EXERCISE 4

The data in `ESE06_ex4.csv` report life-time measurements for electric circuits. Design an X-bar and S control chart.

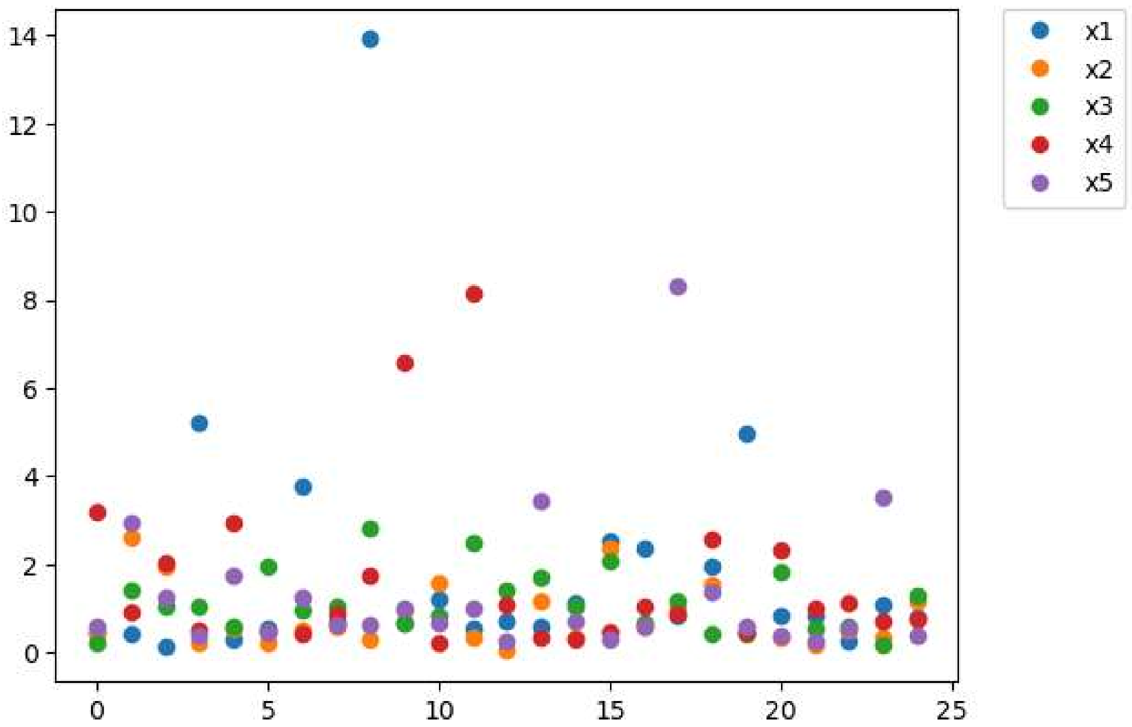
In [ ]:
```python
# Import the necessary libraries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from scipy import stats
import qda

# Import the dataset
data = pd.read_csv('ESE06_ex4.csv')

# Inspect the dataset
data.head()
```

Out[ ]:

	x1	x2	x3	x4	x5
0	0.473	0.405	0.213	3.187	0.572
1	0.430	2.623	1.415	0.915	2.933
2	0.148	1.938	1.057	2.019	1.256
3	5.209	0.211	1.047	0.492	0.388
4	0.308	0.536	0.570	2.951	1.741

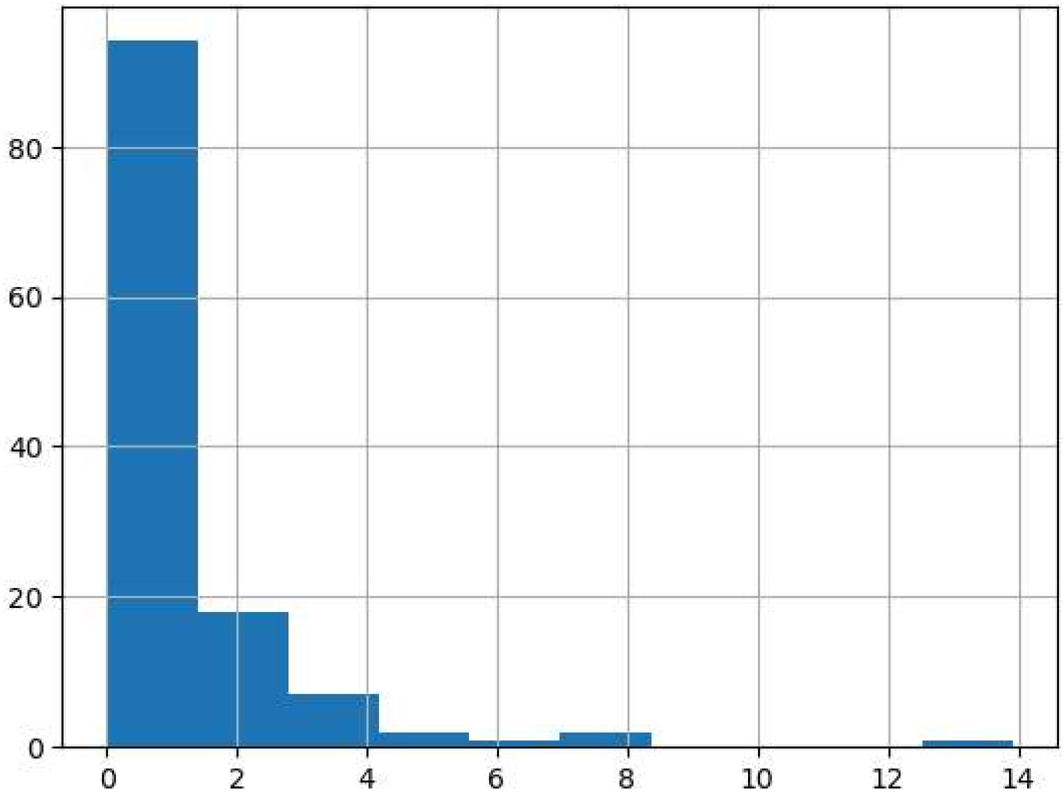
Perform some data snooping first.

