Exercise 3 (max score 4)

A univariate S control chart is used to monitor the variability of a dimensional quality characteristic in a turning process (sample size n=6). The quality characteristic in known to be normally distributed with a mean value of 0.4 mm and a standard deviation of 0.03 mm. Considering a false alarm rate α=0.01:

a) Compute the Type II error for the S chart when the process mean shifts to 0.8;
b) Compute the Type II error for the S chart when the standard deviation of the process becomes 0.07 mm.

Exercise 3

The S control chart with known parameters is:

$$LCS = \mu_S + k\sigma_S = c_4\sigma + k\sqrt{1-c_4^2}\,\sigma$$
$$LC = \mu_S = c_4\sigma$$
$$LCI = \mu_S - k\sigma_S = c_4\sigma - k\sqrt{1-c_4^2}\,\sigma$$

Alpha	0,01
Alpha/2	0,005
K=z_alpha/2	2,576
n	6
c4(6)	0,9515

S-chart		
UCL	CL	LCL
0,05232	0,02855	0,00477

a) The alternative hypothesis is

H₁: $\mu_{new} = \mu_0 + \delta$, where $\delta = 0.4$

The Type II error for the S chart can be computed as follows:

$$\beta_S = P(LCL_S \leq S \leq UCL_S | H_1) = P\left(\frac{(n-1)}{\sigma^2} LCL_S{}^2 \leq \frac{(n-1)}{\sigma^2} S^2 \leq \frac{(n-1)}{\sigma^2} UCL_S{}^2\right)$$

This is constant (it is not a function the process mean). The result is β_S =0,99.

a) The alternative hypothesis is

H₁: $\sigma_{new} = \sigma + \delta$, where $\delta = 0.04$

The Type II error for the S chart can be computed as follows:

$$\beta_S = P(LCL_S \leq S \leq UCL_S | H_1) = P\left(\frac{(n-1)}{\sigma_{new}{}^2} LCL_S{}^2 \leq \frac{(n-1)}{\sigma_{new}{}^2} S^2 \leq \frac{(n-1)}{\sigma_{new}{}^2} UCL_S{}^2\right)$$

The result is β_S =0.2682.

Exercise 3 (max score 4)

A linear model $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$ is fitted to a time series consisting of n=150 observations and the following result was obtained:

Term	Coef	SE Coef	T-value	P-value
β_0	0.0121	0.0585	0.21	0.838
β_1	-0.8692	0.0947	-9.18	0.000
β_2	-0.536	0.142	-3.78	0.001

Estimate the familywise 95% confidence interval for the significant coefficients of the model.

The confidence interval for linear model coefficients is defined as follows:

$\hat{\beta}_i - t_{\frac{\alpha}{2},n-p} se(\hat{\beta}_i) \leq \beta_i \leq \hat{\beta}_i + t_{\frac{\alpha}{2},n-p} se(\hat{\beta}_i)$.

[redacted]

In this case, the constant term is not significant, whereas both β_1 and β_2 are significant. The Bonferroni's correction should be used, such as, being $\alpha' = 0.05$, the individual confidence interval is designed with $\alpha = \frac{\alpha'}{2} = 0.025$.

Being:

- n=150
- p=3
- $t_{\frac{\alpha}{2},n-p}$=2.265

The 95% familywise confidence intervals are:

$$-1.0847 \leq \beta_1 \leq -0.6547$$

$$-0.6547 \leq \beta_2 \leq -0.21437$$

Exercise 3 (max score 3)

Let X_1 and X_2 be two quality characteristics that are known to be normal and independent, with means μ_1 and μ_2, and variances σ_1^2 and σ_2^2. We may be interested in keeping under control the process by monitoring the ratio between the variances of the two variables.

