

Backward Propagation

$$1. \frac{dL}{dA_1} = e(A_1 - y) = 2(0.885 - 1) \approx -0.115$$

$$2. \frac{dL}{dz_2} = \frac{dL}{dA_1} \cdot \text{sigmoid}'(z_1) \text{ where } \text{sigmoid}'(z) = \sigma(z)(1 - \sigma(z))$$

$$\sigma'(z_1) = \sigma(z_1)(1 - \sigma(z_1)) \approx 0.885 \cdot (1 - 0.885) \approx 0.0115$$

$$\frac{dL}{dz_2} = \frac{dL}{dA_1} \cdot \sigma'(z_1) \approx -0.115 \cdot 0.0115 \approx -0.00132$$

$$3. \frac{dL}{dw_2} = dz_2 \cdot A_1^T = -0.00132 \cdot [3, 5, 12, 1, 20, 3]$$

$$dw_2 = [-1.4 \cdot 10^{-6}, -4.5 \cdot 10^{-6}, -7.5 \cdot 10^{-6}]$$

$$4. \frac{dL}{db_2} = \frac{dL}{dz_2} = -0.00132$$

$$5. \frac{dL}{dA_1} = w_2^T \cdot \frac{dL}{dz_2} = [0.2, 0.4, 0.6] \cdot (-0.00132)$$

$$dA_1 \approx [-7.4 \cdot 10^{-8}, -1.5 \cdot 10^{-7}, -2.2 \cdot 10^{-7}]$$

$$6. \frac{dL}{dz_1} = \frac{dL}{dA_1} \cdot \text{ReLU}'(z_1) \text{ where } \text{ReLU}'(x) = 1 \text{ if } x > 0, \text{ else } 0$$

$$\frac{dL}{dz_1} = dA_1 \cdot \text{ReLU}'(z_1), \text{ all } z_1 > 0, \text{ ReLU}' = 1, \text{ then } dz_1 = dA_1$$

$$4. \frac{dL}{dw_1} = \frac{dL}{dz_1} \cdot X^T$$

$$dW_{row1} = -7.4 \cdot 10^{-8} \cdot [20, 2, 4] = [-1.5 \cdot 10^{-6}, 2.2 \cdot 10^{-7}, -3 \cdot 10^{-7}]$$

$$dW_{row2} = -7.5 \cdot 10^{-8} \cdot [20, 2, 4] = [-3 \cdot 10^{-6}, -4.5 \cdot 10^{-7}, -6 \cdot 10^{-7}]$$

$$dW_{row3} = -2.2 \cdot 10^{-7} \cdot [20, 2, 4] = [4.4 \cdot 10^{-6}, 6.6 \cdot 10^{-7}, 8.8 \cdot 10^{-7}]$$

$$8 \quad \frac{dL}{db_1} = \frac{dL}{dz_1} = [-7.4 \cdot 10^{-8}, -1.5 \cdot 10^{-7}, 2.2 \cdot 10^{-7}]$$