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Module : Engineering Mathematics 2B / Mathematics 2B/ Computing Mathematics 2 – EG1761/2008/2681/2916/2961 EGB/D/F/H/J/M207 IT1201/1531/1631/1761 CLB/C/G201

Topic 10 : Hypothesis Testing with One Sample

Objectives :

At the end of this lesson, the student should be able to:

1. formulate a hypothesis test by using its characteristics such as formulating the null and alternate hypothesis, identifying the correct test statistics and applying the critical regions

2. evaluate its reliability by explaining the type I and II errors

3. evaluate the hypothesis of a population mean by using the z-test or t-test

TOPIC 10: Hypothesis Testing with One Sample

10.1 Introduction to Hypothesis Testing

- Suppose a car manufacturer advertises that its new hybrid car has a mean mileage of 50 miles per gallon. This statement may be true but it has yet been proven. Such a statement is known as a statistical hypothesis.

- One way of testing the above hypothesis is to literally test all the hybrid cars made by this manufacturer; which is both impractical and non-economical. The more sensible approach is to test the validity by considering random samples taken from the population of this hybrid cars.

- In this chapter, you will learn how to test a claim or a hypothesis about a population parameter, based on the information obtained from a random sample. In this module, we are only concerned with testing the population mean.

10.2.1 Stating a Hypothesis

- A statement about a population parameter is called a **statistical hypothesis**. To test a population parameter, you must state a **pair** of hypotheses – one that represents the claim and the other its complement. When one of these hypotheses is false, the other must be true. Either hypotheses – the **null hypothesis** or the **alternative hypothesis** may represent the original claim.

Definition

1. A **null hypothesis** H_0 is a statistical hypothesis that contains a statement of equality, such as $\leq$, $=$ or $\geq$.

2. The **alternate hypothesis** H_a is the complement of the null hypothesis. It is a statement that must be true if H_0 is false and it contains statement of strict inequality, such as $>$, $\neq$ or $<$.

- To write the null and alternative hypotheses, translate the claim made about the population parameter from a verbal statement to a mathematical statement. Then write its complements. For instance, if the claim value is *k* and the population parameter is μ, then some possible pairs of null and alternative hypotheses are:

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1st possible pair	2nd possible pair	3rd possible pair
$\begin{cases} H_0 : \mu \le k \\ H_1 : \mu > k \end{cases}$	$\begin{cases} H_0 : \mu \ge k \\ H_1 : \mu < k \end{cases}$	$\begin{cases} H_0 : \mu = k \\ H_1 : \mu \ne k \end{cases}$

- Thereafter, we will examine the sampling distribution and determine whether or not a sample statistic is unusual.

Example 10.2-1

Write the following claims as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

(a) A university publicises that the proportion of its students who graduate in 4 years is 82%.

(b) A water faucet manufacturer announces that the mean flow rate of a certain type of faucet is less than 2.5 gallons per minute.

(c) A cereal company advertises that the mean weight of the contents of its 20-ounce size cereal boxes is more than 20 ounces.

Solution:

(a) The claim "the proportion … is 82%" can be written as $p = 0.82$. Its complement is $p \ne 0.82$. Since $p = 0.82$ contains the statement of equality, it becomes the null hypothesis. In this case, the null hypothesis is also the claim. Hence,

$$\begin{cases} H_0 : p = 0.82 \quad \text{(claim)} \\ H_1 : p \ne 0.82 \end{cases}$$

10.2.2 Types of Errors

- No matter which hypothesis represents the claim, we always begin a hypothesis test by assuming that the equality condition in the null hypothesis is true. So, when we perform a hypothesis test, we make one of two decisions:
  1. Reject the null hypothesis; or
  2. Fail to reject the null hypothesis.

  Since our decision is based on a sample rather than the entire population, there is always the possibility that we will make the wrong decision.

- The only way to be absolutely certain of whether H_0 is true or false is to test the entire population. Otherwise, we might reject H_0 when it is actually true or fail to reject H_0 when it is actually false.

Definition

1. A **type I** error occurs if the null hypothesis is rejected when it is true.
2. A **type II** error occurs if the null hypothesis is not rejected when it is false.

The following table shows the four possible outcomes of a hypothesis test.

