,2168 0,4232 27,02 26,64 26,5496 0,4704 27,32 27,02 26,64 26,5496 0,4704 27,32 27,09 27,32 26,9848 0,2052 26,15 27,19 26,9016 -0,7516 26,28 26,15 26,236 0,044 26,84 26,28 26,319 20,5208 26,71 26,894 26,6776 0,0324 26,89 26,71 26,5944 0,2956 26,87 26,89 26,7096 0,1604 26,35 26,35 26,364 -0,134 25,59 26,23 26,35 26,364 -0,134 25,59 25,876 0,6224 23,74 26,5 26,46 2-2,72 25,83 23,74 24,6936 1,1364 25,47 25,59 25,876 25,66 25,72 25,9608 -0,3608 25,38 25,74 25,9608 -0,3608 25,38 25,76 25,59 25,876 0,6024 26,44 26,13 26,23 27,48 26,13 26,23 27,48 26,13 26,23 27,48 26,14 26,14 26,14 26,14 26,14 26,14 26,14 26,14 26,14 26,14 26,15 26,24 4,24 26,13 26,232 -1,7832 26,36 24,44 25,1416 1,2184 24,56 26,36 24,44 25,1416 1,2184 24,56 26,36 24,44 25,1416 1,2184 24,56 26,36 24,44 25,1416 1,2184 24,56 26,36 24,44 25,1416 1,2184 24,56 26,36 26,3704 -1,8104 25,94 25,68 25,14 25,8456 0,1544 24,64 26 26,24 4 25,2696 1,3704 25,14 26,64 26,5496 -1,4096 25,58 25,14 25,5896 0,0904 24,6 25,244 0,846 25,03 26,09 24,6 25,244 0,846 25,03 26,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 26,09 24,6 25,244 0,846 25,09 26,09 24,6 25,244 0,846 25,09 26,09 24,6 25,244 0,846 25,09 26,09 24,6 25,244 0,846 25,09 26,09 24,6 25,244 0,846 25,09 26,09 24,6 25,244 0,846 25,09 26,09 24,6 25,244 0,846 25,09 26,09 24,6 25,244 0,846 25,09 26,09 24,6 25,244 0,846 25,09 26,09 26,1976 -1,1676 25,78 25,09 26,009 24,6 25,244 0,846 25,09 26,09 26,1976 -1,1676 25,78 25,09 26,009 24,6 25,244 0,846 25,09 26,09 26,1976 -1,1676 25,78 25,09 26,009 24,6 25,244 0,846 25,09 26,09 26,1976 -1,1676 25,78 25,09 26,009 24,6 25,244 0,846 25,09 26,009 26,1976 -1,1676 25,78 25,09 26,009 26,1976 -1,1676 25,78 25,09 26,001 26,1464 -1,1		26,47	26,97	26,7608	-0,2908
27,02 26,64 26,5496 0,4704 27,32 27,02 26,7928 0,5272 27,19 27,32 26,9848 0,2052 26,15 27,19 26,9016 -0,7516 26,28 26,15 26,3192 0,5208 26,71 26,84 26,3192 0,5208 26,71 26,84 26,63192 0,5208 26,71 26,84 26,6776 0,0324 26,89 26,71 26,5944 0,2956 26,87 26,89 26,7096 0,1604 26,35 26,87 26,6968 -0,3468 26,23 26,35 26,364 -0,134 25,59 26,23 26,2872 -0,6972 25,53 23,74 24,6936 1,1364 25,47 25,83 26,0312 -0,5612 25,72 25,47 25,8008 -0,3608 25,38 25,6 25,884 -0,504 25,66 25,72 25,9608 -0,3608 25,38 25,6 25,884 -0,504 25,67 25,38 25,7432 -0,0732 25,16 25,67 25,38 26,3704 -1,8104 26,36 24,44 25,1416 1,2184 24,56 26,36 26,3704 -1,8104 25,9 24,56 25,2184 0,6816 25,9 24,56 25,2184 0,6816 25,9 24,56 25,2184 0,6816 25,14 26,64 25,2696 1,3704 25,14 26,64 25,2696 1,3704 25,14 26,64 25,2696 1,3704 25,14 26,64 25,2696 1,3704 25,14 26,64 25,2696 1,3704 25,14 26,64 25,2696 1,3704 25,14 26,64 25,244 0,846 25,03 26,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,09 24,6 25,244 0,846 25,28 25,78 25,935 2 -1,3352 26,09 24,6 25,244 0,846 25,28 25,78 25,935 2 -1,3352 26,09 24,6 25,244 0,846 25,28 25,78 25,999 2 -0,7492 25,93 25,25 25,66 0,27 24,84 25,93 26,095 2 -1,2552 26,01 24,84 25,93 26,095 2 -1,2552		26,12	26,47	26,4408	-0,3208
27,32   27,02   26,7928   0,5272   27,19   27,32   26,9848   0,2052   26,15   27,19   26,9016   -0,7516   26,28   26,15   26,236   0,044   26,84   26,84   26,676   0,0324   26,84   26,677   26,84   26,6776   0,0324   26,87   26,89   26,71   26,5944   0,2956   26,87   26,6968   -0,3468   26,23   26,35   26,364   -0,134   25,59   26,23   26,2872   -0,6972   26,5   25,59   25,8776   0,6224   23,74   26,5   26,46   -2,72   25,83   23,74   24,6936   1,1364   25,47   25,83   25,67   25,988   -0,5048   25,67   25,47   25,883   25,72   25,9608   -0,3608   25,38   25,6   25,884   -0,504   25,67   25,38   25,7432   -0,0732   25,16   25,67   25,38   25,7432   -0,0732   25,16   25,67   25,388   25,7432   -0,0732   25,16   25,67   25,9288   -0,7688   26,13   25,16   25,6024   0,5276   24,44   26,13   26,2232   -1,7832   26,36   24,44   25,1416   1,2184   24,56   26,36   26,3704   -1,8104   25,99   24,56   25,544   25,946   -1,4096   25,54   25,9586   0,0904   24,64   26   26,14   -1,5   26,64   24,64   25,2696   1,3704   25,14   26,64   26,5496   -1,4096   25,68   25,14   25,5896   0,0904   24,6   25,68   25,244   0,846   25,03   26,099   24,6   25,244   0,846   25,03   26,099   24,6   25,244   0,846   25,08   25,14   25,5896   0,0904   24,6   25,68   