Chapter 2: Set Theory

Objective

The objective of this chapter is to

1. be able to understand the terminology of sets
2. apply the algebra and the laws of algebra of sets.
3. apply the use of counting principles.

Content

2.1 Introduction to sets

The idea of set is often used in daily life events, it is used **to classify and group objects** together. Often the group of objects contains certain **distinctive or similar properties**.
Eg. plants of the **same species**. Different types of food.

Example 1

(a) The 26 letter in English alphabets $a,b,c,d,e,f.......,z$ forms a set.
(b) Students registered in different **diploma** courses constitute different sets.
(c) The types of meals offered by say Burger King also form a set of meals.

Note:
We normally called the objects in the set an element. For example, the 5th element of set (a) in Example 1 is e.

2.1.1 Sets and Elements

A set is simply a collection of objects. A particular element of a set is often called **a member**.

Sets can be represented by symbols either in braces or Venn Diagrams.

Braces
It is a common practice to denote sets by having the members **enclosed in braces** or **curly brackets**. The sets themselves are represented by **Capital Letters**.

Example 2

Set 1: The set V consists of all vowels in the English alphabets, $V = \{a, e, i, o, u\}$.
Set 2: The set of all odd numbers less than 13, is $A = \{1, 3, 5, 7, 9, 11\}$.
Set 3: The set of Z consisting of all numbers which are greater than 7 is
$Z = \{z \mid z > 7\}$

Note: $\mid$ means **such that**.

Notation:
In standard books the Greek letter $\in$, (may also be called **epsilon**) is used to denote **a member of** and $\notin$ means **does not belong** to.

Example 3

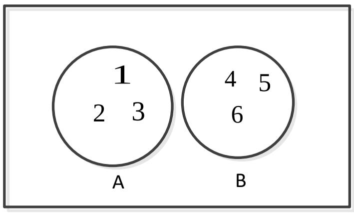
For Set 1 in Example 2, we say $a \in V$ or a belongs to V or a is a member of V.
For Set 2 in Example 2, we can also say $2 \notin A$ or 2 **does not belong** to A or 2 is **not a member of** A.

2.1.2 Venn Diagrams

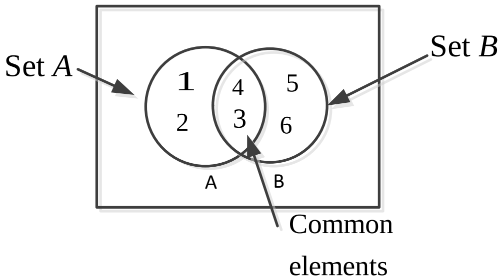
Venn Diagrams (it is sometimes called Venn-Euler Diagrams are very useful in visual representation of sets.

Example 4

(a) If set $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$, they can be represented by a Venn Diagram as shown below.

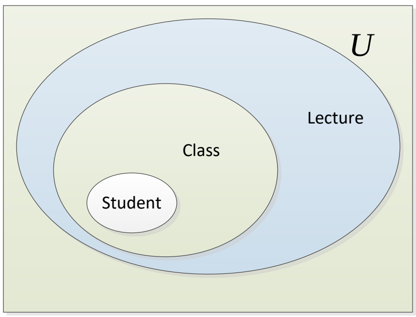


(b) If set $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, they can be represented by a Venn Diagram as shown below. Elements $\{3, 4\}$ is shared and is only written once. The common elements are enclosed by both set A and set B.



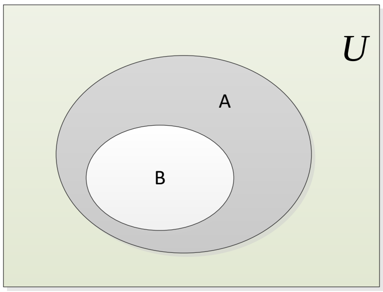
2.1.3 Subsets

A subset of a set A is a set in which it consists of **some** or **all possible** elements of A.

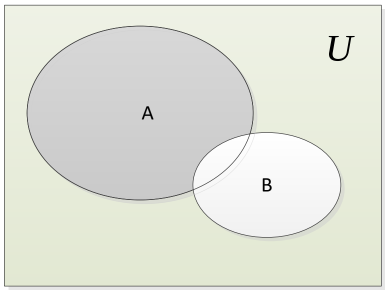


$Student \subset Class \subset Lecture$

Notation:
Thus if a set B is a subset of A, we say that B **is contained** in A or sometimes we say A **contains** B and we write $B \subset A$ or $A \supset B$.



