Course : Diploma in Electronic Systems
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Module : Engineering Mathematics 2B / Mathematics 2B/ Computing Mathematics 2 – EG1761/2008/2681/2916/2961 EGB/D/F/H/J/M207 IT1201/1531/1631/1761 CLB/C/G201

Topic 4 : Probability

Objectives :

At the end of this lesson, the student should be able to:

1 describe probability experiments
2 calculate classical and conditional probabilities
3 distinguish between independent and dependent events
4 apply the multiplication rule
5 identify mutually exclusive events
6 apply the addition rule

Topic 4: Probability

4.1 Sample space and events

- Suppose we throw a die once and look at the number that appears on the top face. The **set of all possible outcomes** is {1, 2, 3, 4, 5, 6}. Suppose we are interested in getting an even number, then the set of outcomes of this **event** is {2, 4, 6}.

- **Sample space** is the **set of all possible outcomes** in an experiment.
  Event is a **subset** of the sample space.

Example 4.1-1

An unbiased coin is tossed three times, list out the sample space and the event in which there is exactly one head.

Solution:

4.2 Probability of single events

- Probability is a measure of how likely an event will happen in an experiment.

- In an experiment, if each outcome in the sample space is equally likely to happen:

$$P(\text{Event}) = \frac{\text{no. of outcomes in event}}{\text{no. of outcomes in sample space}}$$

Example 4.2-1 (List of outcomes)

Following Example 4.1-1, calculate the probability that there is exactly one head.

Solution:

Example 4.2-2 (Frequency Table)

The table below shows the gender and blood pressure categories of 300 participants.

Blood Pressure	Female	Male	Row Total
Normal	39	25	**64**
Pre-hypertension	61	50	**111**
High Stage 1	42	47	**89**
High Stage 2	20	16	**36**
Column Total	**162**	**138**	**300**

A participant is randomly chosen. Calculate the probability that

(a) the participant is male,

(b) the participant has high stage 2 blood pressure.

Solution:

Example 4.2-3 (Counting)

A four-member research team is to be chosen from 6 men and 5 women. What is the probability that the team formed has more men than women?
(Refer to Example 3.4-1)
Solution:
Number of teams without restriction = 330

Number of teams with more men than women = 115

P (team has more men than women) =

4.3 Probability involving multiple events

4.3.1 Complement Event

- Given an event E, its complement event, E' is the set of outcomes in the sample space that is not in E.

- $P(E) = 1 - P(E')$

Example 4.3.1-1

Referring to Example 4.2-3, find the probability that the team formed has at most two men.

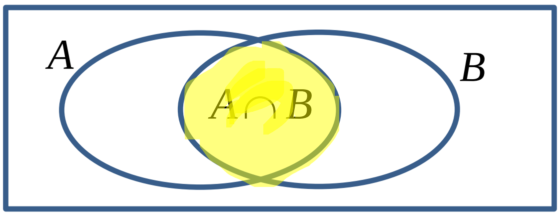
Solution:

Let E = { team of 4 has at most two men }. Observe that E' = { team of 4 has more men than women }.

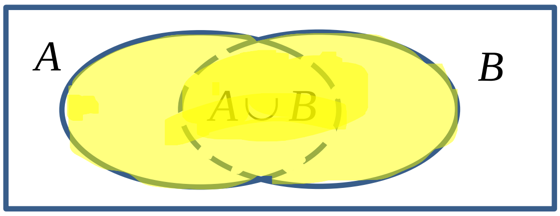
$\therefore P(E) =$

4.3.2 Intersection and union of two events

- Let A, B be two events. The **intersection** of them is known as "A and B" (notation: $A \cap B$) refers to the set of outcomes that is common to **both** A **and** B.



- The **union** of *A* and *B* is known as "A or B" (notation: $A \cup B$) refers to the set of outcomes that is **either** in A **or** B.