25,244   0,846   25,03   26,099   24,6   25,244   0,846   25,03   26,099   24,6   25,244   0,846   25,03   25,14   25,5896   0,0904   24,6   25,68   25,244   0,846   25,03   26,099   26,1976   -1,1676   25,78   25,03   25,5192   0,2608   25,25   25,66   0,27   24,84   25,93   25,5192   0,2608   25,25   25,78   25,03   25,5192   0,2608   25,25   25,66   0,27   24,84   25,93   26,0952   -1,2552   26,01   24,84   25,3976   0,6124   25,02   26,01   26,1464   -1,1264   25,02   26,01   26,1464   -1,1264   25,02   26,01   26,1464   -1,1264   25,02   26,01   26,1464   -1,1264   25,02   26,01   26,1464   -1,1264   25,02   26,01   26,1464   -1,1264   25,02   26,01   26,1464   -1,1264   25,02   26,01   26,1464   -1,1264   25,02   26,01   26,1464   -1,1264   25,02   26,01   26,1464   -1,1		26,64	26,12	26,2168	0,4232
27,19 27,32 26,9848 0,2052 26,15 27,19 26,9016 -0,7516 26,28 26,15 26,236 0,044 26,84 26,28 26,3192 0,5208 26,71 26,84 26,6776 0,0324 26,89 26,71 26,5944 0,2956 26,87 26,89 26,7096 0,1604 26,35 26,87 26,6968 -0,3468 26,23 26,35 26,364 -0,134 25,59 26,23 26,2872 -0,6972 26,5 25,59 25,8776 0,6224 23,74 26,5 26,46 -2,72 25,83 23,74 24,6936 1,1364 25,47 25,83 26,0312 -0,5612 25,72 25,47 25,808 -0,3808 25,38 25,6 25,84 -0,504 25,67 25,38 25,7432 -0,0732 25,16 25,67 25,988 -0,7688 26,13 25,16 25,6024 0,5276 24,44 26,13 26,232 -1,7832 24,44 26,13 26,232 -1,7832 24,44 26,13 26,232 -1,7832 24,44 26,13 26,232 -1,7832 25,16 25,67 25,988 -0,6816 25,9 24,56 25,144 0,6816 25,9 24,56 25,44 25,1416 1,2184 24,56 26,36 26,3704 -1,8104 25,9 24,56 25,44 25,2696 1,3704 25,14 26,64 26,5496 -1,4096 25,14 26,64 26,5496 -1,4096 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 0,0904 25,09 24,6 25,244 0,846 25,09 24,6 25,549 25,5192 0,2608 25,25 25,78 25,9992 -0,7492 25,93 25,25 25,66 0,27 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,3976 0,6124 25,02 26,01 24,84 25,3976 0,6124 25,02 26,01 24,84 25,3976 0,6124		27,02	26,64	26,5496	0,4704
27,19 27,32 26,9848 0,2052 26,15 27,19 26,9016 -0,7516 26,28 26,15 26,236 0,044 26,84 26,28 26,3192 0,5208 26,71 26,84 26,6776 0,0324 26,89 26,71 26,5944 0,2956 26,87 26,89 26,7096 0,1604 26,35 26,87 26,6968 -0,3468 26,23 26,35 26,364 -0,134 25,59 26,23 26,2872 -0,6972 26,5 25,59 25,8776 0,6224 23,74 26,5 26,46 -2,72 25,83 23,74 24,6936 1,1364 25,47 25,83 26,0312 -0,5612 25,72 25,47 25,808 -0,3808 25,38 25,6 25,84 -0,504 25,67 25,38 25,7432 -0,0732 25,16 25,67 25,988 -0,7688 26,13 25,16 25,6024 0,5276 24,44 26,13 26,232 -1,7832 24,44 26,13 26,232 -1,7832 24,44 26,13 26,232 -1,7832 24,44 26,13 26,232 -1,7832 25,16 25,67 25,988 -0,6816 25,9 24,56 25,144 0,6816 25,9 24,56 25,44 25,1416 1,2184 24,56 26,36 26,3704 -1,8104 25,9 24,56 25,44 25,2696 1,3704 25,14 26,64 26,5496 -1,4096 25,14 26,64 26,5496 -1,4096 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 0,0904 25,09 24,6 25,244 0,846 25,09 24,6 25,549 25,5192 0,2608 25,25 25,78 25,9992 -0,7492 25,93 25,25 25,66 0,27 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,3976 0,6124 25,02 26,01 24,84 25,3976 0,6124 25,02 26,01 24,84 25,3976 0,6124		27,32	27,02	26,7928	0,5272
26,28 26,15 26,236 0,044 26,84 26,28 26,3192 0,5208 26,71 26,84 26,6776 0,0324 26,89 26,71 26,5944 0,2956 26,87 26,89 26,7096 0,1604 26,35 26,87 26,6968 -0,3468 26,23 26,35 26,287 26,2872 -0,6972 25,59 26,23 26,2872 -0,6924 23,74 26,5 26,46 -2,72 25,83 23,74 24,6936 1,1364 25,47 25,83 26,0312 -0,5612 25,72 25,47 25,8008 -0,3608 25,6 25,72 25,9608 -0,3608 25,6 25,72 25,9608 -0,3608 25,6 25,72 25,9608 -0,3608 25,6 25,72 25,9608 -0,7688 26,13 25,16 25,67 25,9288 -0,7688 26,13 25,16 25,67 25,928 -0,7688 26,13 25,16 25,67 25,928 -0,788 26,36 24,44 25,1416 1,2184 24,56 26,36 26,3704 -1,8104 25,9 24,56 25,2184 0,6816 25,54 25,9 26,076 -0,536 25,68 25,14 25,2696 1,3704 25,14 26,64 26,5496 -1,4096 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 0,0904 25,78 25,03 25,5192 0,2608 25,25 25,78 25,992 -0,7492 25,93 25,25 25,66 0,27 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,93 26,0952 -1,2552		27,19	27,32	26,9848	0,2052