Being known that under in-control conditions $\frac{\sigma_1^2}{\sigma_2^2} = 1.1$,

a) estimate the probabilistic control limits for the control chart on the ratio between σ_1^2 and σ_2^2 (assume $\alpha = 0.01$ and the same sample size, $n = 6$, for both X_1 and X_2);
b) compute the Type II error when the ratio between σ_1^2 and σ_2^2 is twice the ratio under in-control conditions.

Being known that, under the given assumptions:

$S^2 \sim [\sigma^2/(n-1)]\chi^2(n-1)$

Then:

$$P(F_{1-\frac{\alpha}{2},n-1,n-1} \le \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \le F_{\frac{\alpha}{2},n-1,n-1}) = 1-\alpha,$$

and the probabilistic control limits for the monitored statistic $\frac{\sigma_1^2}{\sigma_2^2}$ can be computed as:

$$UCL = \frac{\sigma_1^2}{\sigma_2^2} F_{\frac{\alpha}{2},n-1,n-1} = 16{,}434$$

$$CL = \frac{\sigma_1^2}{\sigma_2^2} = 1.1$$

$$LCL = \frac{\sigma_1^2}{\sigma_2^2} F_{1-\frac{\alpha}{2},n-1,n-1} = 0{,}0736$$

The Type II error when $H_1: \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2} = 2\frac{\sigma_{0,1}^2}{\sigma_{0,2}^2}$ can be estimated as follows:

$$\beta = P\left(\frac{S_1^2}{S_2^2} \in [LCL, UCL] \middle| \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2}\right) = P\left(\frac{S_1^2}{S_2^2} < UCL \middle| \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2}\right) - P\left(\frac{S_1^2}{S_2^2} < LCL \middle| \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2}\right)$$

$$P\left(\frac{S_1^2}{S_2^2} < UCL \middle| \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2}\right) = P\left(\frac{S_1^2}{S_2^2}\frac{\sigma_{0,2}^2}{\sigma_{0,1}^2}\frac{1}{2} < \frac{\sigma_{0,1}^2}{\sigma_{0,2}^2}\frac{\sigma_{0,2}^2}{\sigma_{0,1}^2}\frac{1}{2}F_{\frac{\alpha}{2},n-1,n-1}\right) = P\left(F_{n-1,n-1} < \frac{1}{2}F_{\frac{\alpha}{2},n-1,n-1}\right) = 0.9772$$

$$P\left(\frac{S_1^2}{S_2^2} < LCL \middle| \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2}\right) = P\left(\frac{S_1^2}{S_2^2}\frac{\sigma_{0,2}^2}{\sigma_{0,1}^2}\frac{1}{2} < \frac{\sigma_{0,1}^2}{\sigma_{0,2}^2}\frac{\sigma_{0,2}^2}{\sigma_{0,1}^2}\frac{1}{2}F_{1-\frac{\alpha}{2},n-1,n-1}\right) = P\left(F_{n-1,n-1} < \frac{1}{2}F_{1-\frac{\alpha}{2},n-1,n-1}\right) = 0.00099$$

Therefore:

$\beta = 0{,}99621.$

Excercise 3 (max score 5)

Show the expression of the Operating characteristic of a Range control chart with n=2. Assume $LCL = 0{,}0024,\ UCL = 4{,}5327$ and $\sigma_R = 1$ compute the OC curve for λ equal 1 and 1,5 where λ is the ratio of the new to the old standard deviations..
(Suggestion: let X be a random variable with continuous distribution and cumulative distribution F. The absolute value of X, $|X|$, has the cumulative distribution function $G(x) = F(x) - F(-x)$, for $x > 0$)

Excercise 3

Range: $R = \max(X1 - X2) = |X1 - X2|$.

The suggestion:
let X be a random variable with continuous distribution and cumulative distribution F. The absolute value of X, $|X|$, has the cumulative distribution function $G(x) = F(x) - F(-x)$, for $x > 0$
It can be rewritten as:
$P(|X| \leq x) = P(X \leq x) - P(X \leq -x) = P(-x \leq X \leq x)$.