- Addition formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Example 4.3.2-1

Refer to the table in Example 4.2-2, find the probability that

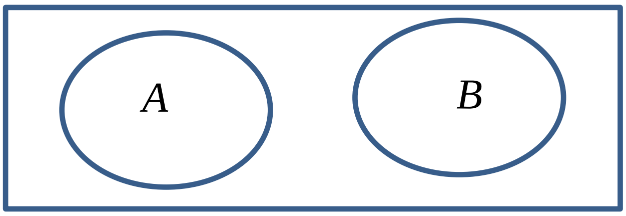
(i) a participant is a female and has high stage 2 blood pressure,

(ii) a participant is a male or has pre-hypertension.

Solution:

4.3.3 Mutually exclusive events

- Two events A and B are **mutually exclusive** if they share no common outcome. I.e. $\underline{P(A \cap B) = 0}$.



Example 4.3.3-1

Suppose we draw a card from a standard deck of poker cards. Find the probability that the card is a "4" or an ace.

Solution:

4.3.4 Conditional events

- The **conditional** event A given B (notation: $A \mid B$) refers to the event that A will occur based on the knowledge that ***B* has occurred**.

- For example, we draw two cards from a deck of 52 poker cards without replacement. Let B be the event that the first card is an ace of heart, A be the event that the second card drawn is an ace. Then $A \mid B$ = {ace of spade, ace of diamond, ace of club}

- $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

Example 4.3.4-1

Two ordinary dice are thrown. Let A be the event that the numbers shown on both dice are equal, B be the event that the total sum of the two numbers is 8. Calculate $P(A \mid B)$ and $P(B \mid A)$.

Solution:

A = { (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }

B = { (2, 6), (6, 2), (3, 5), (5, 3), (4, 4) }

$A \cap B =$ { (4, 4) }

$P(A \mid B) =$

4.3.5 Independent Events

- Two events A and B are **<u>independent</u>** if the probability of one event occurring does not affect the probability of the other event occurring.

- Mathematically, A and B are **<u>independent</u>** if and only if either condition holds
  (a) $P\left(A \cap B\right) = P(A)P(B)$
  (b) $P(A \mid B) = P(A)$

Example 4.3.5-1

The probability of a successful appendicitis operation is 98%. Find the probability that

(i) out of three operations, all are successful.

(ii) out of two operations at least one is unsuccessful.

Solution: Assume that the outcomes of the operations are independent of each other.

4.4 Tree diagram and multiplication rule

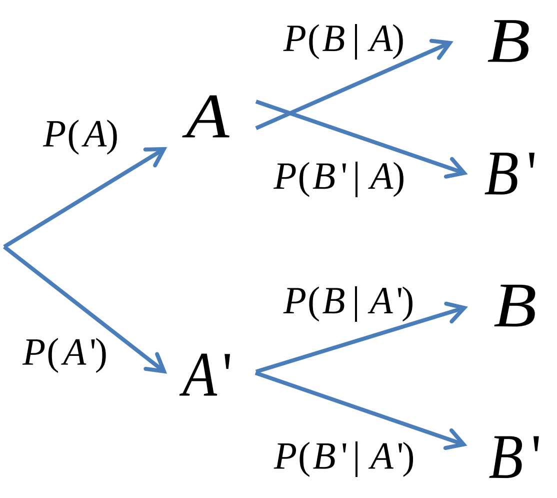
- Recall from Section 4.3.4: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$. We can rearrange the terms to

  obtain $P(A \cap B) = P(A|B) * P(B)$

  This is also known as the **multiplication rule**.

- A **tree diagram** is useful to calculate probabilities involving experiments happening in stages with multiple events happening one after another. For example, we want to calculate the event whether a person smoke followed by whether he or she has lung cancer.

- An example of tree diagram looks like this:



We can use the tree diagram to compute various probabilities such as
$P(A \cap B) = P(A) * P(B|A)$

$$P(B) = P(A \cap B) + P(A' \cap B)$$
$$= [P(A) * P(B|A)] + [*P(A') * P(B|A')]$$

(i.e. add up all the "branches" leading to event B).