In [ ]:
```python
# Make a scatter plot of all the columns against the index
plt.plot(data['x1'], linestyle='none', marker='o', label = 'x1')
plt.plot(data['x2'], linestyle='none', marker='o', label = 'x2')
plt.plot(data['x3'], linestyle='none', marker='o', label = 'x3')
plt.plot(data['x4'], linestyle='none', marker='o', label = 'x4')
plt.plot(data['x5'], linestyle='none', marker='o', label = 'x5')
# place the legend outside the plot
plt.legend(bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
plt.show()
```



Some outliers are present.

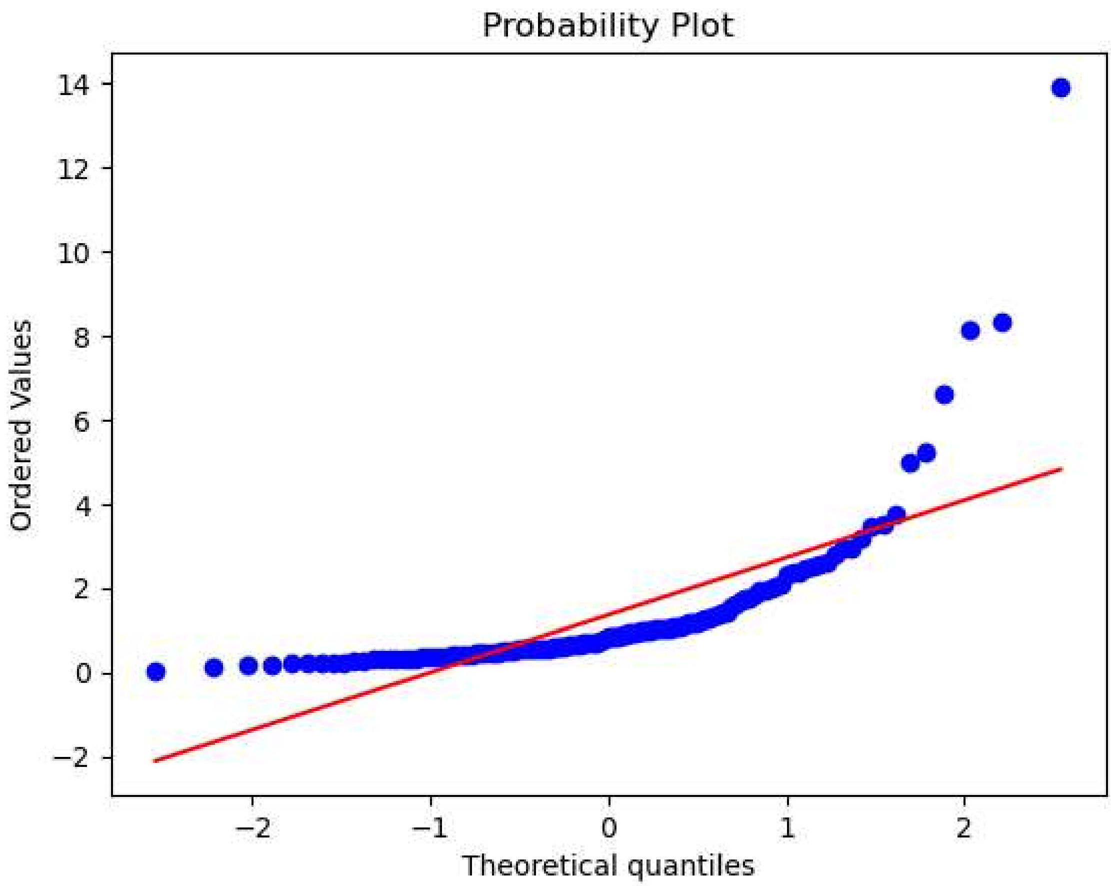
In [ ]:
```
# Stack the data into a single column
data_stack = data.stack()

# Plot a histogram of the data_stack
data_stack.hist()
plt.show()
```