26,84 26,28 26,3192 0,5208 26,71 26,84 26,6776 0,0324 26,89 26,71 26,5944 0,2956 26,87 26,89 26,7096 0,1604 26,35 26,87 26,6968 -0,3468 26,23 26,35 26,364 -0,134 25,59 26,23 26,2872 -0,6972 26,5 25,59 25,8776 0,6224 23,74 26,5 26,46 -2,72 25,83 23,74 24,6936 1,1364 25,47 25,83 26,0312 -0,5612 25,72 25,47 25,808 -0,3608 25,6 25,72 25,9608 -0,3608 25,6 25,72 25,9608 -0,3608 25,6 25,72 25,9608 -0,3608 25,6 25,72 25,984 -0,504 25,67 25,38 25,7432 -0,0732 25,16 25,67 25,38 25,7432 -0,0732 24,44 26,13 26,2232 -1,7832 26,36 24,44 25,1416 1,2184 24,56 26,36 26,3704 -1,8104 25,9 24,56 25,2184 0,6816 25,54 25,9 26,076 -0,536 26,64 24,64 25,2696 1,3704 25,14 26,64 24,64 25,2696 1,3704 25,14 26,64 24,64 25,2696 1,3704 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 0,0904 25,68 25,14 25,5896 0,0904 24,6 25,68 25,14 25,5896 0,0904 25,03 26,09 26,1976 -1,1676 25,78 25,03 25,5192 0,2608 25,25 25,78 25,992 -0,7492 25,93 25,25 25,66 0,27 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,9376 0,6124 25,02 26,01 24,84 25,3976 0,6124 25,02 26,01 24,84 25,3976 0,6124		26,15	27,19	26,9016	-0,7516
26,71		26,28	26,15	26,236	0,044
26,89 26,71 26,5944 0,2956 26,87 26,89 26,7096 0,1604 26,35 26,87 26,6968 -0,3468 26,23 26,35 26,364 -0,134 25,59 26,23 26,2872 -0,6972 26,5 25,59 25,8776 0,6224 23,74 26,5 26,46 -2,72 25,83 23,74 24,6936 1,1364 25,47 25,83 26,0312 -0,5612 25,72 25,47 25,8008 -0,3808 25,6 25,72 25,9608 -0,3608 25,38 25,6 25,884 -0,504 25,67 25,38 25,7432 -0,0732 25,16 25,67 25,9288 -0,7688 26,13 25,16 25,6024 0,5276 24,44 26,13 26,232 -1,7832 26,36 24,44 25,1416 1,2184 24,56 26,36 26,3704 -1,8104 25,9 24,56 25,5184 0,6816 25,9 24,56 25,5284 0,6816 225,9 24,56 25,2184 0,6816 225,14 26,64 26,5496 1,3704 225,14 26,64 26,5496 -1,4096 225,14 26,64 26,5496 -1,4096 225,03 26,09 24,6 25,244 0,846 25,03 26,09 24,6 25,244 0,846 25,03 26,09 26,1976 -1,1676 25,78 25,93 25,5192 0,2608 25,25 25,78 25,9992 -0,7492 25,93 25,25 25,66 0,27 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,3976 0,6124 25,02 26,01 26,1464 -1,1264		26,84	26,28	26,3192	0,5208
26,87		26,71	26,84	26,6776	0,0324
26,35		26,89	26,71	26,5944	0,2956
26,23 26,35 26,364 -0,134 25,59 26,23 26,2872 -0,6972 26,5 25,59 25,8776 0,6224 23,74 26,5 26,46 -2,72 25,83 23,74 24,6936 1,1364 25,47 25,83 26,0312 -0,5612 25,72 25,47 25,8008 -0,0808 25,66 25,72 25,9608 -0,3608 25,38 25,6 25,884 -0,504 25,67 25,38 25,7432 -0,0732 25,16 25,67 25,38 25,7432 -0,0732 25,16 25,67 25,9288 -0,7688 26,13 25,16 25,6024 0,5276 24,44 26,13 26,2232 -1,7832 26,36 24,44 25,1416 1,2184 24,56 26,36 26,3704 -1,8104 25,9 24,56 25,2184 0,6816 25,54 25,9 26,076 -0,536 26 25,54 25,8456 0,1544 24,64 26 26,14 -1,5 26,64 24,64 25,2696 1,3704 25,14 26,64 26,5496 -1,4096 25,68 25,14 25,5896 0,0904 24,6 25,68 25,9352 -1,3352 26,09 24,6 25,244 0,846 25,03 26,09 26,1976 -1,1676 25,78 25,03 25,5192 0,2608 25,25 25,78 25,9992 -0,7492 25,93 25,25 25,66 0,27 24,84 25,93 26,0952 -1,2552 26,01 24,84 25,3976 0,6124 25,02 26,01 26,1464 -1,1264		26,87	26,89	26,7096	0,1604
25,59   26,23   26,2872   -0,6972		26,35	26,87	26,6968	-0,3468
25,59   26,23   26,2872   -0,6972		26,23	26,35	26,364	-0,134
26,5   25,59   25,8776   0,6224					-0,6972
23,74					
25,83				26,46	-2,72
July  25,47   25,83   26,0312   -0,5612   25,72   25,47   25,8008   -0,0808   25,66   25,72   25,9608   -0,3608   25,38   25,6   25,884   -0,504   25,67   25,38   25,7432   -0,0732   25,16   25,67   25,9288   -0,7688   26,13   25,16   25,6024   0,5276   24,44   26,13   26,2232   -1,7832   26,36   24,44   25,1416   1,2184   24,56   26,36   26,3704   -1,8104   25,9   24,56   25,2184   0,6816   25,54   25,9   26,076   -0,536   24,64   26   25,54   25,8456   0,1544   24,64   26   26,14   -1,5   26,64   24,64   25,2696   1,3704   25,14   26,64   26,5496   -1,4096   25,68   25,14   25,5896   0,0904   24,6   25,68   25,9352   -1,3352   26,09   24,6   25,244   0,846   25,03   26,09   26,1976   -1,1676   25,78   25,03   25,5192   0,2608   25,25   25,78   25,9992   -0,7492   25,93   25,25   25,66   0,27   24,84   25,93   26,0952   -1,2552   26,01   24,84   25,3976   0,6124   25,02   26,01   26,1464   -1,1264		25,83		24,6936	1,1364
25,72				26,0312	-0,5612