This relationship between 2 sets is called the **Inclusion Relation.**



If B **does not contain in** A, it is denoted as $B \not\subset A$.

Note: Hence we can ONLY say B is a subset of A if $x \in B \Rightarrow x \in A$.

Example 5

(a) Consider the set of English alphabets $A = \{a, b, c, \ \ldots, x, , y, z\}$. We can say that $V = \{a, e, i, o, u\}$ is a subset of A. Hence, we write as $V \subset A$ or V is a subset of A.

(b) Consider the sets $A = \{1, 3, 4, 5, 8, 9\}$ and $B = \{1, 2, 3, 5, 7\}$ and $C = \{1, 5\}$. We can say that $C \subset A$ and $C \subset B$. However, $B \not\subset A$ as elements 2 and 7 do not belong to A.

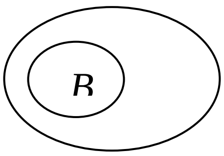
Remarks:

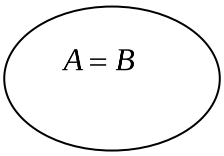
(a) Every set A is a subset of itself.

(b) Some books call B is a proper subset of A if:
(i) B is a subset of A $(B \subset A)$ and
(ii) B is not equal to $A (B \neq A)$.

(c) In some books 'B is a subset of A' is denoted by $B \subseteq A$ while 'B is a proper subset of A' is denoted by $B \subset A$. However, in this module we do not distinguish between a subset and a proper subset.

The relationship $B \subseteq A$ can be pictured in 2 ways as shown below.





2.1.4 Special Subsets

(a) **Null set** or **Empty set**

(i) A set with no element is called an empty set or a null set and is denoted by $\varnothing$.

(ii) Null set, $\varnothing$, is a subset of all sets.
$\varnothing \subset \{1, 2, 3\}$; $\varnothing \subset \{a, b\}$ and $\varnothing \subset \varnothing$.

(b) **Subset of itself**

Consider two sets $E = \{2, 4, 6\}$ and $F = \{6, 2, 4\}$. We can say that E is a subset of F since each member 2, 4 and 6 belonging to E also belongs to F. In fact, $E = F$. In a similar manner, it can be shown that every set is a subset of itself, i.e, the elements of E belongs to E.

2.2 Algebra of Sets

2.2.1 Equality of Sets

Two sets are equal if and only if they have the same members or elements.

Note:
The elements in a set need not be in the same order when you write them. For example, $\{a, b, c\} = \{c, a, b\}$.

Incidentally, noticed that **if** $A \subset B$ **and** $B \subset A$**, then *A* and *B* must be equal** and vice versa. In other words:

$$A = B \text{ if and only if } A \subset B \text{ and } B \subset A$$

(**Note**: the order of the elements are not important here)

2.2.2 Union and Intersection

UNION

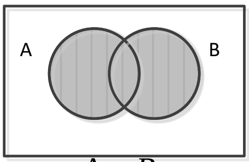
The **UNION** of two sets A and B, denoted by $A \cup B$, is the set of all elements that belong to A **or** B (**or to both**). We write $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

Example 6

Consider $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, $\Rightarrow \quad A \cup B = \{1, 2, 3, 4, 5, 6\}$

Note:
We do not repeat writing the elements found in both sets. For example, elements 3 and 4 can be found in both A and B but we do not write $A \cup B = \{1,2,3,3,4,4,5,6\}$.