	Truth of H_0	
Decision	H_0 is true	H_0 is false
Do not reject H_0	Correct decision	**Type II error**
Reject H_0	**Type I error**	Correct decision

Example 10.2-2

The USDA limit for salmonella contamination for chicken is 20%. A meat inspector reports that the chicken produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. When will a type I or II error occur? Which is more serious?

Solution:

Let p be the proportion of chicken that is contaminated.

$$\begin{cases} H_0 : p \leq 0.2 \\ H_1 : p > 0.2 \quad \text{(claim)} \end{cases}$$

Type I error occurs when the actual proportion of contaminated chicken is less than or equal to 0.2 but we decided to **reject the null hypothesis.**

Type II error occurs when the actual proportion of contaminated chicken is greater than 0.2 but we **do not reject the null hypothesis**.

Type II error is more serious because we are allowing chicken that exceeded USDA contamination limit to be sold to consumers; which could result in sickness and death.

Example 10.2-3

A company specialising in parachute assembly states that its main parachute failure rate is not more than 1%. You perform a hypothesis test to determine if its claim is false. When will a type I or type II error occur? Which is more serious?

Solution:

10.2.3 Level of Significance

Definition

In a hypothesis test, the **level of significance** is your maximum allowable probability of making a **type I error**. It is denoted by α.

- By setting the level of significance at a small value, you are saying that you want the probability of rejecting a true null hypothesis to be small. Three commonly used level of significance are $\alpha = 0.10$, $\alpha = 0.05$ and $\alpha = 0.01$.
- The probability of a type II error is denoted by β.

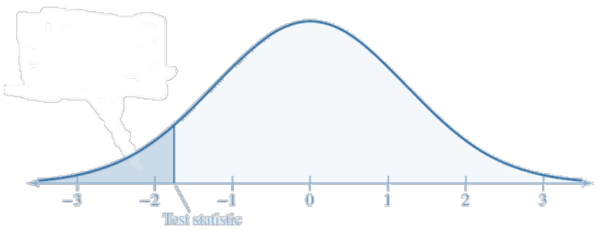
10.2.4 Types of Test and the Rejection Criteria

- Knowing the type of hypothesis tests helps us to decide the criteria for rejecting the null hypothesis. The region of the sampling distribution that favours the alternative hypothesis H_a (i.e. the rejection of H_0) determines the type of test. There are three types of hypothesis tests—a left-, right-, or two-tailed test.

Type 1: Left-tailed test

$H_0 : \mu \geq k$

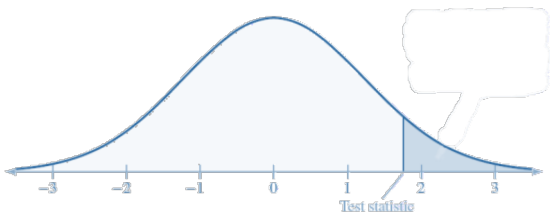
$H_a : \mu < k$



Type 2: Right-tailed test

$H_0 : \mu \leq k$

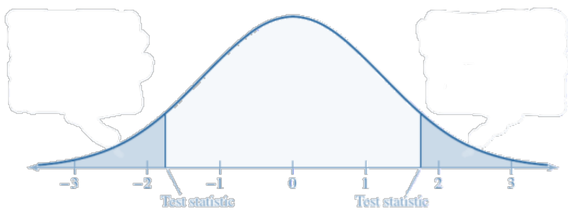
$H_a : \mu > k$



Type 3: Two-tailed test

$H_0 : \mu = k$

$H_a : \mu \neq k$



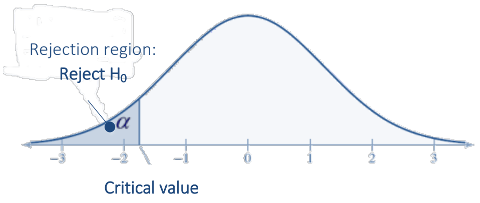
Definition

A rejection region of the sampling distribution is the range of values for which the null hypothesis is not probable. A critical value separates the rejection region from the non-rejection region.