25,6   25,72   25,9608   -0,3608   25,38   25,6   25,884   -0,504   25,67   25,38   25,7432   -0,0732   25,16   25,67   25,9288   -0,7688   26,13   25,16   25,6024   0,5276   24,44   26,13   26,232   -1,7832   26,36   24,44   25,1416   1,2184   24,56   26,36   26,3704   -1,8104   25,9   24,56   25,2184   0,6816   25,54   25,9   26,076   -0,536   24,64   26   25,54   25,8456   0,1544   24,64   26   26,14   -1,5   26,64   24,64   25,2696   1,3704   25,14   26,64   26,5496   -1,4096   25,68   25,14   25,5896   0,0904   24,6   25,68   25,9352   -1,3352   26,09   24,6   25,244   0,846   25,03   26,09   26,1976   -1,1676   25,78   25,03   25,5192   0,2608   25,25   25,78   25,9992   -0,7492   25,93   25,25   25,66   0,27   24,84   25,93   26,0952   -1,2552   26,01   24,84   25,3976   0,6124   25,02   26,01   24,84   25,3976   0,6124   25,02   26,01   26,1464   -1,1264				25,8008	-0,0808
25,38       25,6       25,884       -0,504         25,67       25,38       25,7432       -0,0732         25,16       25,67       25,9288       -0,7688         26,13       25,16       25,6024       0,5276         24,44       26,13       26,2232       -1,7832         26,36       24,44       25,1416       1,2184         24,56       26,36       26,3704       -1,8104         25,9       24,56       25,2184       0,6816         25,9       24,56       25,2184       0,6816         25,54       25,9       26,076       -0,536         26,64       25,54       25,8456       0,1544         24,64       26       26,14       -1,5         26,64       24,64       25,2696       1,3704         25,14       26,64       26,5496       -1,4096         25,68       25,14       25,5896       0,0904         24,6       25,68       25,9352       -1,3352         26,09       24,6       25,244       0,846         25,03       26,09       26,1976       -1,1676         25,78       25,9992       -0,7492         25,93       25,25		25,6		25,9608	-0,3608
25,67   25,38   25,7432   -0,0732		25,38	25,6		-0,504
25,16   25,67   25,9288   -0,7688   26,13   25,16   25,6024   0,5276   24,44   26,13   26,2232   -1,7832   26,36   24,44   25,1416   1,2184   24,56   26,36   26,3704   -1,8104   25,9   24,56   25,2184   0,6816   25,54   25,9   26,076   -0,536   26,3704   -2,546   25,54   25,9   26,076   -0,536   26,64   24,64   26   26,14   -1,5   26,64   24,64   25,2696   1,3704   25,14   26,64   26,5496   -1,4096   25,68   25,14   25,5896   0,0904   24,6   25,68   25,9352   -1,3352   26,09   24,6   25,244   0,846   25,03   26,09   26,1976   -1,1676   25,78   25,03   25,5192   0,2608   25,25   25,78   25,9992   -0,7492   25,93   25,25   25,66   0,27   24,84   25,93   26,0952   -1,2552   26,01   24,84   25,3976   0,6124   25,02   26,01   26,1464   -1,1264			25,38	25,7432	-0,0732
26,13       25,16       25,6024       0,5276         24,44       26,13       26,2232       -1,7832         26,36       24,44       25,1416       1,2184         24,56       26,36       26,3704       -1,8104         25,9       24,56       25,2184       0,6816         25,54       25,9       26,076       -0,536         26       25,54       25,8456       0,1544         24,64       26       26,14       -1,5         26,64       24,64       25,2696       1,3704         25,14       26,64       26,5496       -1,4096         25,68       25,14       25,5896       0,0904         24,6       25,68       25,9352       -1,3352         26,09       24,6       25,244       0,846         25,03       26,09       26,1976       -1,1676         25,78       25,03       25,5192       0,2608         25,25       25,78       25,9992       -0,7492         25,93       25,25       25,66       0,27         24,84       25,93       26,0952       -1,2552         26,01       24,84       25,3976       0,6124         25,02					-0,7688
24,44       26,13       26,232       -1,7832         26,36       24,44       25,1416       1,2184         24,56       26,36       26,3704       -1,8104         25,9       24,56       25,2184       0,6816         25,54       25,9       26,076       -0,536         26       25,54       25,8456       0,1544         24,64       26       26,14       -1,5         26,64       24,64       25,2696       1,3704         25,14       26,64       26,5496       -1,4096         25,68       25,14       25,5896       0,0904         24,6       25,68       25,9352       -1,3352         26,09       24,6       25,244       0,846         25,03       26,09       26,1976       -1,1676         25,78       25,03       25,5192       0,2608         25,25       25,78       25,9992       -0,7492         25,93       25,25       25,66       0,27         24,84       25,93       26,0952       -1,2552         26,01       24,84       25,3976       0,6124         25,02       26,01       26,1464       -1,1264				25,6024	0,5276