In [ ]:
```
# We can use the Shapiro-Wilk test
_, p_value_SW = stats.shapiro(data_stack)
```

```python
print('p-value of the Shapiro-Wilk test: %.3f' % p_value_SW)

# QQ-plot
stats.probplot(data_stack, dist="norm", plot=plt)
plt.show()
```

```
p-value of the Shapiro-Wilk test: 0.000
```

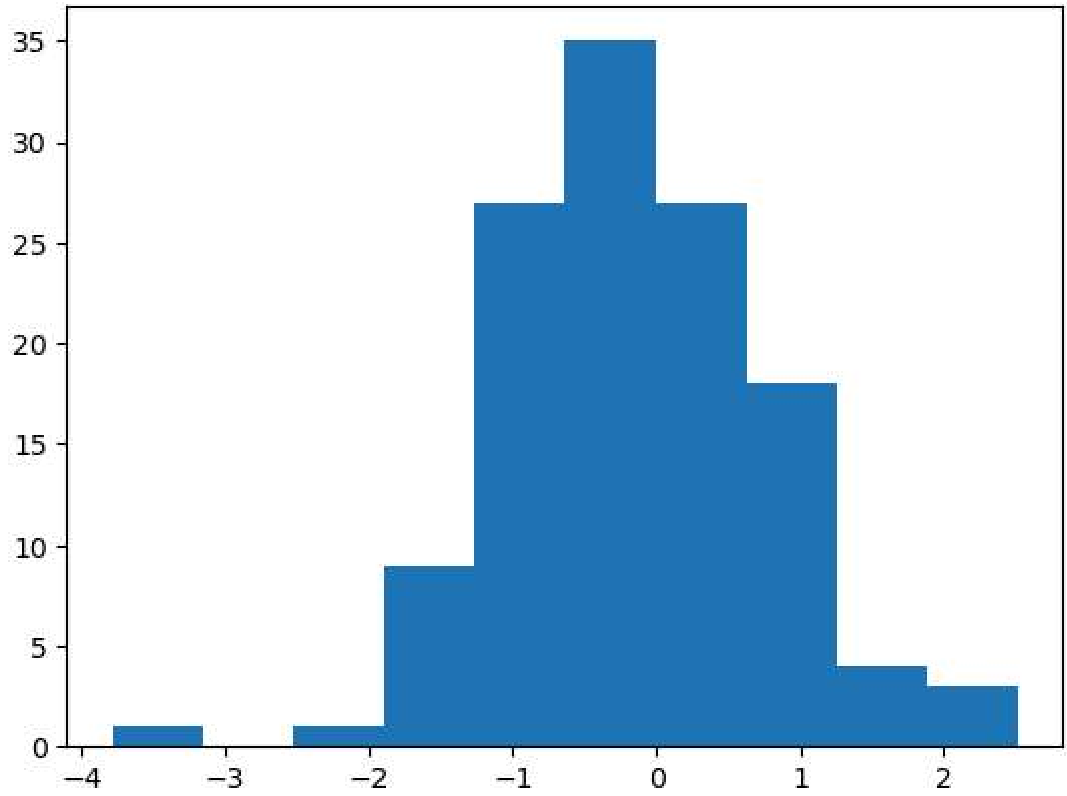


The data are skewed and not normal. Let's try to transform them.

> Let's transform the data to make it more normal using the Box-Cox transformation.
>
> Remember the Box-Cox transformation is defined as:
>
> $$x_{BC,i} = \begin{cases} \frac{x_i^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \ln x_i & \text{if } \lambda = 0 \end{cases}$$