When we wish to express $A \cup B$ in Venn diagram, we will shade the Venn diagram as shown below:



$A \cup B$

Exercise 2.1

Given that $A = \{x \mid x \text{ is a multiple of } 3\}$, $B = \{x \mid x \text{ is a multiple of } 4\}$ and $C = \{x \mid x \text{ is a multiple of } 2\}$, find the following for values of x if x is a positive integer and less than 20:

(a) $A \cup B$

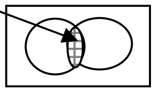
(b) $A \cup C$

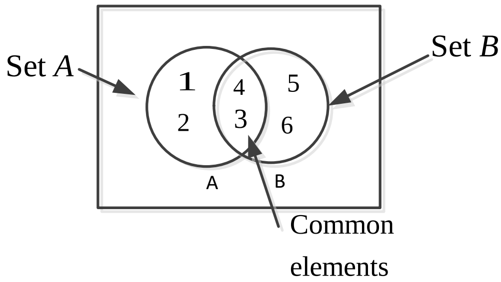
(c) $B \cup C$

INTERSECTION

The **INTERSECTION** of two sets A and B, denoted by $A \cap B$, is the set of all elements that belong to both A and B ONLY. We write $A \cap B = \{x \mid x \in A \; and \; x \in B\}$.

$A \cap B$ (only the intersection is shaded)





<u>Example 7</u>

Consider $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$,

$\Rightarrow \quad A \cap B = \{3, 4\}$.

<u>Exercise 2.2</u>

Consider the set A where $A = \{x \mid x \text{ is a multiple of } 2\}$, $B = \{x \mid x \text{ is a multiple of } 6\}$
Find $A \cap B$ given that x is a positive integer and less than 20 .

2.2.3 Null or Empty Sets

A null or empty set contains no elements.

Consider the following sets:

$$A = \{1, 2, 3, 4, 5\}, B = \{2, 4, 7\} \text{ and } C = \{8, 9\} .$$

Then a) $A \cup B = A$ or $B = \{1, 3, 5, 2, 4, 7\}$ $\Rightarrow$ **Union**

b) $A \cap B = A$ and $B = \{2, 4\}$ $\Rightarrow$ **Intersection**

c) $A \cup C = A$ or $B = \{1, 3, 2, 4, 5, 8, 9\}$ $\Rightarrow$ **Union**

d) $A \cap C = \varnothing =$ Null set $\Rightarrow$ **Empty set**

2.2.4 Universal Set

The set of all objects relevant to a particular application is called the **Universal Set**. We shall use the symbol S **or** U **to denote the Universal Set**.

Exercise 2.3

1) List all the possible outcomes Universal Set, S, of throwing a dice.

2) List the Universal Set S *of* possible outcomes of the tossing two coins.

2.2.5 Complement Set

Definition: $A^C = \{x \mid x \in S \;\&\; x \notin A\}$.

The **complement** of A is A' or A^c

Thus, the universal set S is a union of set A and its complement. $S = A \cup A^C$
A and A^C are said to be **mutually exclusive**.

Note: It also follows that $A^C \cap A = \varnothing$.

Example 8

Let the universal set S be the set of all positive integers and A be the set of all positive integers greater than 10. Then, the complement of A i.e. $A' = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

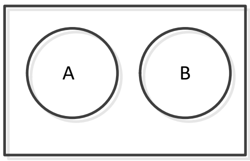
Exercise 2.4

Let S be a universal set consisting of the first seven letters of the English alphabets, that is, $S = \{a, b, c, d, e, f, g\}$. If $A = \{a, b, c, d\}$, find the complement of A.

2.2.6 Disjoint sets

The **null set** is $\varnothing$.

So if $A \cap B = \varnothing$, then it means that A and B are **disjoint**



Example 9

If $P = \{a, b, c\}$ and $Q = \{e, f, g, h\}$, then both P and Q are disjoint.

The sets $\{1, 3, 5, 7, 9\}$ and $\{2, 4, 6, 8\}$ are also disjoint since there is no common element.

Exercise 2.5

(1) If set $A = \{1,\ 3,\ 5,\ 7,\ 9\}$ and set $B = \{2,\ 4,\ 6,\ 8\}$, find the following if S is the universal set of integers from1 to 9.

(a) $A \cup B$

(b) $A^C \cap B$

(c) $A^C \cap B^C$

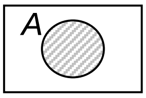
(2) Let the universal $S = \{a, b, c, d, e\}$ with $A = \{a, b, d\}$ and $B = \{b, d, e\}$. Find (a) $A \cup B$ (b) $B \cap A$ (c) B^C

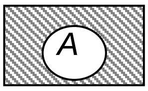
(d) $A^C \cap B$ (e) $A \cup B^C$ (f) $A^C \cap B^C$

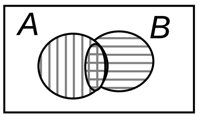
(g) $(A \cap B)^C$ (h) $(A \cup B)^C$

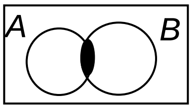
Exercise 2.6

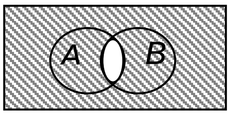
(1) In an online registration for prescribed electives, some students register for 'Accounting (A)' and some students 'Business (B)'. What events are represented by the shaded region? What does the set $A' \cup B$ represent?