July  26,36  24,44  25,1416  1,2184  24,56  26,36  26,3704  -1,8104  25,9  24,56  25,2184  0,6816  25,54  25,9  26,076  -0,536  26,14  -1,5  26,64  24,64  26,64  26,5496  -1,4096  25,14  26,64  24,64  25,2696  1,3704  25,14  26,64  26,5496  -1,4096  25,68  25,14  25,5896  0,0904  24,6  25,68  25,14  25,5896  0,0904  24,6  25,03  26,09  24,6  25,244  0,846  25,03  26,09  26,1976  -1,1676  25,78  25,78  25,93  25,25  25,66  0,27  24,84  25,93  26,0952  -1,2552  26,01  24,84  25,02  26,01  26,1464  -1,1264				26,2232	-1,7832
July       24,56       26,36       26,3704       -1,8104         25,9       24,56       25,2184       0,6816         25,54       25,9       26,076       -0,536         26       25,54       25,8456       0,1544         24,64       26       26,14       -1,5         26,64       24,64       25,2696       1,3704         25,14       26,64       26,5496       -1,4096         25,68       25,14       25,5896       0,0904         24,6       25,68       25,9352       -1,3352         26,09       24,6       25,244       0,846         25,03       26,09       26,1976       -1,1676         25,78       25,03       25,5192       0,2608         25,25       25,78       25,9992       -0,7492         25,93       25,25       25,66       0,27         24,84       25,93       26,0952       -1,2552         26,01       24,84       25,3976       0,6124         25,02       26,01       26,1464       -1,1264		26,36	24,44	25,1416	
July       25,9       24,56       25,2184       0,6816         25,54       25,9       26,076       -0,536         26       25,54       25,8456       0,1544         24,64       26       26,14       -1,5         26,64       24,64       25,2696       1,3704         25,14       26,64       26,5496       -1,4096         25,68       25,14       25,5896       0,0904         24,6       25,68       25,9352       -1,3352         26,09       24,6       25,244       0,846         25,03       26,09       26,1976       -1,1676         25,78       25,03       25,5192       0,2608         25,25       25,78       25,9992       -0,7492         25,93       25,25       25,66       0,27         24,84       25,93       26,0952       -1,2552         26,01       24,84       25,3976       0,6124         25,02       26,01       26,1464       -1,1264			26,36	26,3704	-1,8104
July  25,54  25,9  26,076  -0,536  26,14  24,64  26,25,54  25,2696  1,3704  25,14  26,64  26,64  25,2696  1,3704  25,14  26,64  25,5896  0,0904  24,6  25,68  25,14  25,5896  0,0904  24,6  25,68  25,9352  -1,3352  26,09  24,6  25,03  26,09  24,6  25,244  0,846  25,03  26,09  26,1976  -1,1676  25,78  25,78  25,93  25,5192  0,2608  25,25  25,78  25,9992  -0,7492  25,93  25,93  25,25  25,66  0,27  24,84  25,93  26,0952  -1,2552  26,01  24,84  25,3976  0,6124  25,02  26,01  26,1464  -1,1264					0,6816
July         26         25,54         25,8456         0,1544           24,64         26         26,14         -1,5           26,64         24,64         25,2696         1,3704           25,14         26,64         26,5496         -1,4096           25,68         25,14         25,5896         0,0904           24,6         25,68         25,9352         -1,3352           26,09         24,6         25,244         0,846           25,03         26,09         26,1976         -1,1676           25,78         25,03         25,5192         0,2608           25,25         25,78         25,9992         -0,7492           25,93         25,25         25,66         0,27           24,84         25,93         26,0952         -1,2552           26,01         24,84         25,3976         0,6124           25,02         26,01         26,1464         -1,1264	11	25,54			