- To find the critical value(s) that defines the rejection region, we need to establish the type of hypothesis test, the level of significance and the sampling distribution. The critical value is denoted by:
  1. z_c if the sampling distribution follows normal distribution
  2. t_c if the sampling distribution follows student-t distribution

- Case 1: Left-tailed test

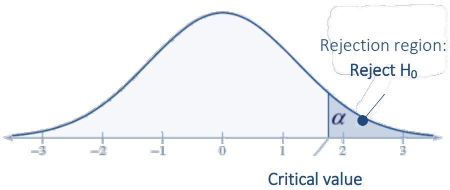
  The rejection region is the area on the left of the critical value, i.e. $z < z_c$ or $t < t_c$



- Case 2: Right-tailed test

  The rejection region is the area on the right of the critical value.

  i. e. $z > z_c$ or $t > t_c$



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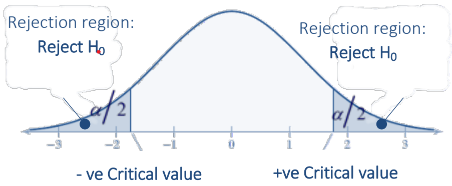
- Case 3: Two-tailed test

  The rejection region is the area on the left of the negative critical value and to the right of the positive critical value.

  i.e. $\{z < -z_c \text{ or } z > z_c\}$ or $\{t < -t_c \text{ or } t > t_c\}$



Example 10.2-4

In each of the claims, state the null and alternative hypotheses, determine if the test is a left-, right- or two-tailed test. At $\alpha = 0.10$, sketch a normal sampling distribution and find the critical value(s).Assume that the population follows a normal distribution.

(i) A consumer analyst reports that the mean life of a certain type of automobile battery is 74 months.

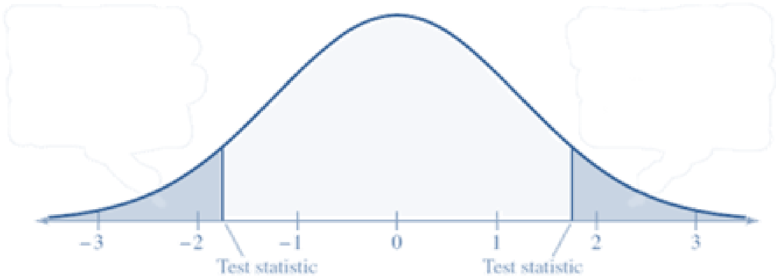
(ii) A radio station publicises that its proportion of the local listening audience is greater than 39%.

Solution:

(i) Null hypothesis : ____________________________

Alternative hypothesis : ____________________________

Type of test : ____________________________



(ii)

10.2.5 Test Statistics and Making Decision

- To use the rejection region to make a conclusion in a hypothesis test:

Case 1: If a test statistic falls **in the rejection region**, we **reject** null hypothesis.

Case 2: If a test statistic falls **outside of the rejection region**, we **fail to reject** the null hypothesis.

- The following table will help you to interpret your decision:

Decision	Claim: Claim is H_0	Claim: Claim is H_a
Reject H_0	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Fail to reject H_0	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

- The **test statistic** for the statistical test for a population mean is the sample mean, $\bar{x}$ and the standardized test statistic is denoted by

(i) z if the sampling distribution follows normal distribution (or $n \geq 30$)

(ii) t if the sampling distribution follows student-t distribution (or $n < 30$)

(iii) The standardized test statistic $= \dfrac{\text{sample mean - hypothesized mean}}{\text{standard error}}$

- When testing a population mean,

	Population Variance, σ^2	Sample size, n	Test Statistic
Testing a single sample value	known	–	$z = \dfrac{x-\mu}{\sigma}$
Testing a mean	known	any n	$z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$
Testing a mean	unknown	$n \geq 30$	$z = \dfrac{\bar{x}-\mu}{s/\sqrt{n}}$
Testing a mean	unknown	$n < 30$	$t = \dfrac{\bar{x}-\mu}{s/\sqrt{n}}$ with $df = n-1$

Example 10.2-5 (Large Sample)

The CEO of a firm claims that the mean work day of the firm's accountants is less than 8.5 hours. A random sample of 35 of the firm's accountants has a mean work day of 8.2 hours with a standard deviation of 0.5 hour. At $\alpha = 0.01$, test the CEO's claim.