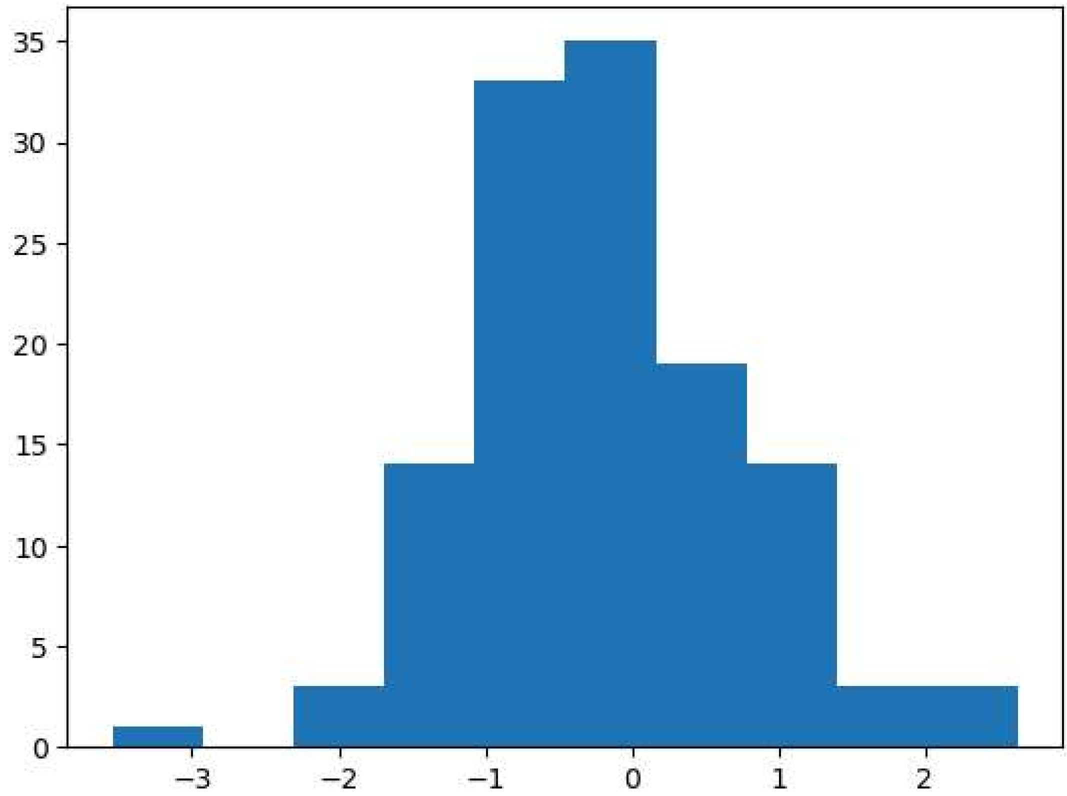
In [ ]:
```python
# Box-Cox transformation and return the transformed data
[data_BC, lmbda] = stats.boxcox(data_stack)

print('Lambda = %.3f' % lmbda)

# Plot a histogram of the transformed data
plt.hist(data_BC)
plt.show()
```

```
Lambda = -0.037
```



By default, the Box-Cox function used Lambda = -0.037. A more interpretable (and very close to optimum) value is Lambda = 0.

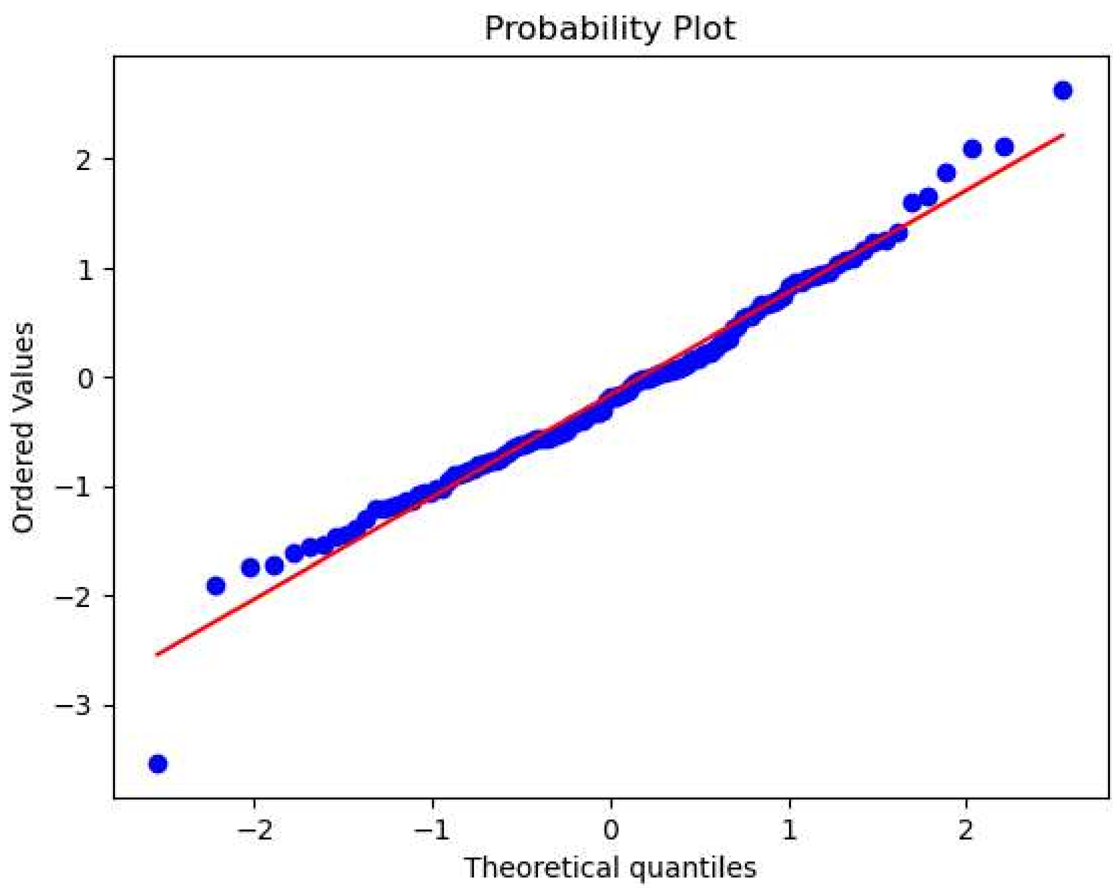
In [ ]:
```
# Use lambda = 0 for Box-Cox transformation and return the transformed data
data_BC = stats.boxcox(data_stack, lmbda=0)

# Plot a histogram of the transformed data
plt.hist(data_BC)
plt.show()
```



