Aerial Suppression of Airborne Platforms (ASAP): Coordinated Capture of a Threat UAS via Stochastic Reachability

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I. Introduction and Motivation

The capability, speed, size, and widespread availability of small unmanned aerial systems (UAS) makes them a serious potential security concern. Potential threat vehicles have a small cross-section, and are difficult to reliably detect using purely ground-based systems (e.g. radar or electro-optical). We propose Aerial Suppression of Airborne Platforms (ASAP), an approach that uses defensive UAS, in coordination with ground based systems, to defend fixed or mobile facilities, people, and assets. This approach requires accurate prediction of possible future locations of threat UAS, to maximize capture probability.

While many approaches have been proposed to predict future positions of a moving UAS threat [1], [2], several popular options presume a worst-case scenario, with adversarial threats, and construct controllers using a differential game framework. These methods provide conservative solutions and do not scale easily with the dimensionality of the state-space. We use a stochastic approach based on forward stochastic reachable sets, in which a suite of UAS pursue a stochastically moving UAS threat. Our approach 1) provides exact probabilistic guarantees, 2) does not create an unnecessarily large set of possible threat positions, and 3) can be calculated in near real-time.

II. Problem Description and Approach

The ASAP system includes three major elements: (i) detection and identification of a potential threat UAS, (ii) tracking and assessment, and (iii) neutralization of the threat UAS, ideally by capturing it. We focus here on the problem of capture. We seek an admissible open-loop controller for the team of controllable UAS pursuers P_i, $i \in \{1, \cdots, N\}$ which maximizes the probability of capturing the stochastic uncontrolled UAS threat G within the time horizon T.

We assume UAS dynamics that are linearized about hovering equilibrium, for both the threat and pursuer vehicles.

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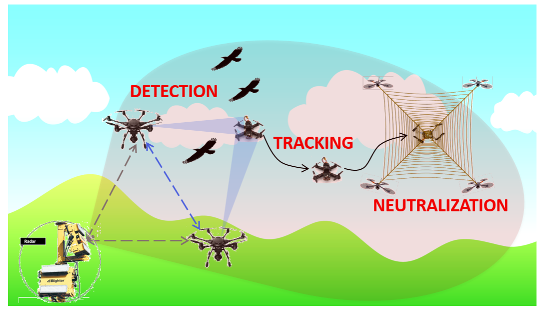

Fig. 1: ASAP approach to capture a threat UAS.

We presume that the threat UAS objective is to approach an asset at location $\bar{x}_a$, and for simplicity, presume that the threat UAS implements a LQR controller. Deviations from this controller are modeled by assuming the control input is perturbed by an IID random process with known probability density $\psi_{\boldsymbol{w}}$. The threat UAS G dynamics are thus given by

$$\begin{aligned} \boldsymbol{x}_G[t+1] &= A\boldsymbol{x}_G[t] + B(u_G + \boldsymbol{w}[t]) && (1)\\ &= A\boldsymbol{x}_G[t] + BK(\boldsymbol{x}_G[t] - \bar{x}_a) + B\boldsymbol{w}[t] && (2)\end{aligned}$$

with state $\boldsymbol{x}_G[t] \in \mathcal{X} \subseteq \mathbb{R}^{12}$, input $u_G = K(\boldsymbol{x}_G[t] - \bar{x}_a) \in \mathbb{R}^4$ for a LQR control gain K, disturbance $\boldsymbol{w}[t] \in \mathcal{W} \subseteq \mathbb{R}^4$, and matrices A, B of appropriate dimensions. We model the pursuer UAS P_i as

$$\bar{x}_{P_i}[t+1] = A\bar{x}_{P_i}[t] + B\bar{u}_{P_i}[t] \quad (3)$$

A. Forward Stochastic Reachability

The probability that the state at time τ is in $\mathcal{S} \subseteq \mathcal{X}$, given $\boldsymbol{x}_G[0] = \bar{x}_0$, is $\mathbb{P}^{\tau,\bar{x}_0}_{\boldsymbol{x}_G}\{\boldsymbol{x}_G[\tau] \in \mathcal{S}\} = \int_{\mathcal{S}} \psi_{\boldsymbol{x}_G}(\bar{y}; \tau, \bar{x}_0) d\bar{y}$. We have recently developed computationally efficient methods to exactly calculate the forward stochastic reach probability density (FSRPD) $\psi_{\boldsymbol{x}_G}(\cdot)$ via Fourier transforms for arbitrary disturbances ([3, Thm. 1]).