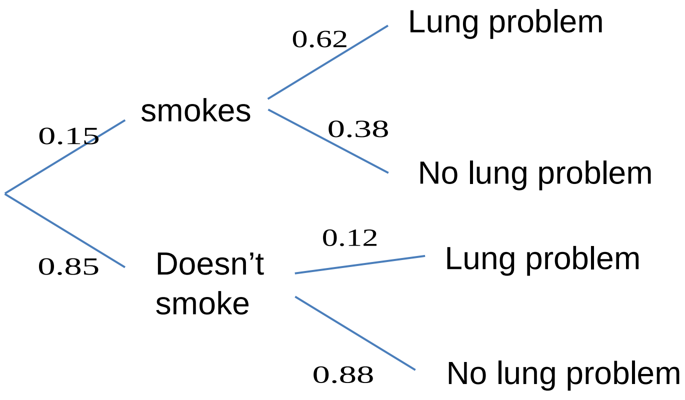
Example 4.4.1

15% of Singaporean adult smokes cigarettes. It is found that 62% of the smokers and 12% of non-smokers develop lung problem by age 60.

(a) Find the probability that a randomly selected 60-year adult has lung problem.

(b) Given that a randomly selected 60-year adult has lung problem, what is the probability that he smokes?

Solution:



(a) $P(\text{lung problem}) =$

(b) $P(\text{smokes} | \text{lung problem}) =$

Example 4.4-2

Machine A, B and C makes components. Machine A makes 20% of the components, machine B makes 30% of the components and machine C makes the rest. The probability that a component is faulty is 0.07 when is made by machine A, 0.06 when made by machine B and 0.05 when made by machine C. A component is picked at random. Calculate the probability that the component is

(a) made by machine A and is faulty.

(b) made by machine B given that it is faulty.

Solution:

4.5 More probability examples (Self Practice)

Example 4.5-1

The blood samples given by donors over one week were being catalogued according to the types of blood, including the positive and negative Rhesus factor. The 2 by 4 matrix of Rhesus factor against the blood type is given below:

		Blood Type				
		O	A	B	AB	Total
Rhesus Factor	Positive	156	139	37	12	**344**
	Negative	28	25	8	4	**65**
	Total	**184**	**164**	**45**	**16**	**409**

Find the probability that a randomly selected donor has

(i) type A blood,

(ii) positive Rhesus factor,

(iii) type A blood and is positive Rhesus factor,

(iv) type O blood or is negative Rhesus factor,

(v) type B blood given that it is positive Rhesus factor,

(vi) positive Rhesus factor given that it is type AB blood.

Solution:

Ans: (i) $\frac{164}{409}$, (ii) $\frac{344}{409}$, (iii) $\frac{139}{409}$, (iv) $\frac{221}{409}$, (v) $\frac{37}{344}$, (vi) $\frac{3}{4}$

Example 4.5-2

A, B and C are three random events. A and B are mutually exclusive, A and C are independent. $P(A)=\frac{1}{5}$, $P(B)=\frac{1}{10}$, $P(A \text{ or } C)=\frac{7}{15}$ and $P(B \text{ or } C)=\frac{23}{60}$ are given.

(a) Find $P(A \text{ or } B)$, $P(C)$, and $P(B \text{ and } C)$.

(b) State whether B and C are independent.

Solution:

(a) A and B are mutually exclusive $\Rightarrow P(A \cap B) =$

$P(A \text{ or } B) =$

A and C are independent $\Rightarrow P(A \cap C) = P(A) * P(C)$

(b) $P(B \text{ and } C) =$

$P(B) * P(C) =$

Ans: (a) $P(A \text{ or } B)=\frac{3}{10}$, $P(C)=\frac{1}{3}$, $P(B \text{ and } C)=\frac{1}{20}$, (b) not independent

Tutorial 4: Probability

A Self Practice Questions

1 A quiz has 3 true/false questions. Suppose you are randomly selecting the answers and have equal chance of being correct for each question. Let *CCW* indicate that you were correct on the first two questions and wrong on the third.