> Now the data seem to follow a normal distribution. Let's verify this by testing the normality.

In [ ]:
```
# We can use the Shapiro-Wilk test
_, p_value_SW = stats.shapiro(data_BC)
print('p-value of the Shapiro-Wilk test: %.3f' % p_value_SW)

# QQ-plot
stats.probplot(data_BC, dist="norm", plot=plt)
plt.show()
```

```
p-value of the Shapiro-Wilk test: 0.107
```



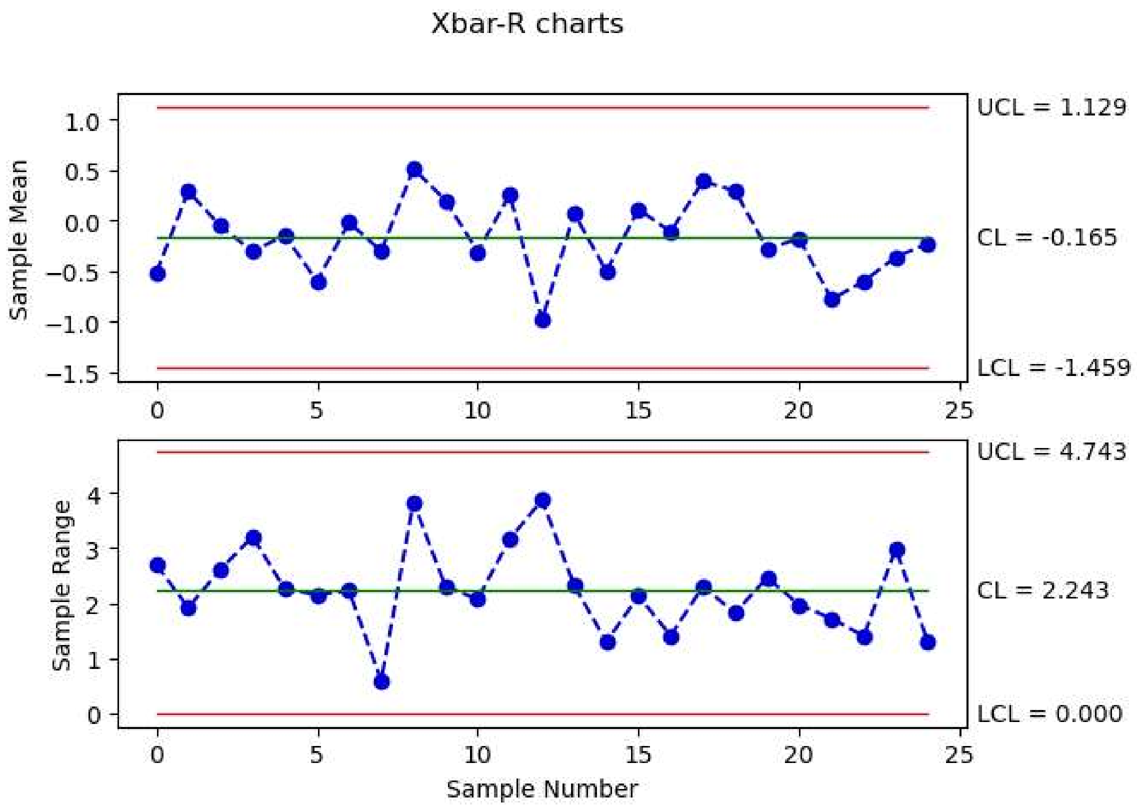
> Normality is verified. We can now use the X-bar and R chart on the transformed data.