24,64       26       26,14       -1,5         26,64       24,64       25,2696       1,3704         25,14       26,64       26,5496       -1,4096         25,68       25,14       25,5896       0,0904         24,6       25,68       25,9352       -1,3352         26,09       24,6       25,244       0,846         25,03       26,09       26,1976       -1,1676         25,78       25,03       25,5192       0,2608         25,25       25,78       25,9992       -0,7492         25,93       25,25       25,66       0,27         24,84       25,93       26,0952       -1,2552         26,01       24,84       25,3976       0,6124         25,02       26,01       26,1464       -1,1264	July				0,1544
26,64       24,64       25,2696       1,3704         25,14       26,64       26,5496       -1,4096         25,68       25,14       25,5896       0,0904         24,6       25,68       25,9352       -1,3352         26,09       24,6       25,244       0,846         25,03       26,09       26,1976       -1,1676         25,78       25,03       25,5192       0,2608         25,25       25,78       25,9992       -0,7492         25,93       25,25       25,66       0,27         24,84       25,93       26,0952       -1,2552         26,01       24,84       25,3976       0,6124         25,02       26,01       26,1464       -1,1264		24,64	26		-1,5
25,14       26,64       26,5496       -1,4096         25,68       25,14       25,5896       0,0904         24,6       25,68       25,9352       -1,3352         26,09       24,6       25,244       0,846         25,03       26,09       26,1976       -1,1676         25,78       25,03       25,5192       0,2608         25,25       25,78       25,9992       -0,7492         25,93       25,25       25,66       0,27         24,84       25,93       26,0952       -1,2552         26,01       24,84       25,3976       0,6124         25,02       26,01       26,1464       -1,1264			24,64	25,2696	1,3704
25,68     25,14     25,5896     0,0904       24,6     25,68     25,9352     -1,3352       26,09     24,6     25,244     0,846       25,03     26,09     26,1976     -1,1676       25,78     25,03     25,5192     0,2608       25,25     25,78     25,9992     -0,7492       25,93     25,25     25,66     0,27       24,84     25,93     26,0952     -1,2552       26,01     24,84     25,3976     0,6124       25,02     26,01     26,1464     -1,1264					-1,4096
24,6     25,68     25,9352     -1,3352       26,09     24,6     25,244     0,846       25,03     26,09     26,1976     -1,1676       25,78     25,03     25,5192     0,2608       25,25     25,78     25,9992     -0,7492       25,93     25,25     25,66     0,27       24,84     25,93     26,0952     -1,2552       26,01     24,84     25,3976     0,6124       25,02     26,01     26,1464     -1,1264					0,0904
26,09     24,6     25,244     0,846       25,03     26,09     26,1976     -1,1676       25,78     25,03     25,5192     0,2608       25,25     25,78     25,9992     -0,7492       25,93     25,25     25,66     0,27       24,84     25,93     26,0952     -1,2552       26,01     24,84     25,3976     0,6124       25,02     26,01     26,1464     -1,1264					-1,3352
25,03     26,09     26,1976     -1,1676       25,78     25,03     25,5192     0,2608       25,25     25,78     25,9992     -0,7492       25,93     25,25     25,66     0,27       24,84     25,93     26,0952     -1,2552       26,01     24,84     25,3976     0,6124       25,02     26,01     26,1464     -1,1264					0,846
25,25     25,78     25,9992     -0,7492       25,93     25,25     25,66     0,27       24,84     25,93     26,0952     -1,2552       26,01     24,84     25,3976     0,6124       25,02     26,01     26,1464     -1,1264		25,03		26,1976	-1,1676
25,25     25,78     25,9992     -0,7492       25,93     25,25     25,66     0,27       24,84     25,93     26,0952     -1,2552       26,01     24,84     25,3976     0,6124       25,02     26,01     26,1464     -1,1264		25,78	25,03	25,5192	0,2608
25,93     25,25     25,66     0,27       24,84     25,93     26,0952     -1,2552       26,01     24,84     25,3976     0,6124       25,02     26,01     26,1464     -1,1264					
24,84     25,93     26,0952     -1,2552       26,01     24,84     25,3976     0,6124       25,02     26,01     26,1464     -1,1264				25,66	0,27
26,01     24,84     25,3976     0,6124       25,02     26,01     26,1464     -1,1264					
25,02 26,01 26,1464 -1,1264		26,01		25,3976	0,6124
					·
		26,61	25,02	25,5128	1,0972