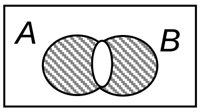
(a) 

(b) 

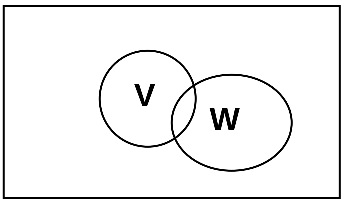
(c) 

(iv) 

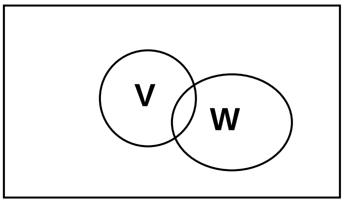
(e) 

(f) 

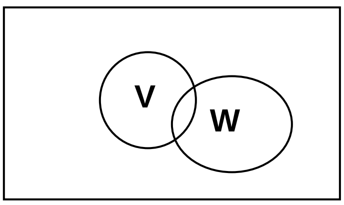
(2) In the Venn diagrams below, shade $V \cap W$, $V^C \cup W$, $V \cap W^C$.



$V \cap W$



$V^C \cup W$



$V \cap W^C$

2.3 Laws of the Algebra of Sets

1. Idempotent Laws	$A \cup A = A$	$A \cap A = A$
2. Associative Laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
3. Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
4. Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
5. Identity Laws	$A \cup \varnothing = A$ $A \cup S = S$	$A \cap S = A$ $A \cap \varnothing = \varnothing$
6. Involution Law	$(A^C)^C = A$	
7. Complement Laws	$A \cup A^C = S$ $S^C = \varnothing$	$A \cap A^C = \varnothing$ $\varnothing^C = S$
8. De Morgan's Laws	$(A \cup B)^C = A^C \cap B^C$	$(A \cap B)^C = A^C \cup B^C$

From the laws above, we can derive the following on unions and intersections

Unions	**Intersections**
$A \cup A = A$	$A \cap A = A$
$A \cup S = S$	$A \cap \varnothing = \varnothing$
$A \cup \varnothing = A$	$A \cap S = A$
$A \cup A' = S$	$A \cap A' = \varnothing$
$A \cup B = B \cup A$	$A \cap B = B \cap A$

Exercise 2.7

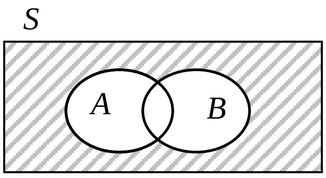
(1) Simplify $A \cap (A^C \cup A)$.

(2) Prove $(A \cap B^C) \cap (A^C \cap B) = \varnothing$

(3) Prove $(A \cup B) \cap (A \cup B^C) = A$

(4) Show that $(A^C \cap B) \cup (A^C \cap B^C) = A^C$

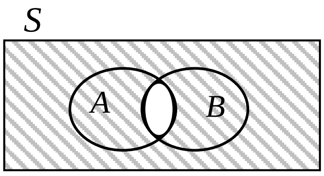
2.3.1 De Morgan's Laws



$\left(A\cup B\right)' = A'\cap B'$

This can also be written as

$S-\left(A\cup B\right)$



$\left(A\cap B\right)' = A'\cup B'$

This can also be written as

$S-\left(A\cap B\right)$

<u>Exercise 2.8</u>

Using the laws of algebra of sets, elevate the following questions.

(1) Prove $A\cap(A^C\cup B)^C = A\cap B^C$

(2) Show $(A\cap B)\cap(A^C\cup B^C) = \varnothing$

(3) Simplify $(A\cap B^C)^C\cup A$

2.4 Cardinality and counting principle

The number of elements in a set A is called the **cardinality** or **size** of A.
This number is denoted as $|A|$. Some books use n(A)

Example 10

(a) Let S be the set of letters in the English Alphabet. Then the cardinality is $|S| = 26$. **This set** is said to be finite **as it has a finite number of elements. i.e. the number of elements can be counted.**

(b) Consider $Y = \{y \mid y > 3\}$. Thus, the set Y is an infinite set.

