AS.110.420 Homework 2

LJ Gonzales, Jed's section

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Problem 1 Given X styles the shop owner has $\binom{X}{3}$ ways to have 3 distinct styles on display. We want this number to at least be equal to 52 so that each week can be assigned such a combination, where having undisplayed combinations at the end of the year is fine. We have $\frac{X!}{(X-3)!3!} \geq 52$, which was solved graphically to reveal $X \geq 7.8$. The shop owner must have at least 8 different dress styles to accomplish his goal.

Problem 2 In the 8-long sequence $O_1O_2O_3\dots O_8$ there are 8 places to choose where to place the 3 indistinguishable heads. The rest will be tails. Hence $\binom{8}{3}$ or 56 sequences.

Problem 3

1. There are 3 people out of 12 to choose for Task 1, 3 more from the remaining for Task 2, etc.. hence $\binom{12}{3}\binom{9}{3}\dots\binom{3}{3} = \frac{12!}{3!3!3!3!} = 369600$ ways.
2. Similarly the first choice only has $\binom{4}{3}\binom{9}{3}\dots\binom{3}{3} = \frac{4!}{3!} \times \frac{9!}{3!3!3!} = 6720$ distinct assigments.
3. There are 4! ways to arrange people a-d, and a further $\frac{9!}{3!3!3!}$ to arrange the rest, for a total of 40320 assignments.
4. We can follow a similar assignment to part a but this time the name of the team is irrelevant, effectively overcounting by 4!. We then have $\frac{12!}{3!3!3!3!4!}$.
5. Each of a,b,c,d will be on different teams, so everyone else will effectively be on one of "Team A", "Team B", etc¿.. Hence $\frac{8!}{2!2!2!2!} = 2520$ teams.

Problem 4

1. This is equivalent to a 30-letter anagram of word containing 10 of each $\{1, 2, 3\}$, or $\frac{30!}{10!10!10!}$, or about 5.55×10^{12}.
2. There are 3 ways to choose the digit that is repeated 12 times rather than 9. There are then $\binom{30}{12}$ ways to put that number in the placeholders of the 30-digit number. There are $\binom{18}{9}$ places to put the next number, regardless of what it is. Hence $3 \times \frac{30!}{12!9!9!}$, or about 1.26×10^{13}.

Problem 5 We consider that pizzas are uniquely defined by their topping choices and quantity (for instance, a 3-mushroom 2-pepperoni 1-tomato pizza is the same whether the tomato slice is across or next to the pepperoni). We can then think of this as choosing to put 6 "stars" representing our choices in 21 categories that may hold more than 1 star, giving $\binom{6+21-1}{6} = 230230$ possible pizzas.

Problem 6

1. Stars and bars with 3 stars and 5 bars: $\binom{3+5}{3} = 56$.
2. Same as a) with stars representing die and 6 boxes representing their outcomes. 56 outcomes.
3. Same as a) with stars representing 1's, and 6 containers for each variable. If a variable contains more than one 1 its integer value woulc change with its sum. 56 choices.

Problem 7

1. Here our stars represent the 55 electrons to be allocated, and 6 "container" energy levels to place them in. We have $\binom{55+6-!}{5} = 3478761$ possible arrangements.
2. The amount of electrons to be allocated in each energy level sums to 55, so there will be no empty spots. There are $\binom{55}{2}$ ways to have electrons in the first energy level, $\binom{53}{8}$ ways to arrange the rest in the second, etc, for a total of $\frac{55!}{2!8!18!18!8!1!} \approx 3.84 \times 10^{36}$

Problem 8

1. Without replacement implies this is just $\binom{5}{2} = 10$. If instead we care about the order that they come out (AF different from FA), this would be $5 * 4 = 20$.
2. We can think of this as a stars and bars problem, where we have 2 choices to put in some arrangement out of 5 choices, for a total of $\binom{2+5-1}{2} = 15$ possibilities. If we instead decide that the order of outcome is important (but two consecutive letters indistinguishable), we need to split into cases to get 25 possible choices. This is because there are 5 ways to get two of the same letters and $(5)_2 = 20$ of different pairings.
3. The number of outcomes (not probability) is now the same than if we were sampling with replacement, since there are sufficient letters to go around. 15/25 depending on your definition of outcome.
4. Still the same 15/25, only difference in practice is that double letters would come out much more often than usual.