Least Squares and SLAM
Odometry Calibration

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Part of the material of this course is taken from the Robotics 2 lectures given by G.Grisetti, W.Burgard, C.Stachniss, K.Arras, D. Tipaldi and M.Bennewitz

Least Squares Minimization

- The minimization algorithm proceeds by repeatedly performing the following steps:
  - Linearizing the system around the current guess ***x*** and computing the following quantities for each measurement

$$\mathbf{e}_i = \mathbf{e}_i(\mathbf{x}) \qquad \mathbf{J}_i = \left.\frac{\partial \mathbf{e}_i(\mathbf{x}+\boldsymbol{\Delta}\mathbf{x})}{\partial \boldsymbol{\Delta}\mathbf{x}}\right|_{\boldsymbol{\Delta}\mathbf{x}=0}$$

  - Computing the terms for the linear system

$$\mathbf{b}^T = \sum_i \mathbf{e}_i^T \boldsymbol{\Omega}_i \mathbf{J}_i \qquad \mathbf{H} = \sum_i \mathbf{J}_i^T \boldsymbol{\Omega}_i \mathbf{J}_i$$

  - Solving the system to get a new optimal increment

$$\boldsymbol{\Delta}\mathbf{x}^* = -\mathbf{H}^{-1}\boldsymbol{\Delta}\mathbf{x}$$

  - Updating the previous estimate

$$\mathbf{x} \leftarrow \mathbf{x} + \boldsymbol{\Delta}\mathbf{x}^*$$

Odometry Calibration

- We have a robot which moves in an environment, gathering the odometry measurements $\boldsymbol{u_i}$,affected by a systematic error.
- For each $\boldsymbol{u_i}$ we have a ground truth $\boldsymbol{u^*_i}$
- There is a function $\boldsymbol{f_i(x)}$ which, given some bias parameters $\boldsymbol{x}$, returns a an unbiased odometry for the reading $\boldsymbol{u_i'}$ as follows

$$\mathbf{u}_i' = f_i(\mathbf{x}) = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} \mathbf{u}_i$$

Odometry Calibration (cont)

- The state vector is

$$\mathbf{x} = \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{21} & x_{22} & x_{23} & x_{31} & x_{32} & x_{33} \end{pmatrix}^T$$

- The error function is

$$\mathbf{e}_i(\mathbf{x}) = \mathbf{u}_i^* - \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} \mathbf{u}_i$$

- Its derivative is:

$$\mathbf{A}_i = \frac{\partial \mathbf{e}_i(\mathbf{x})}{\partial \mathbf{x}} = -\begin{pmatrix} u_{i,x} & u_{i,y} & u_{i,\theta} & & & & & & \\ & & & u_{i,x} & u_{i,y} & u_{i,\theta} & & & \\ & & & & & & u_{i,x} & u_{i,y} & u_{i,\theta} \end{pmatrix}$$

Exercise

- Write a program to calibrate the odometry
- We provide an input file obtained from a real robot.
- Format of z.dat:
  - Every line is a single odometry measurement
    - u'_x u'_y u'_t u_x u_y u_t
  - **u'** and **u** are respectively the true and the measured odometry of the system in relative coordinates (e.g. motion of the robot between two consecutive frames).

Exercise (in sequential steps)

- Load the measurement matrix
- Write a function ***A***=*v2t(**u**)* that given a transformation expressed as a vector $\boldsymbol{u}=[\boldsymbol{u}_x\ \boldsymbol{u}_y\ \boldsymbol{u}_t]$ returns an homogeneous transformation matrix ***A.***
- Write a function ***u***=*t2v**(A)*** dual of the previous one.
- Write a function ***T***=*compute_odometry_trajectory**(U)*** that computes a trajectory in the global frame by chaining up the measurements (rows) of the Nx3 matrix U. *Hint: use the two functions defined above. Test it on the input data by displaying the trajectories.*
- Define the error function $\boldsymbol{e_i(X)}$ for a line of the measurement matrix. Call it *error_function(**i,X,Z**)*.
- Define the Jacobian function for the measurement ***i*** (call it *jacobian**(i,Z)**.*
- Write a function ***X***=*ls_calibrate_odometry**(Z)*** which constructs and solves the quadratic problem. It should return the calibration parameters ***X.***
- Write a function ***Uprime***=*apply_odometry_correction**(X,U)*** which applies the correction to all odometries in the Nx3 matrix U. Test the computed calibration matrix and generate a trajectory.
- Plot the real, the estimated and the corrected odometries.
- In the directory you will find an octave script ‘LsOdomCalib’ which you can use to test your program.