Therefore
Let LCL e UCL be the control limits of the Range control chart, if data are NID with variance $\sigma_1 = \lambda\sigma_0$)

$$\beta = P\left(LCI \leq R \leq LCS \middle| \sigma_1^2\right) = P\left(LCI \leq |X_1 - X_2| \leq LCS \middle| \sigma_1^2\right)$$
$$= P\left(|X_1 - X_2| \leq LCS \middle| \sigma_1^2\right) - P\left(|X_1 - X_2| \leq LCI \middle| \sigma_1^2\right)$$
$$= P\left(-LCS \leq X_1 - X_2 \leq LCS \middle| \sigma_1^2\right) - P\left(-LCI \leq X_1 - X_2 \leq LCI \middle| \sigma_1^2\right)$$

dove

$$X_1 - X_2 \sim N\left(0, 2 \cdot \sigma_1^2\right)$$

λ	β
1	0,9973
1,5	0,9665

Exercise 3 (max score 4)

With respect to the previous exercise, the quality manager wants to redesign the sample size of the control chart in order to be able to detect a change in the level from μ=0,10 mm to μ'=0,11 mm with probability 85% before the fourth sample following the shift. What is the right value of the sample size *n* to be chosen?

Excercise 3

$$1-\beta^4 = 0{,}85 \;\rightarrow \beta = (1-0{,}85)^{\frac{1}{4}} = 0{,}6223$$

Recall that when the sample size changes – *n* – the control limits change as well

$$\begin{matrix} UCL \\ LCL \end{matrix} = \mu \pm \frac{k\sigma}{\sqrt{n}} \rightarrow \beta = \Phi\left(\frac{UCL-\mu'}{\frac{\sigma}{\sqrt{n}}}\right) - \Phi\left(\frac{LCL-\mu'}{\frac{\sigma}{\sqrt{n}}}\right) = 0{,}6223$$

We can find the right n satisfying this equation using excel (considering different possible values of *n* or using the Solver).

n	UCL	LCL	beta
1	0,043859325	0,156141	**0,964551**
2	0,060302548	0,139697	**0,918151**
3	0,067587166	0,132413	**0,858806**
4	0,071929662	0,12807	**0,790176**
5	0,074893127	0,125107	**0,715988**
6	0,077080665	0,122919	**0,639658**
7	0,078780819	0,121219	**0,564065**
8	0,080151274	0,119849	**0,491466**
9	0,081286442	0,118714	**0,423493**
10	0,08224676	0,117753	**0,361203**

The best value is *n*=6.

Exercise 3 (3 points)

A S^2 control chart is used to monitor the variability of a quality characteristic. Determine the minimum sample size n needed to identify, with a probability $P \geq 80\%$, that the standard deviation of the process has increased of a factor 1.5 at the first sample after the shift (use a Type I error $\alpha = 0{,}01$).

Given:

$H_0: \sigma = \sigma_0$
$H_1: \sigma = \lambda\sigma_0$

the probability of signaling an alarm under H_1 with the S^2 control chart is:

$$P = 1 - \beta = 1 - P(S^2 \leq UCL) + P(S^2 \leq LCL) =$$
$$= 1 - P\left(X^2_{n-1} \leq \frac{X^2_{\frac{\alpha}{2},n-1}}{\lambda^2}\right) + P\left(X^2_{n-1} \leq \frac{X^2_{1-\frac{\alpha}{2},n-1}}{\lambda^2}\right)$$