(c) Consider $N = \{x \mid x \text{ is a non-negative integer}\} = \{0,1,2,3,.....\}$. Thus, N is an infinite set.

(d) Consider the set of all positive integers, that is $Z^+ = \{1,2,3,....\}$. Thus, Z^+ is an infinite set.

From example 10 (b) – (d), we can see that a set with **unlimited number** of elements or members is said to be an **infinite set** and it can be **denoted as** $|S| = \infty$. On the other hand, an empty or null set which has no element, we denote as $|\varnothing| = 0$ or $\mathrm{n}(\varnothing) = 0$.

2.4.1 Finite Sets

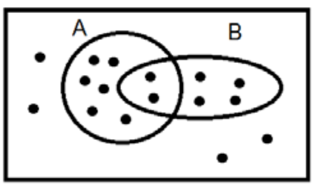
A finite set contains a fix number of elements.

Example 11

A cafe has 5 types of set breakfast set meals. If the set of breakfast meals is represented by B. $B=$\{ Kid's Meal, Healthy Meal, Fast Meal, Hearty Meal, Jumbo Meal\}. The number of meals can be written as $n(\mathrm{B}) = 5$.
The café also serves 10 types of Lunch set meals. If the lunch set meals are represented by L then $n(L) = 10$

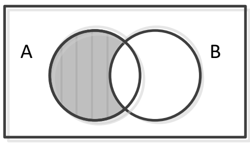
Exercise 2.9

The elements of the Universal set, *S* are black points as given below. Find the cardinality: $|A|, |B|$, $|A \cap B|$, $|A \cup B|$, $|A^C \cap B|$ and $|A^C \cap B^C|$.



2.4.2 Difference of Two sets : *A –B* (Subtraction)

The difference of two sets A and B is the set of elements belonging to A and not B.
Thus $A-B=\{x \mid x\in A \; and \; x\notin B\}$.

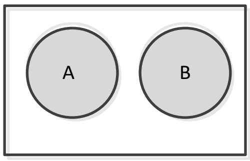


Exercise 2.10

Let $A=\{1,3,5\}$ and $B=\{1,2,3\}$. Find $A-B$, $B-A$ and $n(A-B)$.

2.4.3 Addition of Two sets : *A + B* (Addition)

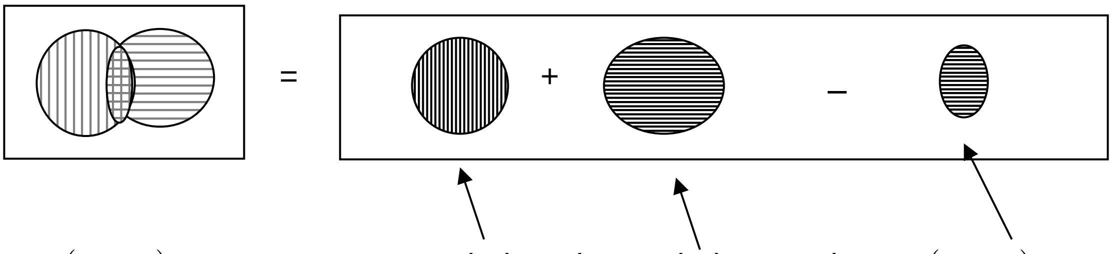
In the case of a disjoint set
$A+B \;=A\cup B$
$|A+B| \;=|A\cup B|$



However for a set which is NOT Disjoint,

$$\boxed{|A\cup B|=|A|+|B|-|A\cap B|}$$

This is because adding sets A and B will add the common elements twice. Note that when we have the union two sets, the common elements are only appearing once and not repeated.



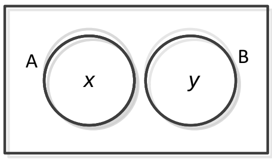
$(A\cup B)$ = (A) plus (B) minus $(A\cap B)$

Exercise 2.11

Let $A=\{1,3,5\}$ and $B=\{1,2,3\}$. Find the $A+B$, $B+A$ and $n(A+B)$.