The resulting special cause control chart with known  $\mu_{\varepsilon}=0$  and known  $\sigma_{\varepsilon}=0.65$  is the following:



The process results to be in-control in June (although some hugging may be present in the second part of the month), whereas it results out-of-control in July (several violations of the limits in the MR control chart; only one violation on the first measurement in July in the I control chart, but a systematic pattern is present, highlighting a possible negative autocorrelation of residuals).

2)

The two control charts on the mean of model coefficients can be designed as follow:

Control chart for the constant term:

$$UCL = E(b_0) + K\sqrt{V(b_0)} = 17.67$$

$$CL = E(b_0) = 9.5$$

$$UCL = E(b_0) + K\sqrt{V(b_0)} = 1.33$$

Control chart for the autoregressive term:

$$UCL = E(b_1) + K\sqrt{V(b_1)} = 1,09$$

$$CL = E(b_1) = 0.64$$

$$UCL = E(b_1) + K\sqrt{V(b_1)} = 0.19$$

where  $K = z_{\alpha/2}$ , being  $\alpha' = 1/ARL_0$  and  $\alpha = \alpha'/2$  according to the Bonferroni's correction, as two control charts are used in parallel for the two model coefficients. Therefore: K = 3.205.

# Model fitting for June's data:

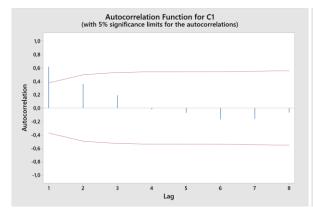
#### Test

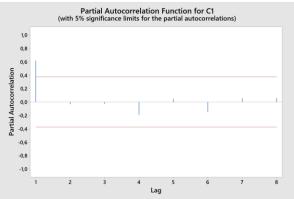
 $\label{eq:hamiltonian} Null \ hypothesis \qquad H_0: \ The \ order \ of \ the \ data \ is \ random \\ Alternative \ hypothesis \ H_1: \ The \ order \ of \ the \ data \ is \ not \ random \\$ 

Number of Runs

Observed Expected P-Value

9 15,73 0,011





# **Regression Analysis: AR1 versus C2**

## **Method**

Rows 1 unused

# **Regression Equation**

X = 9.58 + 0.637 AR1

### **Coefficients**

Term	CoefS	E Coef T	-Value P	-Value	VIF
Consta	ant 9,58	4,02	2,38	0,024	
C2	0,637	0,152	4,20	0,000	1,00

# **Model Summary**

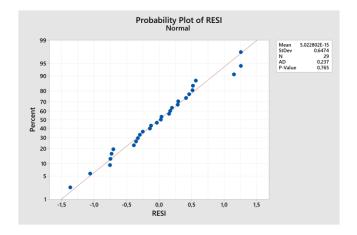
**S** R-sq R-sq(adj) R-sq(pred) 0,659275 39,49% 37,25% 29,07%

0,03321333,1370 31,2370 23,017

# **Analysis of Variance**

Source	DF	Adj SS	Adj MS	F-Value I	P-Value
Regression	1	7,660	7,6602	17,62	0,000
C2	1	7,660	7,6602	17,62	0,000
Error	27	11,735	0,4346		
Lack-of-Fit	25	8,499	0,3400	0,21	0,982
Pure Error	2	3,236	1,6182		
Total	28	19,396			

Check of assumptions on model residuals:



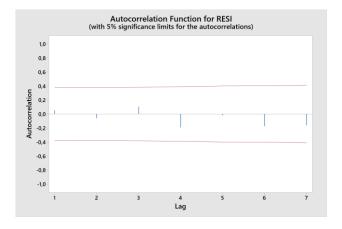
#### **Test**

Null hypothesis  $H_0$ : The order of the data is random Alternative hypothesis  $H_1$ : The order of the data is not random

**Number of Runs** 

#### Observed Expected P-Value

12 15,48 0,187



The model is appropriate.

# Model fitting for July's data:

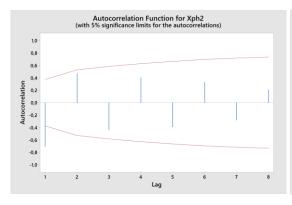
### Test

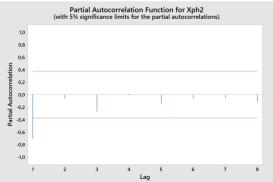
Null hypothesis  $H_0$ : The order of the data is random Alternative hypothesis  $H_1$ : The order of the data is not random

Number of Runs

## Observed Expected P-Value

27 15,73 0,000





### WORKSHEET 1

# Regression Analysis: Xph2 versus ARph2

# **Regression Equation**

Xph2 = 45,08 - 0,768 ARph2

## Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	45,08	3,43	13,15	0,000	
ARph2	-0,768	0,135	-5,71	0,000	1,00

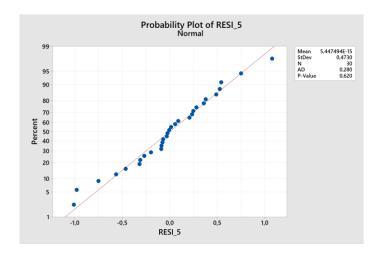
# **Model Summary**

S R-sq R-sq(adj) R-sq(pred) 0,481343 53,80% 52,15% 41,79%

# **Analysis of Variance**

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	7,555	7,5546	32,61	0,000
ARph2	1	7,555	7,5546	32,61	0,000
Error	28	6,487	0,2317		
Total	29	14,042			

Check of assumptions on model residuals:



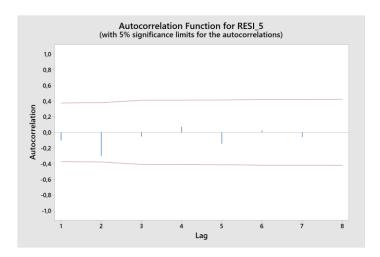
Test

Null hypothesis H<sub>0</sub>: The order of the data is random Alternative hypothesis H<sub>1</sub>: The order of the data is not random

#### **Number of Runs**

# Observed Expected P-Value

19 15,93 0,252



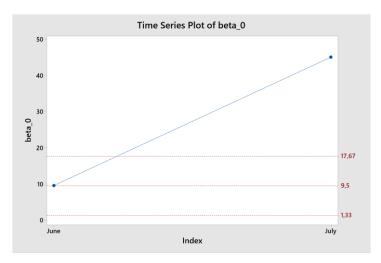
The model is appropriate.

