NANYANG POLYTECHNIC

Course : Diploma in Electronic Systems
Diploma in Telematics & Media Technology
Diploma in Aerospace Systems & Management
Diploma in Electrical Engineering with Eco-Design
Diploma in Mechatronics Engineering
Diploma in Digital & Precision Engineering
Diploma in Aeronautical & Aerospace Technology
Diploma in Biomedical Engineering
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Diploma in Engineering with Business
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Diploma in Financial Informatics
Diploma in Cybersecurity & Forensics
Diploma in Infocomm & Security
Diploma in Chemical & Pharmaceutical Technology
Diploma in Biologics & Process Technology
Diploma in Chemical & Green Technology

Module : Engineering Mathematics 2B / Mathematics 2B/ Computing Mathematics 2 – EG1761/2008/2681/2916/2961 EGB/D/F/H/J/M207 IT1201/1531/1631/1761 CLB/C/G201

Topic 3 : Principles of Counting

Objectives :

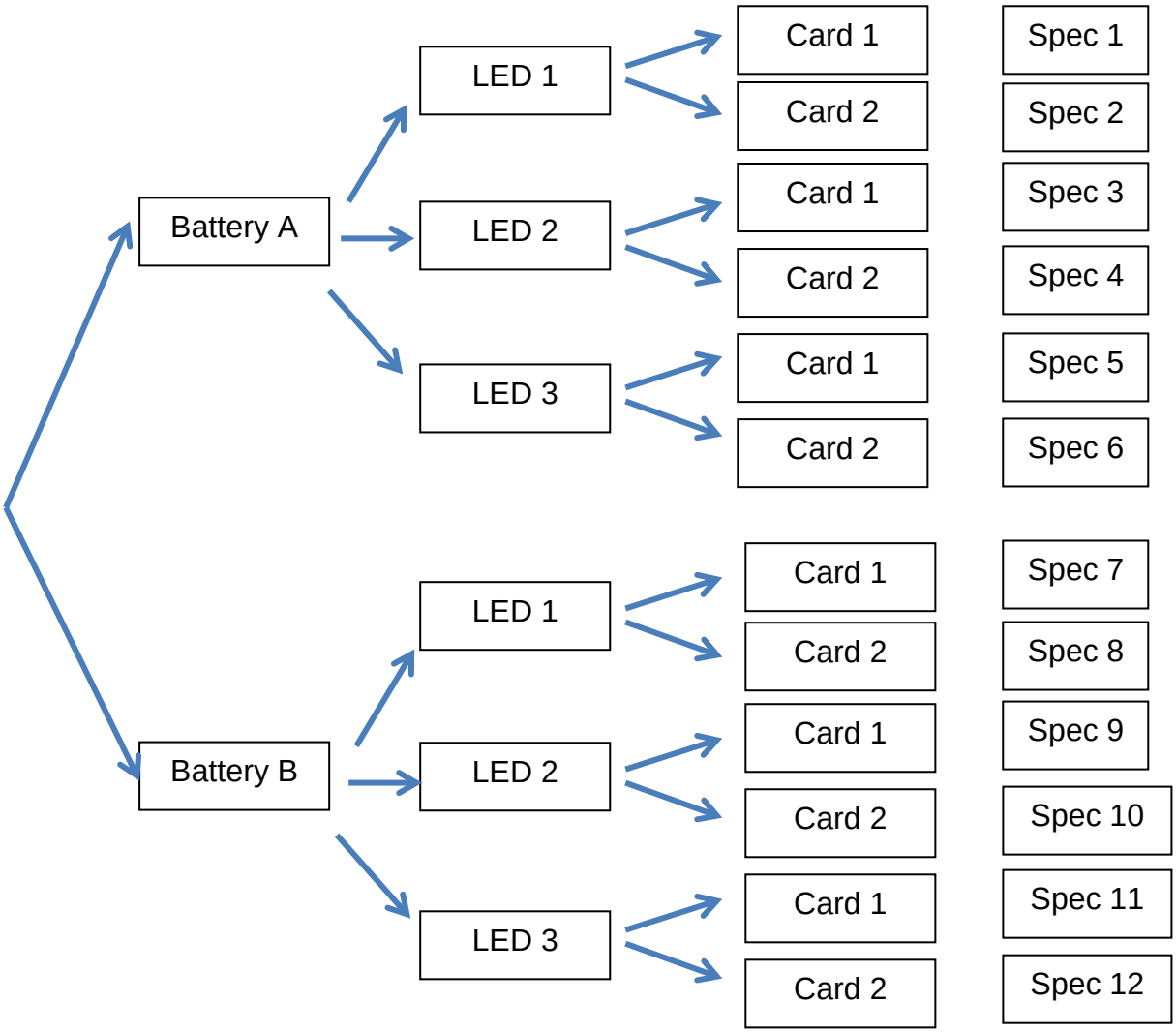
At the end of this lesson, the student should be able to:

1 understand the counting principles
2 explain the difference between permutations and combinations
3 use permutations and combinations in counting problems

Topic 3: Principles of Counting

3.1.1 Multiplication Principle

- Suppose that the hardware you wish to create consists of three parts: (a) a battery, (b) a LED screen and (c) a memory card. There are two types of batteries, three types of LED screens and two types of memory cards available. How many products of different specifications can you create?