In [ ]:
```
# First we need to unstack the data
data_BC_unstack = data_BC.reshape(data.shape)
# and convert it to a DataFrame
data_BC_unstack = pd.DataFrame(data_BC_unstack, columns = data.columns)

# Print out the transformed data
data_BC_unstack.head()
```

Out[ ]:

	x1	x2	x3	x4	x5
0	-0.748660	-0.903868	-1.546463	1.159080	-0.558616
1	-0.843970	0.964319	0.347130	-0.088831	1.076026
2	-1.910543	0.661657	0.055435	0.702602	0.227932
3	1.650388	-1.555897	0.045929	-0.709277	-0.946750
4	-1.177655	-0.623621	-0.562119	1.082144	0.554460

In [ ]:
```
# X-bar and R charts
data_BC_XR = qda.ControlCharts.XbarR(data_BC_unstack)
```



The process is in control. Let's try now with the X-bar and S chart.

$\bar{X} - S$ Control chart for transformed data:

Xbar chart (in *Xbar*-S) K=3

$$\text{UCL} = \hat{\mu} + \text{K}\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} + 3\frac{1}{c_4\sqrt{n}}\bar{s} = \bar{\bar{x}} + A_3(n)\bar{s}$$

$$\text{CL} = \hat{\mu} = \bar{\bar{x}}$$

$$\text{LCL} = \hat{\mu} - \text{K}\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} - 3\frac{1}{c_4\sqrt{n}}\bar{s} = \bar{\bar{x}} - A_3(n)\bar{s}$$

Analogously: *S chart*

parameters

known	*unknown*
$\text{UCL} = B_6(n)\sigma$	$\text{UCL} = B_4(n)\bar{s}$
$\text{CL} = c_4(n)\sigma$	$\text{CL} = \bar{s}$
$\text{LCL} = B_5(n)\sigma$	$\text{LCL} = B_3(n)\bar{s}$

$\bar{X} - S$ Control chart for transformed data:

S chart Known parameters

$$\text{UCL} = \mu_S + \text{K}\,\sigma_S = c_4\sigma + 3\sqrt{1-c_4^{\,2}}\,\sigma = B_6\sigma \Rightarrow B_6 = c_4 + 3\sqrt{1-c_4^{\,2}}$$

$$\text{CL} = \mu_S = c_4\sigma$$

$$\text{LCL} = \mu_S - \text{K}\,\sigma_S = c_4\sigma - 3\sqrt{1-c_4^{\,2}}\,\sigma = B_5\sigma \Rightarrow B_5 = c_4 - 3\sqrt{1-c_4^{\,2}}$$

Unknown parameters

$$\text{UCL} = c_4\hat{\sigma} + 3\sqrt{1-c_4^{\,2}}\,\hat{\sigma} = \bar{s} + 3\frac{\sqrt{1-c_4^{\,2}}}{c_4}\bar{s} = B_4\bar{s} \Rightarrow B_4 = 1 + 3\frac{\sqrt{1-c_4^{\,2}}}{c_4}$$

$$\text{CL} = c_4\hat{\sigma} = \bar{s}$$

$$\text{LCL} = c_4\hat{\sigma} - 3\sqrt{1-c_4^{\,2}}\,\hat{\sigma} = \bar{s} - 3\frac{\sqrt{1-c_4^{\,2}}}{c_4}\bar{s} = B_3\bar{s} \Rightarrow B_3 = 1 - 3\frac{\sqrt{1-c_4^{\,2}}}{c_4}$$

> Let's compute the mean and the range for each sample.
>
> *Note: we need to apply the mean and std functions to each row of the data frame.*

In [ ]:

```
# Make a copy of the data
data_XS = data_BC_unstack.copy()
```

```python
# Add a column with the mean of the rows
data_XS['sample_mean'] = data_BC_unstack.mean(axis=1)
# Add a column with the range of the rows
data_XS['sample_std'] = data_BC_unstack.std(axis=1)

# Inspect the dataset
data_XS.head()
```

Out[ ]:

	x1	x2	x3	x4	x5	sample_mean	sample_std
0	-0.748660	-0.903868	-1.546463	1.159080	-0.558616	-0.519705	1.009216
1	-0.843970	0.964319	0.347130	-0.088831	1.076026	0.290935	0.791392
2	-1.910543	0.661657	0.055435	0.702602	0.227932	-0.052583	1.075037
3	1.650388	-1.555897	0.045929	-0.709277	-0.946750	-0.303121	1.233562
4	-1.177655	-0.623621	-0.562119	1.082144	0.554460	-0.145358	0.930668

> Now compute the grand mean and the mean of the ranges.

