AS.110.420 HW3

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February 2023

Problem 1 We note that B and B^c are mutually exclusive events. Because $A \cap B \subseteq B$ and $A \cap B^c \subseteq B^c$, AB and AB^c are also mutually exclusive events. By the axiom of subbadditivity, it then holds that $P((A \cap B) \cup (A \cap B^c)) = P(AB^c) + P(AB)$. However, we can equivalently write the left hand side of this statement as $P(A \cup (B \cap B^c))$. However because B and B^c are mutually exlusive, their interesection is the null set, and any set $\cup$ the null set is just that set itself. It follows that $P(A) = P(A - B) + P(AB)$, writing the previous statement in a different way. Rearranging, we have the result.

Problem 2

1. Let D be the event that the market goes down, and T the event that trading exceeds 1 billion shares, such that $P(D) = 0.05, P(T) = 0.2, P(D \cap T) = 0.03$. We want to compute $P(D \cap T^c)$. This is just $P(D) - P(D \cap T)$, or 0.02.

2. Given the entire population Ω, let T represent the event that a chosen person is under 20 years old, and E the event that they exercise. The given information tells us $P(E|T) = 0.75, P(T|E) = 0.6, P(E \cup T) = 0.1$. We want to compute $P(E|T^c)$. To do so by Baye's rule requires computing $\frac{P(E \cap T^c)}{P(T^c)}$.

3. Letting R and W represent the event of rain and wind, We know that $P(R) = 0.5, P(W) = 3, P(R^c \cap W^c) = 0.1$. By De Morgan's law the latter is equal to $P((R \cup W)^c) = 1 - P(R \cup W)$. It would then follow that $P(R \cup W) = 0.9$. However, this cannot be correct, since the union of two sets is at most equal to their sums, with equality only in the case that the events are disjunct.

Problem 3 Our sample space Ω is the set $\{H_1H_2, H_1T_2, T_1H_2, T_1T_2\}$, and we seek $P(H_1|(H_1 \cap H_2) = \frac{P(H_1 \cap (H_1 \cap H_2))}{P((H_1 \cap H_2))}$. Note that $H_1 \subset (H_1 \cap H_2)$ so the numerator is just $1/2$. The denominator is the probability of any one of three events in the sample space (excluding TT), or $3/4$. The answer is $\frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$.

Problem 4

1. For the next 3 questions we let R be the event that the second card drawn is the same rank as the first, and C the event that they are the same color. We want to compute $P(C^c|R) = \frac{P(C^c \cap R)}{P(R)}$. We can write the numerator as $\frac{2}{51}$, as there are 2 cards left of the same rank and different

color that could be chosen. The denominator is $\frac{11}{51}$, as there are 11 cards left of the same suit (one having been removed).

2. $P(R|C^c) = \frac{P(R \cap C^c)}{P(C^c)}$, where, by a similar argument as the previous, the numerator is $\frac{2}{51}$ and the denominator $\frac{26}{51}$.

3. $P(R|C) = \frac{P(R \cap C)}{P(C)} = \frac{\frac{1}{51}}{\frac{25}{51}} = \frac{1}{25}$.

Problem 5