Being $\lambda = 1{,}5$ and $\alpha = 0{,}01$, it is possible to estimate this probability for different values of the sample size n leading to:

n	P1	P2	beta	1-beta
2	0,003333	0,938704	0,935371	0,064629
3	0,002225	0,905088	0,902863	0,097137
4	0,001499	0,873168	0,871669	0,128331
5	0,001026	0,84168	0,840654	0,159346
6	0,000714	0,81036	0,809646	0,190354
7	0,000504	0,77919	0,778685	0,221315
8	0,000361	0,748228	0,747867	0,252133
9	0,000262	0,717558	0,717297	0,282703
10	0,000192	0,687271	0,68708	0,31292
11	0,000141	0,657452	0,65731	0,34269
12	0,000105	0,628179	0,628074	0,371926
13	7,88E-05	0,599523	0,599444	0,400556
14	5,94E-05	0,571543	0,571483	0,428517
15	4,51E-05	0,54429	0,544245	0,455755
16	3,43E-05	0,517807	0,517773	0,482227
17	2,63E-05	0,492128	0,492102	0,507898
18	2,02E-05	0,467278	0,467258	0,532742
19	1,56E-05	0,443276	0,443261	0,556739
20	1,21E-05	0,420136	0,420124	0,579876
21	9,38E-06	0,397863	0,397854	0,602146
22	7,31E-06	0,376461	0,376453	0,623547
23	5,72E-06	0,355925	0,355919	0,644081
24	4,48E-06	0,336249	0,336245	0,663755
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

25	3,52E-06	0,317424	0,317421	0,682579
26	2,77E-06	0,299435	0,299433	0,700567
27	2,19E-06	0,282268	0,282266	0,717734
28	1,73E-06	0,265903	0,265901	0,734099
29	1,37E-06	0,250321	0,25032	0,74968
30	1,09E-06	0,235501	0,2355	0,7645
31	8,66E-07	0,22142	0,221419	0,778581

[redacted]

32	6,9E-07	0,208054	0,208053	0,791947
33	5,51E-07	0,195379	0,195379	0,804621
34	4,4E-07	0,183372	0,183371	0,816629
35	3,52E-07	0,172006	0,172005	0,827995
36	2,83E-07	0,161256	0,161256	0,838744
37	2,27E-07	0,151098	0,151098	0,848902
38	1,82E-07	0,141506	0,141506	0,858494
39	1,47E-07	0,132456	0,132456	0,867544
40	1,18E-07	0,123924	0,123924	0,876076

The minimum sample size such that the probability of detecting the change of the process variability is $P \geq 80\%$ is $n = 33$.

Exercise 3 (5 points)

A quality characteristic is measured by sampling products from two different production lines. The two lines are independent and the quality characteristic from both of them is known to be normal with means μ_1 (line 1) and μ_2 (line 2), and variances σ_1^2 and σ_2^2. The head of the quality control department is interested in keeping under control the process by monitoring the ratio between the variances of the quality characteristic measured in the two lines. Being known that under in-control conditions, $\sigma_1^2 = 3{,}5$ and $\sigma_2^2 = 2{,}5$:

1) Estimate the probabilistic control limits for a control chart on the ratio between σ_1^2 and σ_2^2 (assume $\alpha = 0.01$ and the same sample size, $n = 8$, from both production lines).
2) The head of the quality department is interested in changing the sample size in order to tune the performances of the control chart. To this aim, determine the minimum sample size that allows signalling, with a probability at least equal to 70%, that the ratio between the variances is four times the one under in-control conditions.

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1)

Being known that, under the given assumptions:

$S^2 \sim [\sigma^2/(n-1)]\chi^2(n-1)$

Then:

$P(F_{1-\frac{\alpha}{2},n-1,n-1} \leq \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \leq F_{\frac{\alpha}{2},n-1,n-1}) = 1-\alpha,$

and the probabilistic control limits for the ratio between the two variances can be computed as:

$$UCL = \frac{\sigma_1^2}{\sigma_2^2} F_{\frac{\alpha}{2},n-1,n-1} = 12{,}44$$

$$CL = \frac{\sigma_1^2}{\sigma_2^2} = 1{,}4$$

$$LCL = \frac{\sigma_1^2}{\sigma_2^2} F_{1-\frac{\alpha}{2},n-1,n-1} = 0{,}158$$

Where: $\sigma_1^2 = 3{,}5$, $\sigma_2^2 = 2{,}5$, $\alpha = 0.01$ and $n = 8$.