(a) List the sample space.

(b) List the possible outcomes with at least two questions answered correctly.

2 A pair of unbiased die is tossed. Find the probability of getting

(i) a total of 7;

(ii) at most a total of 6.

3 Given that $P(A)=\frac{3}{7}$, $P(B)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{9}$. Find

(i) $P(A')$ (ii) $P(A\cup B)$ (iii) $P(A'\cap B')$

(iv) $P(A\cap B')$ (v) $P(A|B)$

4 A group of files in a medical clinic classifies the patients by gender and by type of diabetes (I or II). The groupings may be shown as follows. The table gives the number in each classification.

		Type of Diabetes	
		I	II
Gender	Male	25	20
	Female	35	20

If one file is selected at random, find the probability that the individual is a

(a) female.

(b) Type II.

(c) Type II, given that the patient is a male.

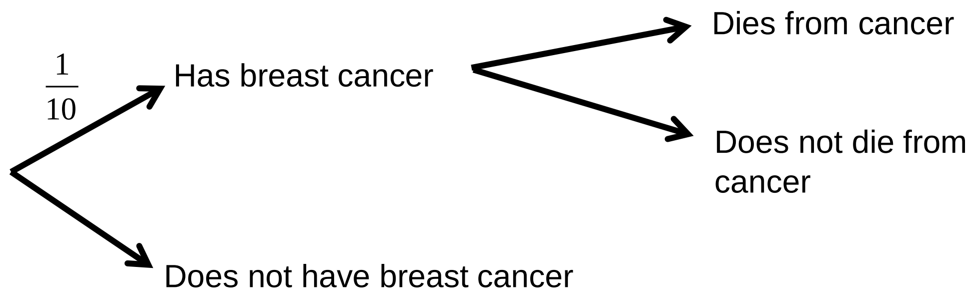
(d) Are the events “Type II” and “a male” independent?

(e) Are the events “Type I” and “a female” mutually exclusive?

5 A study showed that one out of every ten women will get breast cancer. Among those who do, one out of four will die of it.

(i) Complete the tree diagram below.

(ii) Calculate the probability that a randomly chosen woman get breast cancer and not die of it.



B Discussion Questions

1 In a group of 10 persons, 4 have a type *A* personality and 6 have a type *B* personality. If two persons are selected at random from this group, what is the probability that the two will have different personality type?

2 If 3 books are picked at random from shelf containing 6 novels, 5 cook books and 1 computer book, what is the probability that

(a) the computer book is selected?

(b) 2 novels and 1 cook book are selected?

3 In a road show, the compere holds a bag containing 4 movie tickets and 6 concert tickets. 4 tickets are to be drawn at random and given away to 4 lucky winners on stage. Find the probability that

(a) all 4 drawn are concert tickets.

(b) 4 tickets are not of the same type.

(c) at least 2 movie tickets are drawn.

4 Independent events A and B are such that $P(A) = P(B) = p$ and $P\left(A \cup B\right) = \frac{5}{9}$.

Find p and $P(A \cap B)$.

5 Events A and B are such that $P(A)=\frac{1}{3}$, $P(B|A)=\frac{1}{4}$ and $P(A'\cap B')=\frac{1}{6}$. Find

(i) $P(A\cup B)$,

(ii) $P(B)$.

6 The probability that a family owns a car is 0.48, that it owns a 5-room flat is 0.35, and that it owns both a car and a 5-room flat is 0.21. What is the probability that a randomly selected family owns a car or a 5-room flat?

7 1000 people were randomly selected and they were asked whether they are right-handed or left-handed. The following table shows the result of the survey:

	Men	Women
Left-handed	63	50
Right-handed	462	425

(a) A person is selected at random from the sample. Find the probability that the person is
(i) left-handed or a woman;
(ii) right-handed or a man;
(iii) not right handed given the person is a man;
(iv) right-handed woman.

(b) Are the events "being right-handed" and "being a woman" mutually exclusive? Explain.