v2t & t2v

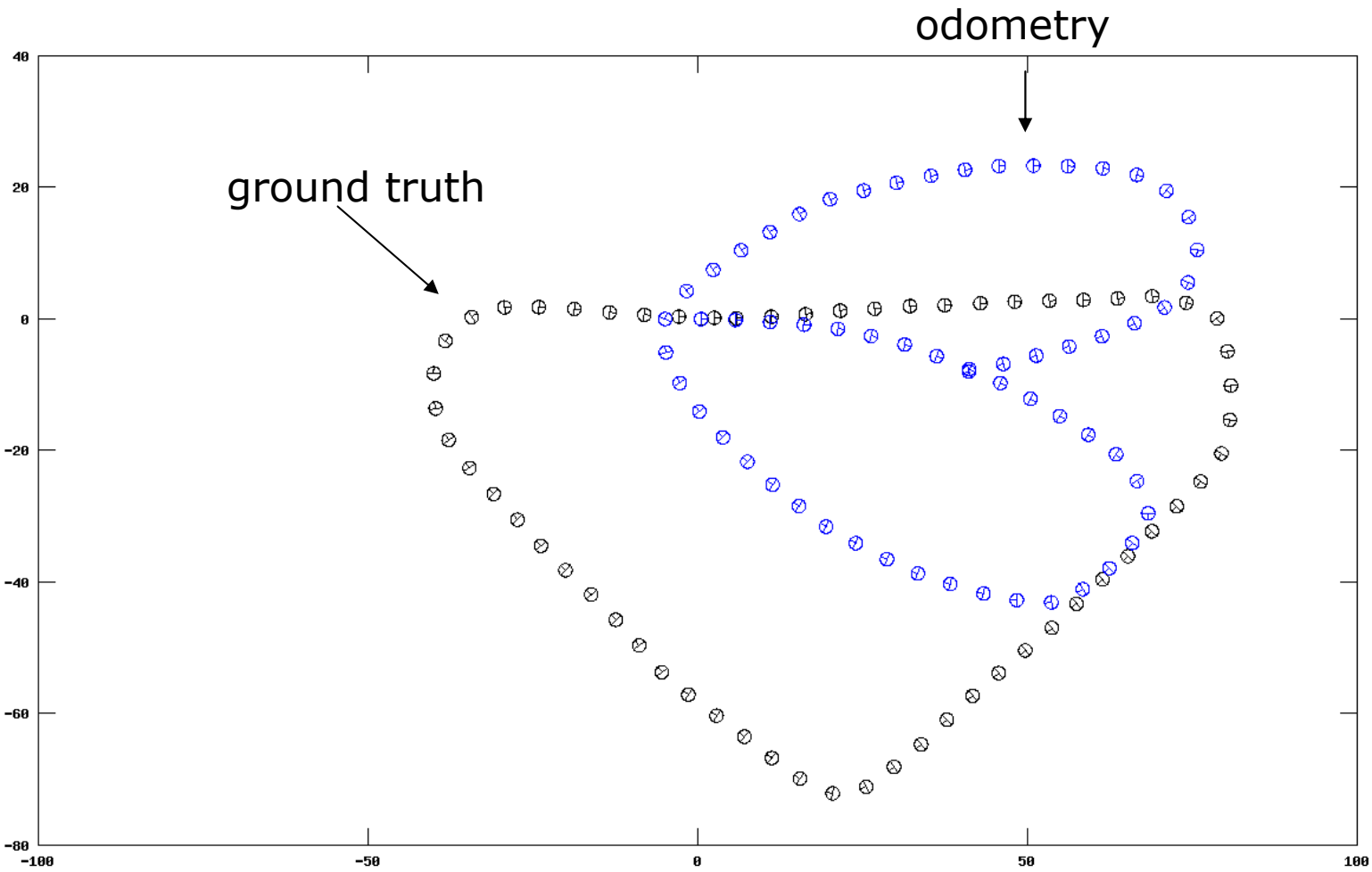
```
function v=t2v(A)
   v(1:2, 1)=A(1:2,3);
   v(3,1)=atan2(A(2,1),A(1,1));
endfunction

function A=v2t(v)
   c=cos(v(3));
   s=sin(v(3));
   A=
   [c, -s, v(1) ;
    s,  c, v(2) ;
    0   0  1  ];
endfunction
```

compute_odometry_trajectory

```
function T=compute_odometry_trajectory(U)
   T=zeros(size(U,1),3);
   P=v2t(zeros(1,3));
   for i=1:size(U,1),
        u=U(i,1:3)';
        P*=v2t(u);
        T(i,1:3)=t2v(P)';
   end
end
```

Trajectories



Error function

```
function e=error_function(i,X,Z)
   uprime=Z(i,1:3)';
   u=Z(i,4:6)';
   e=uprime-X*u;
end
```

Jacobian

```
function A=jacobian(i,Z)
   u=Z(i,4:6);
   A=zeros(3,9);
   A(1,1:3)=-u;
   A(2,4:6)=-u;
   A(3,7:9)=-u;
end
```