In [ ]:

```python
Xbar_mean = data_XS['sample_mean'].mean()
S_mean = data_XS['sample_std'].mean()

print('Mean of the sample mean: %.3f' % Xbar_mean)
print('Mean of the sample range: %.3f' % S_mean)
```

```
Mean of the sample mean: -0.165
Mean of the sample range: 0.909
```

In [ ]:

```python
n = 5
K = 3
A3 = K * 1 / (qda.constants.getc4(n) * np.sqrt(n))
B3 = np.maximum(1 - K * (np.sqrt(1-qda.constants.getc4(n)**2)) / (qda.constants.
B4 = 1 + K * (np.sqrt(1-qda.constants.getc4(n)**2)) / (qda.constants.getc4(n))

# Now we can compute the CL, UCL and LCL for Xbar and S
data_XS['Xbar_CL'] = Xbar_mean
data_XS['Xbar_UCL'] = Xbar_mean + A3 * S_mean
data_XS['Xbar_LCL'] = Xbar_mean - A3 * S_mean

data_XS['S_CL'] = S_mean
data_XS['S_UCL'] = B4 * S_mean
data_XS['S_LCL'] = B3 * S_mean

# Inspect the dataset
data_XS.head()
```

Out[ ]:

	x1	x2	x3	x4	x5	sample_mean	sample_std	Xbar_CL
0	-0.748660	-0.903868	-1.546463	1.159080	-0.558616	-0.519705	1.009216	-0.164846
1	-0.843970	0.964319	0.347130	-0.088831	1.076026	0.290935	0.791392	-0.164846
2	-1.910543	0.661657	0.055435	0.702602	0.227932	-0.052583	1.075037	-0.164846
3	1.650388	-1.555897	0.045929	-0.709277	-0.946750	-0.303121	1.233562	-0.164846
4	-1.177655	-0.623621	-0.562119	1.082144	0.554460	-0.145358	0.930668	-0.164846

> Add two columns to store the violations of the control limits.

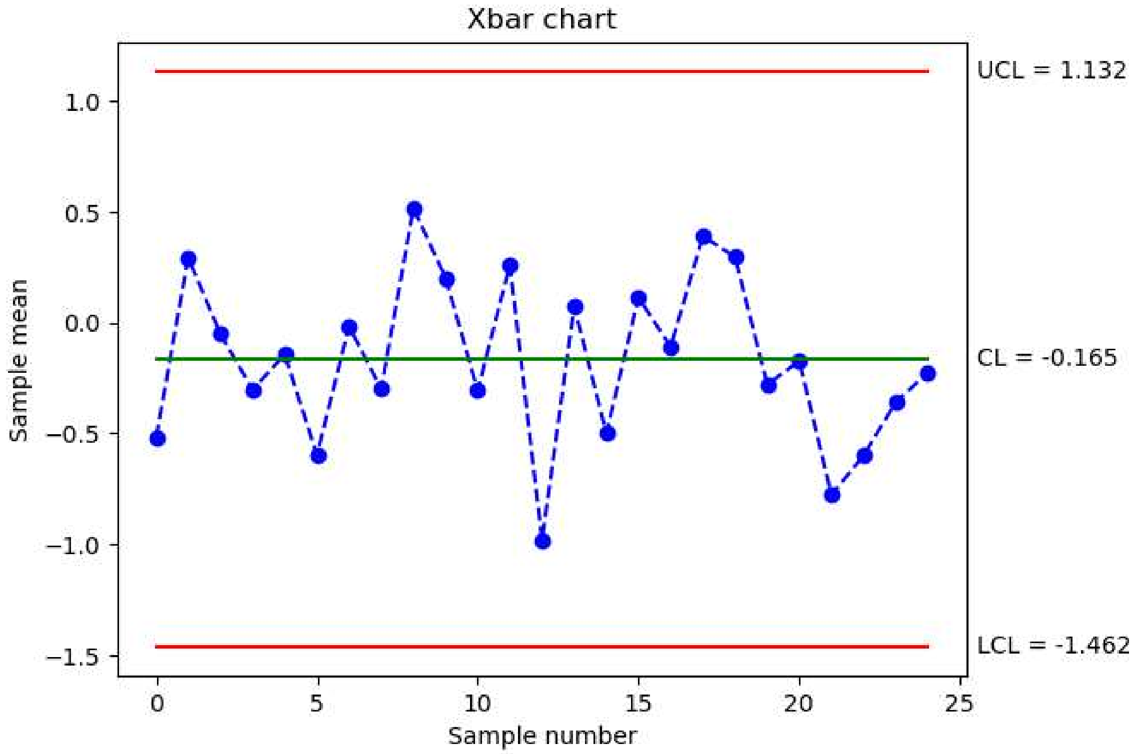
In [ ]:

```
data_XS['Xbar_TEST1'] = np.where((data_XS['sample_mean'] > data_XS['Xbar_UCL'])
                (data_XS['sample_mean'] < data_XS['Xbar_LCL']), data_XS['sample_
data_XS['S_TEST1'] = np.where((data_XS['sample_std'] > data_XS['S_UCL']) |
                (data_XS['sample_std'] < data_XS['S_LCL']), data_XS['sample_std'
```

> Now plot the limits and the data in the charts.