8 Two thousand randomly selected adults were asked if they think they are better off financially than their parents. The following table gives the two-way classification of the responses based on the education levels of the adults and whether they are financially better off, the same, or worse off than their parents.

	Primary	Secondary	Tertiary
Better off	140	450	420
Same	60	250	110
Worse off	200	300	70

Suppose one adult is selected at random from these 2000 adults. Find the probability that the adult is

(i) better off and has secondary education,
(ii) not the same financially,
(iii) worse off or has primary education,
(iv) not better off given secondary education.

9 The table below shows the results of a survey of the 120 cars in a carpark, in which the colour of each car and the gender of the driver were recorded.

	Male	Female
Green	18	12
Blue	48	22
Red	6	14

One of the cars is selected at random.

M is the event that the car selected has a male owner.

G is the event that the car selected is green.

B is the event that the car selected is blue.

R is the event that the car selected is red.

Find the following probabilitites:

(i) $P(M \cup B)$,
(ii) $P(M \mid R')$.
(iii) Determine whether the events *M* and *G* are independent, justifying your answer.

10 A shipment of two boxes, each containing 6 calculators is received by a store. Box 1 contains one defective calculator and box 2 contains two defective calculators. After the boxes are unpacked, a calculator is selected and found to be defective. Find the probability that it came from box 2.

11 A certain virus infects 0.5 % of the population. A test will be positive 80% of the time if the person has the virus and 5 % of the time if the person does not have the virus. Suppose *A* is the event "the person is infected" and *B* is the event "the person tests positive".

(a) Draw a tree diagram to show the outcomes of the tests.

(b) Find the probability that

(i) the person is infected and is tested positive,

(ii) the person is tested positive.

12 Two children, Tan and Mui, are each to be given a pen from a box containing 3 red pens and 5 blue pens. One pen is chosen at random and given to Tan. A green pen is then put in the box. A second pen is chosen at random from the box and given to Mui.

(i) Draw a tree diagram to represent the possible outcomes.

(ii) Find the conditional probability that Mui's pen is blue, given that Tan's pen is red.

(iii) Find the probability that Mui's pen is red.

(iv) Find the conditional probability that Tan's pen is red, given that Mui's pen is blue.

Answers

A1	i	{ *CCC*, *CCW*, *CWC*, *WCC*, *CWW*, *WCW*, *WWC*, *WWW* }								
	ii	{ *CCC*, *CCW*, *CWC*, *WCC* }								
A2	i	$\frac{1}{6}$	ii	$\frac{5}{12}$						
A3	i	$\frac{4}{7}$	ii	$\frac{41}{63}$	iii	$\frac{22}{63}$	iv	$\frac{20}{63}$	v	$\frac{1}{3}$
A4	a	$\frac{11}{20}$	b	$\frac{2}{5}$	c	$\frac{4}{9}$	d	No	e	No
A5	ii	$\frac{3}{40}$								
B1	$\frac{8}{15}$		**B2**	a	$\frac{1}{4}$	b	$\frac{15}{44}$			
B3	a	$\frac{1}{14}$	b	$\frac{97}{105}$	c	$\frac{23}{42}$		**B4**	$\frac{1}{3}$;	$\frac{1}{9}$
B5	a	$\frac{5}{6}$	b	$\frac{7}{12}$						
B6	0.62									
B7	ai	$\frac{269}{500}$	aii	$\frac{19}{20}$	aiii	$\frac{3}{25}$	iv	$\frac{17}{40}$	b	No
B8	i	$\frac{9}{40}$	ii	$\frac{79}{100}$	iii	$\frac{77}{200}$	iv	$\frac{11}{20}$		
B9	i	$\frac{47}{60}$	ii	$\frac{33}{50}$	iii	Independent				
B10	$\frac{2}{3}$									
B11	bi	0.004	bii	0.05375						
B12	ii	$\frac{5}{8}$	iii	$\frac{21}{64}$		iv	$\frac{3}{7}$			