In total there are _____ specifications of the product we can create.

I.e. _____ x _____ x _____ = 12 ways.

- In a counting event whereby it can be broken down into *n* stages. If there is m_1 ways for step 1, m_2 ways for step 2, …, m_n ways for step n. Then by **multiplication principle**, there are a total $m_1 \times m_2 \times ... \times m_n$ ways.

Example 3.1.1-1

A female student is preparing for her graduation dance party and her wardrobe contains 4 blouses, 7 skirts, 6 pairs of shoes and 3 sets of jewellery and 5 handbags. Assuming that all the items can be matched in terms of colours and styles, how many possible ways could she dress herself up?

Solution:

Example 3.1.1-2

A typical PIN (personal identification number) consists of any four letters followed by two numeric digits. How many different PINs are possible if

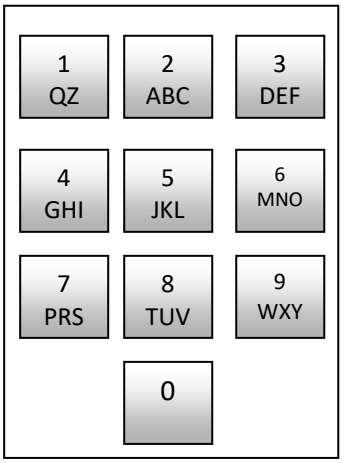
(a) repetition of alphanumeric are allowed;

(b) such repetition is not allowed.

Solution:

Example 3.1.2-3

The diagram below shows the keypad for an automatic teller machine. The same sequence of keys represents a variety of different PINs. For instance, 2133, AZDE and BQ3F are all keyed in exactly the same way.



(a) How many different PINs are represented by the same sequence of keys as 2133?

(b) How many different PINs are represented by the same sequence of keys as 6809?

Solution:

3.1.2 Addition Principle

- Suppose you are considering to buy a laptop from three brands, Acer, Lenovo and Dell. Acer, Lenovo and Dell has three, five and two different models available to choose from. How many choices can you have to buy your laptop?

  Case 1: Buy from Acer: _____ choices
  Case 2: Buy from Lenovo: _____ choices
  Case 3: Buy from Dell: _____ choices

  In total there are _____ choices.

- In a counting event whereby it can be broken down into n **non – overlapping** cases and there are m_1 ways for case 1, m_2 ways for case 2, …, m_n ways for case n. Then by **addition principle**, there are a total $m_1 + m_2 + ... + m_n$ ways.

3.2 Permutation

- Permutation involves **arrangement** of objects whereby if we switch their positions we will get a different outcome.

- Examples include:
  (a) Arranging alphabets / digits to form words or codes,

  (b) Arranging people in a formation to take photographs.

- A summary of the various scenarios on permutation is shown below:

Given n objects and **arrange** r of them, $0 \le r \le n$

n objects all **distinct**

$$ {}_nP_r $$

- Example 3.2-1,2
- ${}_nP_n = n!$
- ${}_nP_r = \dfrac{n!}{(n-r)!}$

$r = n$

n objects **not all distinct**

k_1 objects identical of type 1, k_2 objects identical of type 2,…, k_m objects identical of type m

$$ \frac{n!}{k_1!k_2!...k_m!} $$

- Example 3.2-3

Example 3.2-1

A class of 10 students consist of 6 men and 4 women.

(a) How many ways can all of them arrange themselves in a row?

(b) How many ways can we arrange 6 of them in a row?

Solution:

Example 3.2-2

A biologist has decided to use colours to label the collection of cell specimens in the laboratory. If he has 5 colours (red, blue, green, yellow and pink) to choose from, how many 3-colour codes can he make with no repetitions of each colour selected?

Solution:

Example 3.2-3

(a) How many ways can we arrange all the letters in the word “RANDOM”?
(b) How many ways can we arrange all the letters in the word “ENGINEERING”?

Solution:

3.3 Combination

- Combination involves **selection** of objects whereby the positions of the objects does not matter.