2.4.4 Counting principle

(1) Consider $\mathrm{n(A)} = x, \mathrm{n(B)} = y$.

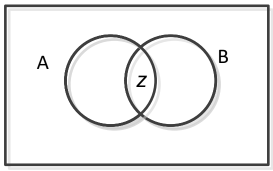


(a) If both sets A and B are disjoint, then

$n(\mathrm{A} \cup \mathrm{B}) = \mathrm{n(A)} + \mathrm{n(B)}$ $\qquad n(\mathrm{A} \cap \mathrm{B}) = \varnothing$

$= x + y$

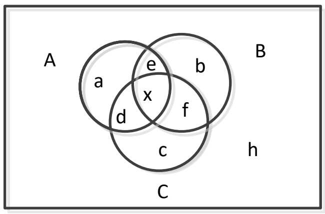
(b) If intersection of set A and set B is not an empty set, then



$n(\mathrm{A} \cup \mathrm{B}) = \mathrm{n(A)} + \mathrm{n(B)} - \mathrm{n(A \cap B)}$ $\qquad n(\mathrm{A} \cap \mathrm{B}) = z$

$= x + y - z$

(2) With reference to the Venn diagram below,



(a) If the value of $n(\mathrm{A} \cap \mathrm{B} \cap C) = x$ is given and the following are given as well:

$n(\mathrm{A} \cap \mathrm{B} \cap C) = x$

$n(\mathrm{A}), \mathrm{n(B)}, \mathrm{n(C)}$

$\mathrm{n(A \cap B)}, \mathrm{n}(\mathrm{A} \cap C), n(\mathrm{B} \cap C)$

$\mathrm{n(S)}$

With the value of *x* given, the values of b, c, d and e, f, g can be found.

$d = \mathrm{n}(\mathrm{A} \cap C) - x$ $\qquad e = \mathrm{n(A \cap B)} - x$ $\qquad f = \mathrm{n}(\mathrm{B} \cap C) - x$

$a = n(A) - d - e - x$ $\qquad b = n(B) - e - f - x$ $\qquad c = n(C) - d - f - x$

Those that are not within the three sets but is part of the universal set is: $h = n(\mathrm{S}) - a - b - c - d - e - f - x$

(b) If the value of $n(\mathrm{A} \cap \mathrm{B} \cap C) = x$ is not given and the following are given as well:

$n(\mathrm{A}), \mathrm{n(B)}, \mathrm{n(C)}$

$\mathrm{n(A \cap B)}, \mathrm{n}(\mathrm{A} \cap C), n(\mathrm{B} \cap C)$

$\mathrm{n(S)}, \mathrm{h}$

Then, $n(S) - h = n(\mathrm{A}) + n(\mathrm{B}) + \mathrm{n(C)} - n(A \cap B) - n(A \cap C) - n(B \cap C) + x$

$x = n(S) - h - n(A) - n(B) - n(C) + n(A \cap B) + n(A \cap C) + n(B \cap C)$

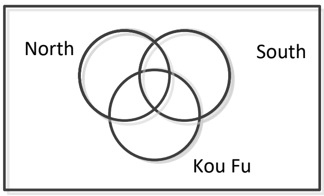
Once $n(\mathrm{A} \cap \mathrm{B} \cap C) = x$ is found, the values of a to f can be calculated.

Exercise 2.12

The Venn diagram below shows the results of a survey among 52 students. The students are to list the canteen they frequent for lunch on week 3 of the semester.

The results of the survey are as follows:

25 students frequent North canteen	n(North)= 25
20 students frequent South canteen	n(South)= 20
25 students frequent Kou Fu	n(Kou Fu)= 25
13 students frequent both North and South canteen	n(North & South)= 13
15 students frequent both Kou Fu and North canteen	n(North & Kou Fu)= 15
12 students frequent both Kou Fu and South canteen	n(South & Kou Fu)= 12
10 students frequent all three canteens	n(North & South & Kou Fu)= 10



(a) Draw a Venn diagram to represent the above results.

(b) Find the number of students who ate only at North Canteen.

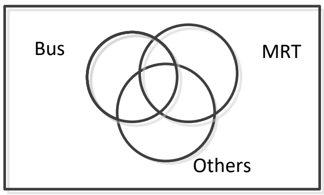
(c) Find the number of students who did not eat at any of the 3 canteens listed.

