

Course : Diploma in Electronic Systems
Diploma in Telematics & Media Technology
Diploma in Aerospace Systems & Management
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Topic 5 : Discrete Probability Distribution

Objectives :

At the end of this lesson, the student should be able to:

- 1 define random variables
- 2 distinguish between discrete and continuous random variables
- 3 define a discrete probability distribution
- 4 compute the mean, variance and standard deviation of a discrete random variable

Topic 5: Discrete Probability Distribution

5.1 Random variable

- A variable is an alphabetical representation of a quantity that can take on various numerical values.
- A random variable, usually denoted by X , Y , ..., is a variable that takes on different values due to random phenomenon (or by chance).
- A random variable can be discrete or continuous (refer to Chapter 1).

Example 5.1-1

- (a) A coin is tossed three times.

If X is a random variable representing the number of heads, then

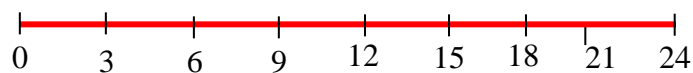
$$X = 0, 1, 2, 3$$

There can be no head or 1 head or 2 heads or 3 heads in the three toss.

- (b) Supposing Y is a random variable representing the time a sales person spends on making calls per day.

The time spends on making calls can be any value (e.g. 2.4 minutes, 49.5 minutes, etc), Y is said to be continuous random variable.

The values of a continuous random variable can be represented as an interval on a number line.



5.2 Discrete probability distribution

- We may not know the exact value of a random variable at any specific moment. However we may calculate the likelihood (probability) that a random variable may take a specific value.
- A **probability distribution** is a table or an equation that links each value of a random variable with its probability of occurrence. The probability distribution of a discrete random variable may be represented using a table.

Example 5.2-1

A fair coin is tossed twice. If X is a random variable representing the number of heads, then construct the probability distribution for X .

Solution:

$$P(X = 0) = P(TT) = \frac{1}{2} \times \frac{1}{2} =$$

$$P(X = 1) = P(TH) + P(HT) = \frac{1}{2} \times \frac{1}{2} \times 2 =$$

$$P(X = 2) = P(HH) = \frac{1}{2} \times \frac{1}{2} =$$

$X = k$	0	1	2
$P(X = k)$			

A **probability distribution** must satisfy the following conditions:

(a) $0 \leq P(X = k) \leq 1$ for all values of k ,

(b) $\sum_{all\ k} P(X = k) = 1$ (sum of all probabilities is 1).

Example 5.2-2

Explain whether each of the following is a discrete probability distribution function.

(a)

$X = k$	5	6	7	8
$P(X = k)$	$\frac{1}{16}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

(b)

$X = k$	1	2	3	4
$P(X = k)$	0.09	0.36	0.49	0.05

Solution:

(a) It is a discrete probability distribution function since

(i) $\text{---} \leq P(X = k) \leq \text{---}$

(ii) $\sum_{k=5}^8 P(X = k) = P(5) + P(6) + P(7) + P(8) = \text{---}$

(b) It is not a discrete probability function since

$$\sum_{k=1}^4 P(X = k) = \text{---} + \text{---} + \text{---} + \text{---} \neq 1$$

5.3 Mean and Variance of a discrete probability distribution

- In Chapter 1 (Section 1.3.1 and 1.4.2), we learnt to calculate the mean and variance for a set of **data values**.
- In this chapter, we will learn to calculate the theoretical **population** mean μ and **population** variance σ^2 from a discrete **probability distribution**.
- The expectation of a random variable (or expected value) is the same as the population mean.

$$\mu = \sum kP(X = k)$$

$$\sigma^2 = \left[\sum k^2 \cdot P(X = k) \right] - \mu^2 \text{ or } \sigma^2 = \sum (k - \mu)^2 P(X = k)$$

Example 5.3-1

Find the mean, variance and standard deviation of the random variable in the following probability distribution:

$X = k$	1	2	3	4	5
$P(X = k)$	0.16	0.22	0.28	0.20	0.14

Solution:

$$\text{Mean, } \mu = \sum kP(X = k) = 1(0.16) + 2(0.22) + 3(0.28) + 4(0.20) + 5(0.14) =$$

$$\sum k^2 P(X = k) = 1^2(0.16) + 2^2(0.22) + 3^2(0.28) + 4^2(0.20) + 5^2(0.14) =$$

$$\text{Variance, } \sigma^2 = \sum k^2 P(X = k) - \mu^2 =$$

$$\text{Standard deviation, } \sigma =$$

Example 5.3-2

The random variable X represents the number of defective tires. The probability distribution of X is given below:

k	0	1	2	3	4
$P(X = k)$	m	0.16	0.06	0.04	0.20

- (a) Find the value of m .
- (b) Compute
- the expectation of X ,
 - the standard deviation of the distribution.

Solution:

- (a) For a probability distribution, $m + 0.16 + 0.06 + 0.04 + 0.2 = 1$

$$m =$$

- (bi) $E(X) = \sum kP(X = k) =$

- (bii) $\sigma^2 = \sum k^2 P(X = k) - \mu^2 =$

$$\text{Standard deviation, } \sigma =$$

Example 5.3-3

The following table shows the distribution of household sizes in a small town.

k	1	2	3	4	5	6
$P(X = k)$	0.266	0.330	0.166	0.140	0.064	0.034

- Show that the distribution is a probability distribution.
- What is the expected size of a household in the town?

Solution:

- (i) Since $0.266 + 0.330 + 0.166 + 0.140 + 0.064 + 0.034 =$

- (ii) $E(X) = \sum kP(X = k) =$

- 4 A charity organisation is selling \$4 raffle tickets as part of a fund-raising programme. The first prize is a computer valued at \$3150, and the second prize is a vacuum cleaner valued at \$450. The remaining 15 prizes are \$25 gift vouchers. The number of tickets sold is 5000.
- (a) Find the expected net gain to the player for one play of the game.
- (b) Interpret your answer to part (a).

Answers

1(a)

No. of children	0	1	2	3
No. of households	60/139	56/139	19/139	4/139

(b) (i) 0.403 (ii) 0.165 (iii) 0.432 (iv)
0.568

2 (a) 0.35 (b) 18.6 months

3 $k = 0.06$; mean = 0.44; standard deviation = 0.852

4 -\$3.21