Homework 1
AS.110.311
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4. We demand $z_1 z_2 = 0$. Writing this in polar form, we have

$$r_1 r_2[cos(\theta_1 + \theta_2) + isin(\theta_1 + \theta_2)] = 0 + 0i$$

We therefore need

$$r_1 r_2 cos(\theta) = 0, r_1 r_2 sin(\theta) = 0.$$

cosine and sine cannot both be 0 so we need at least one of r_1, r_2 to be 0.

9.

$$\frac{(2+3i)(1-2i)}{(1+2i)(1-2i)} - \frac{(8+i)(6+i)}{(6-i)(6+i)}$$
$$= \frac{8-i}{5} - \frac{47+14i}{37}$$
$$= \frac{61}{185} + i(\frac{-107}{185})$$

which is in the form $a + ib$, as sought.

15.

- $i^{4k} = ((i^2)^2)^k = (1)^k = 1$ for all k.
- $i^{4k+1} = i(i^{4k}) = i$ for all k, using the previous exercise.
- $i^{4k+2} = i(i^{4k+1}) = -1$ for all k, using the previous exercise.
- $i^{4k+3} = i(i^{4k+2}) = -i$ for all k, using the previous exercise.

16. We begin by noting:

- $4 = 4(1)$
- $62 = 4(15) + 2$
- $-202 = 4(-51) + 2$
- $-4321 = 4(-1081) + 3$

And using the results of problem 15, we find $1, -1, -1, -i$.

3. Writing each in polar form $z = |z|e^{i\theta}$, we have that $|i| = \sqrt{i(-i)} = 1$ for the first one, using the formula $|z| = \sqrt{zz'}$. Likewise, we have $|2| = 2, |-i| = 1, |-3| = 3$. The point -3 is farthest from the origin.

4.

$$z = 3 - 2i$$
$$-z = -(3 - 2i)$$
$$z' = (3 - 2i)' = 3 + 2i$$
$$-z' = -3 - 2i$$
$$z^{-1} = \frac{z'}{|z|^2} = \frac{-3}{13} + i\frac{2}{13}$$

Repeating the procedure for $z = 2 + 3i, z = -2i$, we have respectively:

$$-z = -2 - 3i,\ z' = 2 - 3i,\ -z' = -2 - 3i,\ z^{-1} = \frac{2}{13} - \frac{3}{13}i.$$

$$-z = 2i,\ z' = 2i,\ -z' = -2i,\ z^{-1} = \frac{1}{2}i.$$