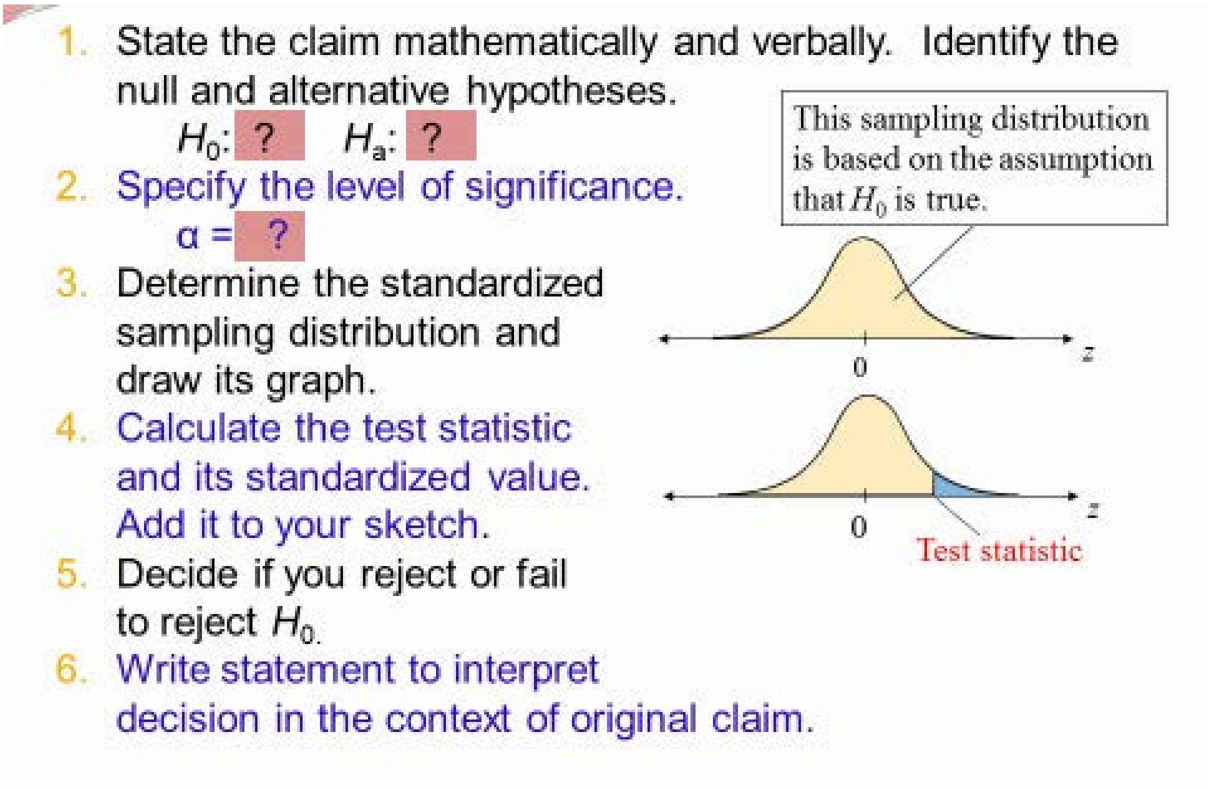
Solution:

Example 10.2-6 (Small Sample)

A used car dealer says that the mean price of a 2010 Honda Pilot LX is at least $\$23,900$. You suspect this claim is incorrect and find that a random sample of 14 similar vehicles has a mean price of $\$23,000$ and a standard deviation of $\$1113$. Is there enough evidence to reject the dealer's claim at $\alpha = 0.05$? Assume the population is normally distributed.

Solution:

In summary, the steps for hypothesis testing are:



Tutorial 10: Hypothesis Testing with One Sample

A Self Practice Questions

A.1 Finding Critical Values for Normal Distribution

Find the critical value(s) for the indicated *z*-test and level of significance.

(a) right-tailed, $\alpha = 0.05$
(b) right-tailed, $\alpha = 0.08$
(c) left-tailed, $\alpha = 0.03$
(d) left-tailed, $\alpha = 0.09$
(e) two-tailed, $\alpha = 0.02$
(f) two-tailed, $\alpha = 0.10$

A.2 Finding Critical Value(s) for Student *t*-distribution

Find the critical value(s) for the indicated *t*-test, level of significance and sample.