- Examples include:
  (a) Forming a team of 5 people out of 10 people,

  (b) Choosing 5 balls from a box full of balls in various colours.

- n **distinct** objects **select** r of them, $0 \le r \le n$. Number of selections:

$$ {}_nC_r = \frac{n!}{(n-r)!r!} $$

- Alternate notation: ${}^nC_r$ or $\binom{n}{r}$.

Example 3.3-1

3 different species of orchid are to be selected from 20 unique species for cross-breeding. How many possible selections can be made?

Solution:

Example 3.3-2

The manager of a marketing department wants to form a four- person committee from the 15 employees in the department. In how many ways can the manager form this committee?

Solution:

3.4 More counting examples

Example 3.4-1 (Cases)

A four-member research team is to be chosen from 6 men and 5 women.

(a) How many teams can be formed if there are no restrictions?

(b) How many teams can be formed if there must be more men than women?

Solution:

Example 3.4-2

Eleven cards each bear a letter, and together they can be made to spell the word "EXAMINATION". Three cards are selected from the eleven cards and the order of selection is not important. Find how many selections can be made

(i) if the three cards all bear different letters,

(ii) if two of the three cards bear the same letter.

Solution:

Example 3.4-3 (Complement & Slot – in Method)

In how many ways can the letters of the word “EXCELLENCE” be arranged if

(i) the four E’s are not all together,

(ii) the four E’s are all separated.

Solution:

Tutorial 3: Counting

A Self Practice Questions

Permutation

1 Using the letters from the word *COMPUTER*, find

(i) the number of words that can be formed using all the letters,

(ii) the number of 4 – letter words that can be formed.

2 Find the number of 3-digit PIN codes that can be formed using the digits $1, 2, 3, 4, 5, 6$ if

(i) no repetitions are allowed,

(ii) repetitions are allowed.

3 In how many distinguishable ways can the letters in the following words be arranged?

(a) PAPAYA (b) PERMUTATIONS

Combination

4 Space shuttle astronauts each consume an average of 3000 calories per day. One meal normally consists of a main dish, a vegetable dish, and two different desserts. The astronauts can choose from 10 main dishes, 8 vegetable dishes, and 13 desserts. How many different meals are possible?

5 In a class of 20 people there are 13 girls and 7 boys. Find the number of ways to form a committee of 8 members if

(i) there are no restrictions,

(ii) the committee is made up of all girls,

(iii) there is exactly 1 boy in the committee,

(iv) there are less than 2 boys in the committee.

6 In a box there are 3 green, 5 red, 7 yellow and 6 blue balls. Find the number of ways to select 2 balls of different colours.

B Discussion Questions

1 (a) How many different ways can three of the letters of the word *BYTES* be chosen and written in a row?
(b) How many different ways can this be done if the first letter must be "*B*"?

2 Janet has 10 different books that she is going to put on her bookshelf. Of these, 4 are Chemistry books, 3 are Biology books, 2 are Statistics books, and 1 Physics book. Janet wants to arrange her books so that all the books dealing with the same subject are together on the shelf. How many different arrangements are possible?

3 In how many ways can three distinct letters and two distinct digits be arranged if
(i) there is no restriction,
(ii) the letters must come first,
(iii) the digits must always be together.

4 Find the number of distinguishable ways the word STATISTICS can be arranged
(i) without conditions,
(ii) if the letter "T"s must be together,
(iii) if no two "T"s are together.

5 A sample of 5 mice is to be chosen from 7 male and 6 female mice. In how many ways can the sample be selected if it must have at least 2 male and 1 female mice?

6 A shipment of 10 microwave ovens contains two defective units. In how many ways can a restaurant buy three of these units and receive
(a) no defective units?
(b) one defective unit?
(c) at least two non-defective units?

7 Four sales representatives for a company are to be chosen to participate in a training program. The company has eight sales representatives, two in each of four regions. In how many ways can the four representatives be chosen if
(a) there are no restrictions?
(b) the selection must include a sales representative from each region?
(c) the selection must be from only two of the four regions?

8 There are 10 students who are going to spend the evenings in 2 groups; one group goes to the Library and the other plays football. In how many ways can the group for football be selected if there must be at least 4 people in each group?

Answers

A1	i	40320	ii	1680				
A2	i	120	ii	216				
A3	a	60	b	239 500 800				
A4	6240							
A5	i	125970	ii	1287	iii	12012	iv	13299
A6	161							
B1	a	60	b	12				
B2	6912							
B3	i	120	ii	12	iii	48		
B4	i	50400	ii	3360	iii	23520		
B5	1155							
B6	a	56	b	56	c	112		
B7	a	70	b	16	c	6		
B8	672							