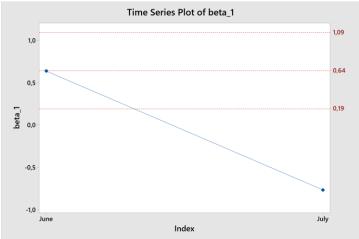
4)

Estimated model coefficients are:

	$b_0$	$b_1$
June	9.58	0.637
July	45.08	-0.768

By using the control chart designed in point 2, the test phase in June and July is the following:





The process in July results to be out-of-control both in terms of  $b_0$  and  $b_1$  coefficients. Indeed, the process in July exhibits a negative autocorrelation pattern, differently from the in-control process model.

The special cause control chart has a higher reactivity than the control chart on estimated model parameters, since it allows signaling an alarm at every measured data point, but it is also prone to false alarms related to random fluctuations within the month. The control chart on estimated model parameters is less reactive, as it allows signalling an alarm just at the end of the month, but it can be more effective if the aim is to detect a change in the underlying model despite the slowest reaction.

In real applications, when control charts on estimated model parameters are used, it is a good practice to include also a control chart on the residuals of the model to avoid information losses and enhance the detection capability of the monitoring tool.

#### Exercise 2

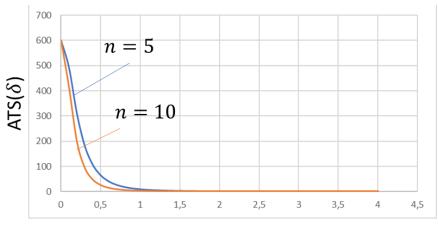
The value of  $K = z_{\alpha/2}$  with  $\alpha = \frac{1}{200} = 0.005$  is: K = 2,807.

The Type II error as a function of the mean shift in standard deviation units is given by:

$$\beta = \Pr(Z \le K - \delta \sqrt{n}) - \Pr(Z \le -K - \delta \sqrt{n})$$
, where  $\delta = \frac{\mu_1 - \mu_0}{\sigma}$ 

Then,  $ATS(\delta) = h \cdot \frac{1}{1-\beta}$  where h = 3 hours.

The  $ATS(\delta)$  curves for n=5 and n=10 are the following:



Shift,  $\delta$ 

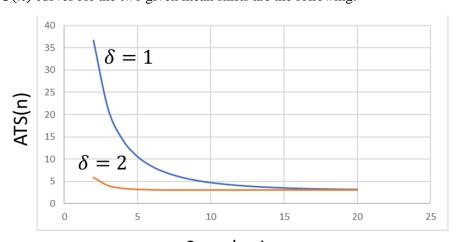
	$\delta = 1$	$\delta = 2$
ATS with n=5	10.56 h	3.15 h
ATS with n=10	4.70 h	3.00 h

b)

Being fixed  $\delta$ , the type II error can be estimated as a function of n with the same expression used in the previous case:

$$\beta = \Pr(Z \le K - \delta\sqrt{n}) - \Pr(Z \le -K - \delta\sqrt{n})$$

The resulting ATS(n) curves for the two given mean shifts are the following:



Sample size, n

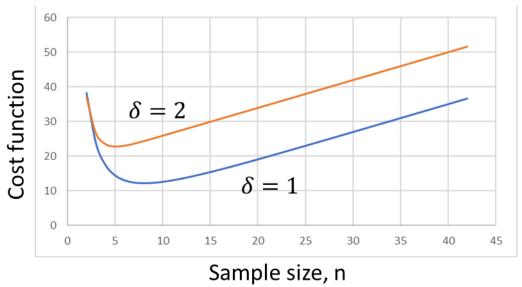
c)

The function to be minimized is the following:

$$C(n) = C1 * ATS(n) + C2 * n$$

For 
$$\delta = 1$$
,  $C(n) = 1ATS(n) + 0.8n$ , whereas for  $\delta = 2$ ,  $C(n) = 6ATS(n) + 0.8n$ .

The cost functions for the two shifts are shown below:



The optimal values of the sample size are:

- n=8 for  $\delta=1$
- n=5 for  $\delta=2$

The late detection cost predominates at smaller values of n, whereas the inspection cost predominates at larger values of n. For a smaller shift, the best compromise is obtained with a larger sample size, due to the higher relative effect of a late detection cost (with higher Type II error), whereas for a larger shift the best compromise is obtained with a smaller sample size.

#### Exercise 3

a)

When the process is in-control, the probability of non-conforming items is  $\gamma = P(D < LSL) + P(D > USL) = 0.0455$ , whereas the probability of producing conforming items is  $1 - \gamma$ .

The cost per each conforming items is therefore  $C_c = 0.1(1 - \gamma) = 0.095 \in$ , whereas the cost per each non-conforming item is  $C_{nc} = 2(\gamma) = 0.091 \in$ .

Being 60 the number of parts produced per hour, when the process is in-control ( $\mu_0 = 20 \text{ mm}$ ) the cost per hour is  $C_{IC} = 60(C_c + C_{nc}) = 11.19 \text{€}/h$ .

b)

When the process is out-of-control, the probability of non-conforming items is  $\gamma = P(D < LSL) + P(D > USL) = 0.16$ , whereas the probability of producing conforming items is  $1 - \gamma$ .

The cost per each conforming items is therefore  $C_c = 0.1(1 - \gamma) = 0.084 \in$ , whereas the cost per each non-conforming item is  $C_{nc} = 2(\gamma) = 0.32 \in$ .

Being 60 the number of parts produced per hour, when the process is out-of-control the cost per hour is  $C_{OOC} = 60(C_c + C_{nc}) = 24.24 \in /h$ .