2)

The probability to detect an increase of the ratio between the variances when $H_1\colon \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2} = 4\frac{\sigma_{0,1}^2}{\sigma_{0,2}^2}$ can be estimated as follows:

$$1-\beta = 1 - P\left(\frac{S_1^2}{S_2^2} \in [LCL, UCL] \middle| \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2}\right) = 1 - \left[P\left(\frac{S_1^2}{S_2^2} < UCL \middle| \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2}\right) - P\left(\frac{S_1^2}{S_2^2} < LCL \middle| \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2}\right)\right]$$

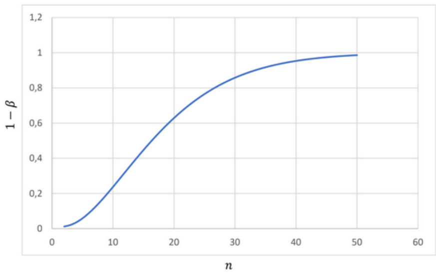
$$P\left(\frac{S_1^2}{S_2^2} < UCL \middle| \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2}\right) = P\left(\frac{S_1^2}{S_2^2}\frac{\sigma_{0,2}^2}{\sigma_{0,1}^2}\frac{1}{4} < \frac{\sigma_{0,1}^2}{\sigma_{0,2}^2}\frac{\sigma_{0,2}^2}{\sigma_{0,1}^2}\frac{1}{4}F_{\frac{\alpha}{2},n-1,n-1}\right) = P\left(F_{n-1,n-1} < \frac{1}{4}F_{\frac{\alpha}{2},n-1,n-1}\right)$$

$$P\left(\frac{S_1^2}{S_2^2} < LCL \middle| \frac{\sigma_{1,1}^2}{\sigma_{1,2}^2}\right) = P\left(\frac{S_1^2}{S_2^2}\frac{\sigma_{0,2}^2}{\sigma_{0,1}^2}\frac{1}{4} < \frac{\sigma_{0,1}^2}{\sigma_{0,2}^2}\frac{\sigma_{0,2}^2}{\sigma_{0,1}^2}\frac{1}{4}F_{1-\frac{\alpha}{2},n-1,n-1}\right) = P\left(F_{n-1,n-1} < \frac{1}{4}F_{1-\frac{\alpha}{2},n-1,n-1}\right)$$

Thus:

$$1-\beta = 1 - \left[P\left(F_{n-1,n-1} < \frac{1}{4}F_{\frac{\alpha}{2},n-1,n-1}\right) - P\left(F_{n-1,n-1} < \frac{1}{4}F_{1-\frac{\alpha}{2},n-1,n-1}\right)\right]$$

This probability as a function of the sample size n can be represented as follows:



The smallest sample size to detect the out-of-control state with a probability ≥70% is $n = 23$.

Exercise 3 (3 points)

A company is interested in observing the stability of the linear drift of a tool. To this aim, for each tool, a linear model is fitted $y = \beta_0 + \beta_1 t + \varepsilon_t$ starting from the same number n of data observed in each drift curve.

Then, for each curve the slope $\hat{\beta}_1 = b_1$ is estimated using ordinary least squares. Assuming $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, and assuming known β_0 and β_1, describe the expression of the control limits for a control chart for monitoring the curve slope of each tool.

Exercise 3 solution

The estimated slope b_1 is a random variable such that:

$E(b_1) = \beta_1, V(b_1) = \frac{\sigma_\varepsilon^2}{S_{xx}}$ where:

- σ_ε^2 is the variance of the normal error term
- $S_{xx} = \sum_{i=1}^{n}(t_i - \bar{t})^2$

By using the Shewart's scheme and assuming known parameters, the control chart for b_1 can be designed as follows:

$$UCL = \beta_1 + z_{\alpha/2}\sqrt{\frac{\sigma_\varepsilon^2}{S_{xx}}}$$

$$CL = \beta_1$$

$$LCL = \beta_1 - z_{\alpha/2}\sqrt{\frac{\sigma_\varepsilon^2}{S_{xx}}}$$

Where α is the Type I error.