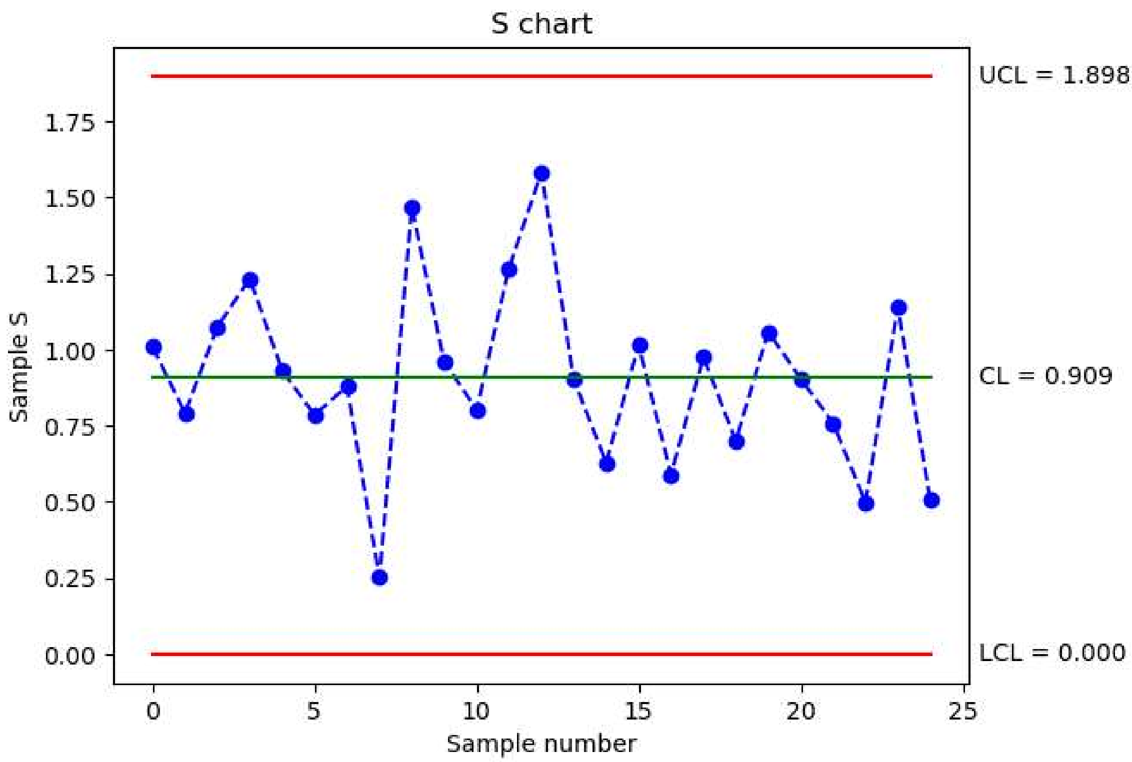
In [ ]:

```
# Plot the Xbar chart
plt.title('Xbar chart')
plt.plot(data_XS['sample_mean'], color='b', linestyle='--', marker='o')
plt.plot(data_XS['Xbar_UCL'], color='r')
plt.plot(data_XS['Xbar_CL'], color='g')
plt.plot(data_XS['Xbar_LCL'], color='r')
plt.ylabel('Sample mean')
plt.xlabel('Sample number')
# add the values of the control limits on the right side of the plot
plt.text(len(data_XS)+.5, data_XS['Xbar_UCL'].iloc[0], 'UCL = {:.3f}'.format(dat
plt.text(len(data_XS)+.5, data_XS['Xbar_CL'].iloc[0], 'CL = {:.3f}'.format(data_
plt.text(len(data_XS)+.5, data_XS['Xbar_LCL'].iloc[0], 'LCL = {:.3f}'.format(dat
# highlight the points that violate the alarm rules
plt.plot(data_XS['Xbar_TEST1'], linestyle='none', marker='s', color='r', markers
plt.show()
```



In [ ]:

```
# Plot the S chart
plt.title('S chart')
plt.plot(data_XS['sample_std'], color='b', linestyle='--', marker='o')
plt.plot(data_XS['S_UCL'], color='r')
plt.plot(data_XS['S_CL'], color='g')
plt.plot(data_XS['S_LCL'], color='r')
plt.ylabel('Sample S')
plt.xlabel('Sample number')
# add the values of the control limits on the right side of the plot
plt.text(len(data_XS)+.5, data_XS['S_UCL'].iloc[0], 'UCL = {:.3f}'.format(data_X
plt.text(len(data_XS)+.5, data_XS['S_CL'].iloc[0], 'CL = {:.3f}'.format(data_XS[
plt.text(len(data_XS)+.5, data_XS['S_LCL'].iloc[0], 'LCL = {:.3f}'.format(data_X
# highlight the points that violate the alarm rules
plt.plot(data_XS['S_TEST1'], linestyle='none', marker='s', color='r', markersize
plt.show()
```



In alternative, you can use the `XbarS` function from the `qda` package.

```
In [ ]: # X-bar and S charts
        data_BC_XS = qda.ControlCharts.XbarS(data_BC_unstack)
```