Quadratic Solver

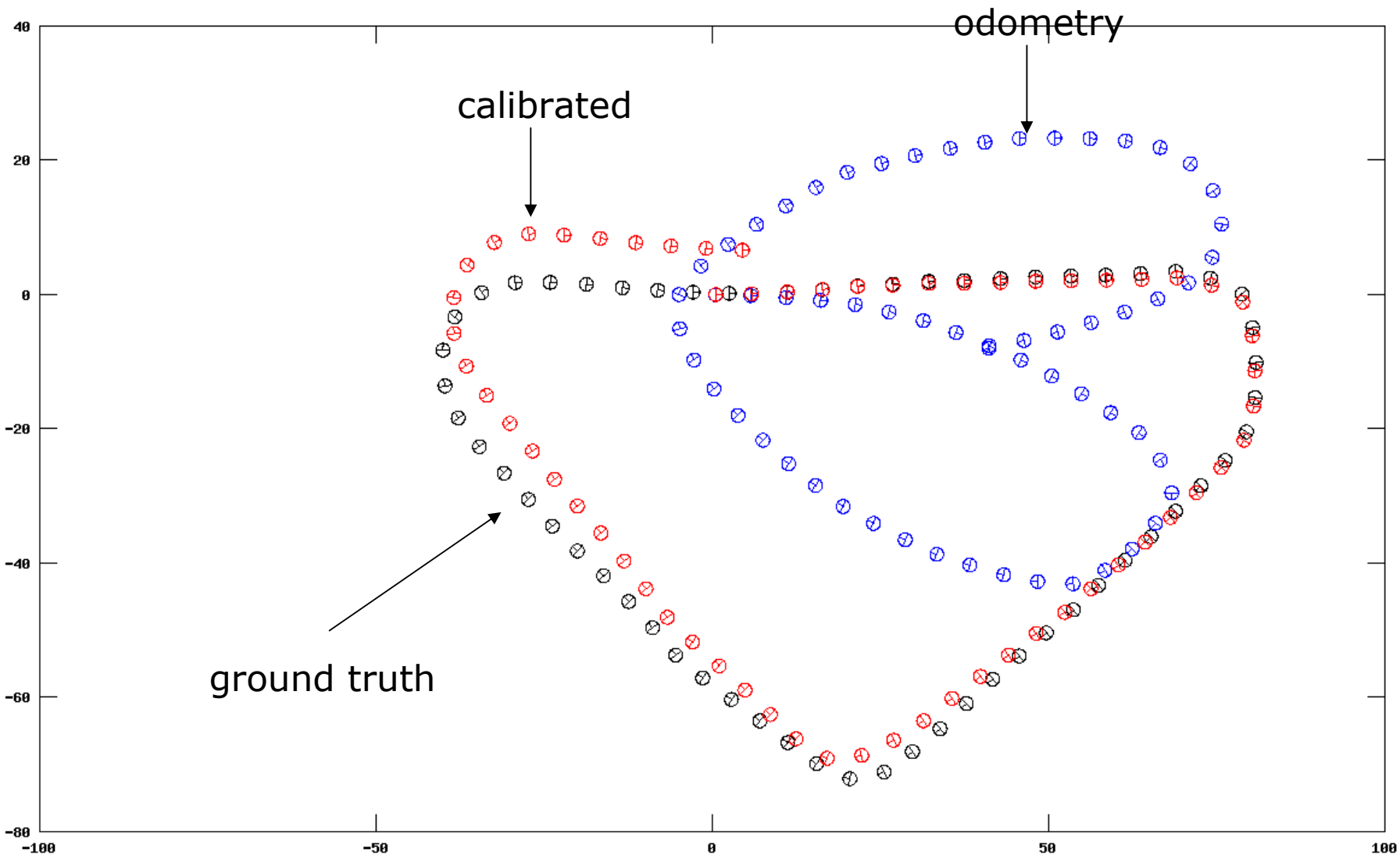
```
function X=ls_calibrate_odometry(Z)
      #accumulator variables for the linear system
      H=zeros(9,9);
      b=zeros(9,1);
      #initial solution (can be anything, se set it to the identity transformation)
      X=eye(3);

      #loop through the measurements and update the
      #accumulators
      for i=1:size(Z,1),
            e=error_function(i,X,Z);
            A=jacobian(i,Z);
            H=H+A'*A;
            b=b+A'*e;
      end
      #solve the linear system
      deltaX=-H\b;
      #this reshapes the 9x1 increment vector in a 3x3 atrix
      dX=reshape(deltaX,3,3)';
      #computes the cumulative solution
      X=X+dX;
end
```

applyOdometryCorrection

```
function C=applyOdometryCorrection(bias, U)
   C=zeros(size(U,1),3);
   for i=1:size(U,1),
        u=U(i,1:3)';
        uc=bias*u;
        C(i,:)=uc';
   end
endfunction
```

Plots


odometry
calibrated
ground truth
40
20
0
-20
-40
-60
-80
-100
-50
0
50
100

Questions ☺

- Did you find this practical useful?
- Would you feel confident to apply least squares to more complex problems?
- Which one of the wheels of the robot was deflated (right or left)?

For the next lecture you must have understood the basic concepts on least square minimization: we will do SLAM.