The control charts can be used to monitor the stability over time of the drift curve slopes for different tools. It can be possibly combined with a control chart on $\hat{\sigma}_\varepsilon^2$, to monitor the model residuals as well.

Exercise 3 (3 points)

An $\bar{X}$ control chart with $K = 3$ is used to monitor the mean of a process. A sample of size $n = 3$ is collected every 4 hours and it is known that in the presence of a shift of the mean δ equal to 1.1 times the standard deviation of the monitored variable X, the average time to signal an alarm is 29.24 hours.

The head of the quality control department wants to reduce this time. She has two options: a) increase the sample size to $n = 6$, or b) reduce the time between consecutive sample collections to 2 h. Compute the average time to signal the mean shift δ for these two options. Which is the best option? Discuss the result.

Exercise 3

Option a) (increase of sample size)

In the presence of a mean shift $\delta = \frac{\mu_1 - \mu_0}{\sigma} = 1{,}1$, the Type II error for an $\bar{X}$ control chart with $n = 6$ and $K = 3$ is:

$$\beta = \Pr(LCL \leq \bar{X} \leq UCL | H_1) = \Phi(3 - \delta\sqrt{6}) - \Phi(-3 - \delta\sqrt{6}) = 0.62$$

Being $\Delta T = 4$ h the time between two consecutive sample collections, the corresponding average time to signal is:

$$ATS = \Delta T \cdot ARL_1 = \Delta T \frac{1}{1-\beta} = 10{,}53 \text{ h}$$

Option b) (reduce of time between two consecutive samples)

The Type II error is the same of the original control chart, i.e.,

$$\beta = \Pr(LCL \leq \bar{X} \leq UCL | H_1) = \Phi(3 - \delta\sqrt{3}) - \Phi(-3 - \delta\sqrt{3}) = 0.8632$$

Being $\Delta T = 2$ h the time between two consecutive sample collections, the corresponding average time to signal is:

$$ATS = \Delta T \cdot ARL_1 = \Delta T \frac{1}{1-\beta} = 14.62 \text{ h}$$

In both cases, the number of items to be inspected per unit time is the same, but option a), i.e., increasing the sample size, is the most efficient as it yields the lowest average time to signal for the given shift.

Exercise 3 (5 points)

An $\bar{X}$ control chart with $K = 3$ and $n = 5$ is used to monitor the mean of a process (samples are collected every 4 hours).

It is known that in the presence of a shift of the mean δ equal to 1,1 times the standard deviation of the monitored variable X, the average time to signal an alarm is 13,58 hours.

a) Determine the minimum sample size n to decrease such average time to signal to 8 h
b) Keeping fixed the original sample size ($n = 5$), determine the minimum time between consecutive sample collections to decrease the average time to signal to 8 h. Discuss the pros and contras of the two options.

a) (increase of sample size)

In the presence of a mean shift $\delta = \frac{\mu_1 - \mu_0}{\sigma} = 1{,}1$, the Type II error for an $\bar{X}$ control chart with $K = 3$ is:

$$\beta = \Pr(LCL \le \bar{X} \le UCL|H_1) = \Phi(3 - \delta\sqrt{n}) - \Phi(-3 - \delta\sqrt{n})$$

Being $\Delta T = 4$ h the time between two consecutive sample collections, the corresponding average time to signal is:

$$ATS = \Delta T \cdot ARL_1 = \Delta T \frac{1}{1-\beta}$$

By recursively increase n, we have:

n	ATS
5	13,5830
6	10,5272
7	8,6156
8	7,3489

Thus, the sample size should be increased to $n = 8$.

b) (reduce of time between two consecutive samples)

The Type II error is the same of the original control chart, i.e.,

$$\beta = \Pr(LCL \le \bar{X} \le UCL|H_1) = \Phi(3 - \delta\sqrt{5}) - \Phi(-3 - \delta\sqrt{5}) = 0{,}706$$

By recursively decrease ΔT, we get that for $\Delta T = 2{,}3$ h, the ATS can be decreased down to about 8 h.