Exercise 2.13

The Venn diagram below shows the results of a survey among 55 NYP students on the mode used by students travelling to school.

The results of the survey are as follows:-

n(Bus)= 30
n(MRT)= 28
n(Others)= 17
n(Bus & MRT)= 8
n(MRT & Others)= 10
n(Bus & Others)= 7



(a) Find the number of students who use bus, MRT and Others to travel to school.

(b) How many students used only the Bus to travel to school?

Tutorial 2

1. Determine the relationships for sets A, B and C using set notations ($\in$, $\cup$, $\cap$, $\subset$, $\neq$, $=$, $-$, $\varnothing$, etc):
   (a) $A=\{2,3,6,8,10,11\}$, $B=\{2,6,8,10\}$ and $C=\{3,11\}$
   (b) $A=\{x: x \text{ is a multiple of } 5\}$, $B=\{x: x \text{ is a multiple of } 10\}$ and
   $C=\{x: x \text{ is a multiple of } 15\}$

2. List the elements of the following sets.
   (a) $A=\{x:\ x \text{ is an odd positive integer between } 10 \text{ to } 20\}$
   (b) $B=\{x:\ x \in \text{set of integers}, x \text{ is postive and is a multiple of } 7\}$
   (c) $C=\{x: x \in \text{set of integers}, x \text{ is a solution of } 2x+1=2\}$

3. Given the Universal set $S=\{x: x \text{ is the first } 10 \text{ English alphabet}\}$, $A=\{a,b,c,d,e\}$ and $B=\{c,d,e,f,g\}$, evaluate the following sets:
   (a) $A\cup B'$ (b) $A'\cap B'$ (c) $A\cap B'$
   (d) $(A\cup B)\cap A'$ (e) $S\cap A$ (f) $S\cap(A\cap B)'$
   (g) $(A\cap B)'\cap B'$

4. A poll of 100 students was taken at a school to find out how they travel to school. The results were as follows:

28 mentioned car pools	9 used car pools and buses
31 took buses	10 used car pools and sometimes their own cars
42 drove to school	6 used buses as well as their own cars.
	4 used all three methods

   (a) Draw a Venn diagram to represent the above results.
   (b) How many students use none of the three methods?
   (c) How many students use car pools exclusively to get to school?
   (d) How many students use buses exclusively to get to school?

5. In XYZ Manufacturing Company, a quality inspector has inspected a sample of 28 components. Among these components, there are:
   17 with assembly faults, 17 with defective parts and 18 with wrong colour coding; 7 with assembly faults and defective parts, 13 with defective parts and wrong colour coding, 9 with wrong colour coding and assembly faults.
   Using Venn diagram, find the number of components with all 3 faults.

6. Prove the following statements by using the laws of algebra of sets:
   (a) $(A\cup S')\cap(A\cup\varnothing'')=A$ (b) $(A'\cup B)'\cap A'=\varnothing$

7. Simplify the following expressions by using the laws of algebra of sets:
   (a) $(A\cap S)'\cup(B\cap\varnothing'')$ (b) $(A'\cap B)'\cap(A\cup B)$

Answers

1. (a) $B \subset A, \quad A - B = C, \quad B \cap C = \varnothing, \quad B \cup C = A$, etc
   (b) $C - B \subset A, \quad C - A \subset B, \quad C \subset A, \quad B \subset A$, etc

2. (a) $A = \{11, 13, 15, 17, 19\}$ (b) $B = \{7, 14, 21, 28....\}$ (c) $C = \{\ \} = \varnothing$

3. (a) $\{a, b, c, d, e, h, i, j\}$ (b) $\{h, i, j\}$ (c) $\{a, b\}$
   (d) $\{f, g\}$ (e) $\{a, b, c, d, e\} = A$ (f) $\{a, b, f, g, h, i, j\}$
   (g) $\{a, b, h, i, j\}$

4. (b) 20 (c) 13 (d) 20

5. 5

7. (a) A' (b) A

Questions from past year examination papers

1) 2013/14S2 EG1740 Sem Exam-Q6b

The table below shows the results of a survey of 80 MIT students on their preference for Chinese, Indian or Malay food. Every student in the survey has to choose at least one type of food.

