

Course :	Diploma in Electronic Systems Diploma in Telematics & Media Technology Diploma in Aerospace Systems & Management Diploma in Electrical Engineering with Eco-Design Diploma in Mechatronics Engineering Diploma in Digital & Precision Engineering Diploma in Aeronautical & Aerospace Technology Diploma in Biomedical Engineering Diploma in Nanotechnology & Materials Science Diploma in Engineering with Business Diploma in Information Technology Diploma in Financial Informatics Diploma in Cybersecurity & Forensics Diploma in Infocomm & Security Diploma in Chemical & Pharmaceutical Technology Diploma in Biologics & Process Technology Diploma in Chemical & Green Technology
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Topic 5 : Discrete Probability Distribution

Objectives :

At the end of this lesson, the student should be able to:

- 1 define random variables
- 2 distinguish between discrete and continuous random variables
- 3 define a discrete probability distribution
- 4 compute the mean, variance and standard deviation of a discrete random variable

Topic 5: Discrete Probability Distribution

5.1 Random variable

- A variable is an alphabetical representation of a quantity that can take on various numerical values.
- A random variable, usually denoted by X, Y, \dots , is a variable that takes on different values due to random phenomenon (or by chance).
- A random variable can be discrete or continuous (refer to Chapter 1).

Example 5.1-1

- (a) A coin is tossed three times.

If X is a random variable representing the number of heads, then

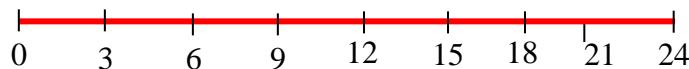
$$X = 0, 1, 2, 3$$

There can be no head or 1 head or 2 heads or 3 heads in the three toss.

- (b) Supposing Y is a random variable representing the time a sales person spends on making calls per day.

The time spent on making calls can be any value (e.g. 2.4 minutes, 49.5 minutes, etc), Y is said to be continuous random variable.

The values of a continuous random variable can be represented as an interval on a number line.



5.2 Discrete probability distribution

- We may not know the exact value of a random variable at any specific moment. However we may calculate the likelihood (probability) that a random variable may take a specific value.
- A **probability distribution** is a table or an equation that links each value of a random variable with its probability of occurrence. The probability distribution of a discrete random variable may be represented using a table.

Example 5.2-1

A fair coin is tossed twice. If X is a random variable representing the number of heads, then construct the probability distribution for X .

Solution:

$$P(X = 0) = P(TT) = \frac{1}{2} \times \frac{1}{2} =$$

$$P(X = 1) = P(TH) + P(HT) = \frac{1}{2} \times \frac{1}{2} \times 2 =$$

$$P(X = 2) = P(HH) = \frac{1}{2} \times \frac{1}{2} =$$

$X = k$	0	1	2
$P(X = k)$			

A **probability distribution** must satisfy the following conditions:

(a) $0 \leq P(X = k) \leq 1$ for all values of k ,

(b) $\sum_{all\ k} P(X = k) = 1$ (sum of all probabilities is 1).

Example 5.2-2

Explain whether each of the following is a discrete probability distribution function.

(a)

$X = k$	5	6	7	8
$P(X = k)$	$\frac{1}{16}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

(b)

$X = k$	1	2	3	4
$P(X = k)$	0.09	0.36	0.49	0.05

Solution:

(a) It is a discrete probability distribution function since

(i) _____ $\leq P(X = k) \leq$ _____

(ii) $\sum_{k=5}^8 P(X = k) = P(5) + P(6) + P(7) + P(8) =$ _____

(b) It is not a discrete probability function since

$$\sum_{k=1}^4 P(X = k) = \text{_____} + \text{_____} + \text{_____} + \text{_____} \neq 1$$

5.3 Mean and Variance of a discrete probability distribution

- In Chapter 1 (Section 1.3.1 and 1.4.2), we learnt to calculate the mean and variance for a set of data values.
- In this chapter, we will learn to calculate the theoretical population mean μ and population variance σ^2 from a discrete probability distribution.
- The expectation of a random variable (or expected value) is the same as the population mean.