Both reducing the sampling interval and increasing the sample size yields about the same number of inspected part per unit time. However, reducing the sampling interval increases the risk of autocorrelation among successively sampled data.

Exercise 3 (3 points)

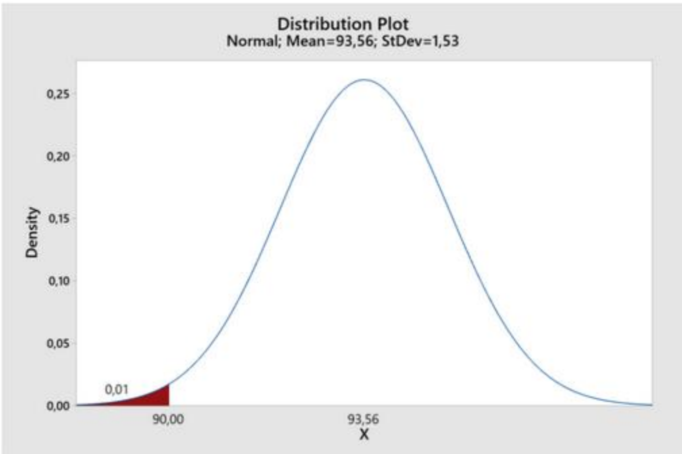
Referring to the application described in Exercise 1, it is known that a pre-heating temperature lower that 90 °C leads to parts whose quality is not conforming with respect to the product specifications. Assuming that the pre-heating temperature follows a normal distribution with mean μ and standard deviation $\sigma = 1{,}53$ °C, estimate the target mean temperature, μ, such that the probability of having non-conforming builds is 1%.

Exercise 3

If the one-sided specification limit LSL = 90 °C corresponds to a non-conforming probability $\gamma = 0{,}01$, it is possible to estimate the mean temperature μ as follows:

$$z_\gamma = \frac{LSL - \mu}{\sigma} = \frac{90 - \mu}{1{,}53} = -2{,}326$$

Which results into: $\mu = 93{,}56\,°C$.



Exercise 3 (3 points)

The hole diameter is monitored in a drilling process. It is known that the diameter follows a normal distribution with mean μ and variance σ^2. The lower and upper specification limits are set at values denoted by LSL and USL, respectively. The cost of a non-conforming hole with too small diameter is $C1$, the cost of a non-conforming hole with too large diameter is $C2$ and $C1 < C2$ (because a too large diameter leads to a waste part, whereas a too small diameter can be fixed with an additional manufacturing step). Assuming that σ^2, LSL, USL, C1 and C2 are known, determine the expression that allows you to set the target value μ that minimizes the costs.

Exercise 3 solution

Since $D \sim N(\mu, \sigma^2)$, the probability of a non-conforming hole on the left side of the distribution (too small diameters) is $\Phi\left(\frac{LSL-\mu}{\sigma}\right)$, whereas the probability of a non-conforming hole on the right side of the distribution (too large diameters) is $1 - \Phi\left(\frac{USL-\mu}{\sigma}\right)$.

Therefore, the function to be minimized is:

$$f = C1 \cdot \Phi\left(\frac{LSL-\mu}{\sigma}\right) + C2 \cdot \left[1 - \Phi\left(\frac{USL-\mu}{\sigma}\right)\right]$$

The target value μ that minimizes the costs can be estimated by solving an optimization problem.