Type of food	Chinese	Indian	Malay	Chinese & Indian	Chinese & Malay	Indian & Malay
Number of students	40	43	40	15	20	18

Using the Venn diagram, find the number of students who chose

(i) all three types of food. (6 marks)

(ii) only Malay food. (2 marks)

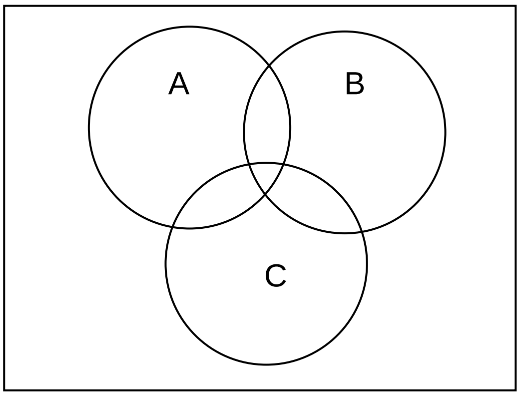
Represent the statement “Students who chose Indian food or Malay food but not Chinese food” using the following methods:

(iii) a Venn diagram. (3 marks)

(iv) Set notation. (2 marks)

2) 2012/13S2 EG1740 Sem Exam-Q2

(a) Copy the Venn diagram that shows the sets A, B and C and shade the set $A \cup (B - C)$. (4 marks)



(b) Using the Laws of Algebra of Sets, show that for any sets A, B and C, $B \cap (A \cap B)^C = B \cap A^C$. (4 marks)

3) 2013/14S2 EG1740 Sem Exam-Q3b

Use the Laws of Algebra of Sets to show that $(A \cap B) \cap \left(A^C \cup B\right)^C = \varnothing$.

(4 marks)

4) 2013/14S1 IT1101/1501/1561/1751/1621 Sem Exam– Q2

A survey was conducted on 140 students to find out their preferred mode of communication with friends. The findings are shown in the table below:

Preferred mode of communication	**Number of students**
Email (E)	50
SMS (S)	65
WhatsApp (W)	75
Email and SMS	28
SMS and WhatsApp	24
All three modes	7
None of the above mode	15

(a) Find the number of students who prefer to use Email and WhatsApp to communicate. (3 marks)

(b) Draw a Venn Diagram with all the information. (5 marks)

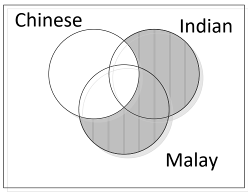
(c) Find the number of students with only one preferred mode of communication. (2 marks)

Answers

1 i) n(all types of food)=10

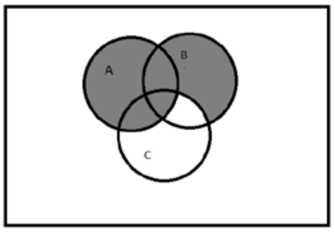
ii) $n(Malay\ food\ only) = 12$

iii)



iv) $x = C^C \cap (M \cup I)$

2a)



b) $B \cap (A \cap B)^C$

$= B \cap (A^C \cup B^C)$ *De Morgan's Law*

$= (B \cap A^C) \cup (B \cap B^C)$ *Distributive Law*

$= (B \cap A^C) \cup \varnothing$ *Complement Law*

$= B \cap A^C$ *Identity Law*

3) $(A \cap B) \cap (A^C \cup B)^C$

$= (A \cap B) \cap (A \cap B^C)$

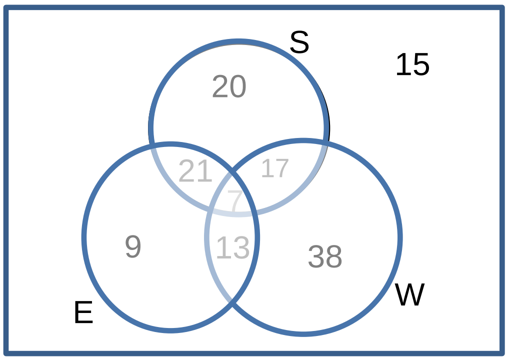
$= (A \cap A) \cap (B \cap B^C)$

$= A \cap \varnothing$

$= \varnothing$

4a) Number of students who prefer Email and Whatsapp = 20

b)



c) 67