$$\mu = \sum k P(X = k)$$

$$\sigma^2 = [\sum k^2 \cdot P(X = k)] - \mu^2 \text{ or } \sigma^2 = \sum (k - \mu)^2 P(X = k)$$

Example 5.3-1

Find the mean, variance and standard deviation of the random variable in the following probability distribution:

$X = k$	1	2	3	4	5
$P(X = k)$	0.16	0.22	0.28	0.20	0.14

Solution:

$$\text{Mean, } \mu = \sum k P(X = k) = 1(0.16) + 2(0.22) + 3(0.28) + 4(0.20) + 5(0.14) =$$

$$\sum k^2 P(X = k) = 1^2(0.16) + 2^2(0.22) + 3^2(0.28) + 4^2(0.20) + 5^2(0.14) =$$

$$\text{Variance, } \sigma^2 = \sum k^2 P(X = k) - \mu^2 =$$

$$\text{Standard deviation, } \sigma =$$

Example 5.3-2

The random variable X represents the number of defective tires. The probability distribution of X is given below:

k	0	1	2	3	4
$P(X = k)$	m	0.16	0.06	0.04	0.20

- (a) Find the value of m .
- (b) Compute
 - (i) the expectation of X ,
 - (ii) the standard deviation of the distribution.

Solution:

- (a) For a probability distribution, $m + 0.16 + 0.06 + 0.04 + 0.2 = 1$

$$m =$$

$$(bi) E(X) = \sum kP(X = k) =$$

$$(bii) \sigma^2 = \sum k^2 P(X = k) - \mu^2 =$$

$$\text{Standard deviation, } \sigma =$$

Example 5.3-3

The following table shows the distribution of household sizes in a small town.

k	1	2	3	4	5	6
$P(X = k)$	0.266	0.330	0.166	0.140	0.064	0.034

- (i) Show that the distribution is a probability distribution.
- (ii) What is the expected size of a household in the town?

Solution:

- (i) Since $0.266 + 0.330 + 0.166 + 0.140 + 0.064 + 0.034 =$

$$(ii) E(X) = \sum kP(X = k) =$$

Tutorial 5: Discrete Probability Distribution

- 1 Randomly selected households from a particular estate were asked on the number of children they have and the following frequency distribution shows the result of the survey:

Number of children	0	1	2	3
Households	300	280	95	20

- (a) Construct a probability distribution table.

(b) Let X denotes the number of children from the particular estate. Find the following probabilities:

(i) $P(X = 1)$

(ii) $P(X \geq 2)$

(iii) $P(X < 1)$

(iv) $P(1 \leq X \leq 3)$

- 2 An electrical appliance company offers its customers a number of different instalment plans. Let the random variable X represents the number of instalments for a randomly selected customer and the probability distribution for X is given below:

x	6	12	24	36
$P(X = x)$	0.20	0.30	k	0.15

- (a) Find the constant value, k .
 - (b) Find the mean of the distribution, X .

- 3 Let the random variable X be the number of errors that a randomly selected page of a book contains. The following table lists the probability distribution of X .

x	0	1	2	3	4
$P(X = x)$	0.73	0.16	k	0.04	0.01

Find the value of k ; hence, find the mean and standard deviation of X .

- 4 A charity organisation is selling \$4 raffle tickets as part of a fund-raising programme. The first prize is a computer valued at \$3150, and the second prize is a vacuum cleaner valued at \$450. The remaining 15 prizes are \$25 gift vouchers. The number of tickets sold is 5000.

(a) Find the expected net gain to the player for one play of the game.

(b) Interpret your answer to part (a).

Answers

1(a)

No. of children	0	1	2	3
No. of households	60/139	56/139	19/139	4/139

(b) (i) 0.403 (ii) 0.165 (iii) 0.432 (iv)
 0.568

2 (a) 0.35 (b) 18.6 months

3 $k = 0.06$; mean = 0.44; standard deviation = 0.852

4 -\$3.21