(a) right-tailed, $\alpha = 0.05, n = 23$
(b) right-tailed, $\alpha = 0.01, n = 11$
(c) left-tailed, $\alpha = 0.025, n = 19$
(d) left-tailed, $\alpha = 0.05, n = 14$
(e) two-tailed, $\alpha = 0.01, n = 27$
(f) two-tailed, $\alpha = 0.05, n = 10$

A.3 Testing the Claim

Test the claim about the population mean *μ* at the given level of significance using the given sample statistics. Assume population is normally distributed for (iii) and (iv).

(i) Claim: $\mu = 40$; $\alpha = 0.05$. Sample statistics: $\bar{x} = 39.2$, $s = 3.23$, $n = 75$

(ii) Claim: $\mu > 1030$; $\alpha = 0.05$. Sample statistics: $\bar{x} = 1035$, $s = 23$, $n = 50$

(iii) Claim: $\mu \neq 52\,200$; $\alpha = 0.05$. Sample statistics: $\bar{x} = 53\,200$, $s = 1200$, $n = 4$

(iv) Claim: $\mu \geq 8\,000$; $\alpha = 0.01$. Sample statistics: $\bar{x} = 7\,700$, $s = 450$, $n = 25$

B Discussion Questions

B1. A report claims that an adult has an average of 130 Facebook friends. A random sample of 50 adults revealed that the average number of Facebook friends was 142 with a standard deviation of 38.2. At 5% significance level, is there enough evidence to reject the claim?

B2. An officer from the utility department claims that the average water usage per household is more than 12 cubic meters per month. To check the claim, a random sample of 40 households was selected and found that the average monthly water usage was 13 cubic meters with a standard deviation of 3 cubic meters. At 1% significance level, is there enough evidence to support the officer's claim?

B3. The management of weight loss club claims that its members lose an average of 3 kg or more within the first month after joining the club. A consumer agency that wanted to check this claim took a random sample of 36 members of this club and found that they lost an average of 2.9 kg with a standard deviation of 0.6 kg within the first month of membership. Test, at 10% significance level, on whether the management's claim is true.

B4. A psychologist claims that the mean age at which children start walking is 12.5 months. To check this claim, you took a random sample of 18 children and found that the mean age at which these children started walking was 12.9 months with a standard deviation of 0.7 month. Using the 10% significance level, can you conclude that the mean age at which all children start walking is 12.5 months? Assume that the ages at which all children start walking have an approximately normal distribution.

B5. A pharmaceutical company claims that the average selling price per tablet of its new drug is less than 45 cents. You have been asked to challenge the claim and so you conducted a random sampling of prices at 10 pharmacies across the country. The results (in cents) are as follow:

33.45	28.99	27.45	42.89	53.91
37.95	48.55	36.80	35.95	40.45

Is there sufficient evidence to support the claim that the average price per tablet is less than 45 cents at the 1% level of significance? Assume that the selling price per tablet is approximately normally distributed.

B6. The average monthly telephone bill was reported to be more than \$50.07. A random sample of 10 people was taken and the following were the monthly charges (in dollars):

55.83, 49.88, 62.98, 70.42, 60.47, 52.45, 49.20, 50.02, 58.60, 51.29

At the 5% significance level, can the claim be supported? Assume all telephone bills to be approximately normal.

Answers

A1a	$z_{0.95} = 1.645$	**A1b**	$z_{0.92} = 1.41$
A1c	$z_{0.03} = -1.88$	**A1d**	$z_{0.09} = -1.34$
A1e	$z_{0.01} = \pm 2.33$	**A1f**	$z_{0.05} = \pm 1.645$
A2a	$t_{0.05,22} = 1.717$	**A2b**	$t_{0.01,10} = 2.764$
A2c	$t_{0.025,18} = -2.101$	**A2d**	$t_{0.05,13} = -1.771$
A2e	$t_{0.01,26} = \pm 2.779$	**A2f**	$t_{0.05,9} = \pm 2.262$

A3i $z = -2.145 < -1.96$, reject H_0

A3ii $z = 1.537 < 1.645$, do no reject H_0

A3iii $t = 1.667 < 3.182$, do no reject H_0

A3iv $t = -3.333 < -2.492$, reject H_0

B1	Reject H_0	**B2**	Do not reject H_0
B3	Do not reject H_0	**B4**	Reject H_0
B5	Do not reject H_0	**B6**	Reject H_0