Assuming $\boldsymbol{w} \sim \mathcal{N}(\bar{\mu}_{\boldsymbol{w}}, \Sigma_{\boldsymbol{w}})$, we know $\boldsymbol{x}_G[\tau; \bar{x}_0] \sim \mathcal{N}(\bar{\mu}[\tau], \Sigma[\tau])$, with mean and covariance

$$\begin{aligned} \bar{\mu}[\tau] &= (A^\tau \bar{x}_0 + \mathscr{C}_\tau \bar{U}) + \mathscr{C}_\tau(\bar{1}_{\tau\times 1} \otimes \bar{\mu}_{\boldsymbol{w}}) && (4)\\ \Sigma[\tau] &= \mathscr{C}_\tau (I_\tau \otimes \Sigma_{\boldsymbol{w}}) \mathscr{C}_\tau^\top && (5)\end{aligned}$$

dependent upon the extended controllability matrix $\mathscr{C}_\tau = [B \ \ AB \ \ A^2B \ \ \ldots \ \ A^{\tau-1}B] \in \mathbb{R}^{12\times(4\tau)}$ and the input sequence $\bar{U} = [u_G^\top[\tau-1] \ u_G^\top[\tau-2] \ \ldots \ u_G^\top[0]]^\top \in \mathbb{R}^{4\tau}$, with $\otimes$ denoting the Kronecker product. Since the Gaussian distribution is log-concave and A is invertible, we can show that $\psi_{\boldsymbol{x}_G}(\bar{y}; \tau, \bar{x}_0)$ is log-concave over $\bar{y} \in \mathcal{X}$ [3].

B. Threat capture via convex optimization

We define the *team catch probability* at time τ for team state $\bar{x}_R = [\bar{x}_{P_1}, \bar{x}_{P_2}, ..., \bar{x}_{P_N}]$ as

$$\mathrm{TeamCatchPr}(\tau, \bar{x}_R) = \max_{P_i}[\mathrm{CatchPr}_{\bar{x}_{P_i}}(\tau, \bar{x}_{P_i}[\tau])] \quad (6)$$

with the *catch probability* for any UAS P_i defined as

$$\begin{aligned}\mathrm{CatchPr}_{\bar{x}_{P_i}}(\tau, \bar{x}_{P_i}[\tau]) &= \mathbb{P}^{\tau}_{\boldsymbol{x}_G}\{\boldsymbol{x}_G[\tau] \in \mathrm{Capture}(\bar{x}_{P_i}[\tau])\} \\ &= \int_{\mathrm{Capture}(\bar{x}_{P_i}[\tau])} \psi_{\boldsymbol{x}_G}(\bar{y};\tau)d\bar{y}. \quad (7)\end{aligned}$$

The set $\mathrm{Capture}(\bar{x}_{P_i}[\tau])$ is a small box, centered at $\bar{x}_{P_i}[\tau]$, from which the pursuer can initiate a successful capture. The team catch probability can be maximized by maximizing the catch probability of each pursuer UAS individually, presuming preventive collision avoidance measures are enacted as appropriate. Hence for every pursuer UAS P_i, we solve

$$\begin{array}{ll}\text{maximize} & \mathrm{CatchPr}_{\bar{x}_{P_i}}(\tau, \bar{x}_{P_i}[\tau]) \\ \text{subject to} & \begin{cases} \tau & \in [0, T] \\ \bar{x}_{P_i}[\tau] & \in \mathrm{Reach}_{P_i}(\tau; \bar{x}_{P_i}[0]) \end{cases}\end{array} \quad (8)$$

The optimal values obtained for each pursuer P_i are then used to evaluate (6). The capture time τ and the position of the UAS P_i at time τ, $\bar{x}_{P_i}[\tau]$, are the decision variables in (8). The forward reachable set for UAS P_i at time τ is $\mathrm{Reach}_{P_i}(\tau; \bar{x}_{P_i}[0]) = \{\bar{y} \in \mathcal{X} | \exists \bar{\pi}_\tau \in \overline{\mathcal{M}}_\tau \text{ s.t. } \bar{x}_{P_i}[\tau] = \bar{y}\}$, with a control policy $\bar{\pi}_\tau = [\bar{u}_{P_i}[0],\ \bar{u}_{P_i}[1],\ \ldots\ \bar{u}_{P_i}[\tau-1]]$. The policy $\bar{\pi}_\tau$ consists of open-loop control actions (dependent only upon the initial state), in the set of admissible control policies $\overline{\mathcal{M}}_\tau$. For a convex capture set and logconcave $\psi_{\boldsymbol{w}}$, we can transform (8) into a convex optimization problem [3]. When the capture set is a polytope, we can solve (8) with a near real-time computation. Algorithm 1 computes the team catch probability.

Algorithm 1 Calculation of team catch probability

Input: Initial threat UAS location $\boldsymbol{x}_G[0]$, probability density function $\psi_{\boldsymbol{w}}(\cdot)$, initial pursuer UAS locations $\bar{x}_{P_i}[0]$, admissible pursuer control set $\overline{\mathcal{M}}_\tau$, UAS dynamics A, B

Output: Team catch probability $\mathrm{TeamCatchPr}(t^*, \bar{x}^*_R)$ and optimal time of intercept t^*

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for τ = 0, 1, 2, ...., T do
    Compute ψ_xG(·; τ) from (4) and (5)
    for all P_i do
        Compute CatchPr_x̄Pi(τ, x̄*_Pi[τ]) from (8)
    end for
    Compute TeamCatchPr(τ, x̄*_R) from (6)
end for
Return t* and TeamCatchPr(t*, x̄*_R) =
max_τ TeamCatchPr(τ, x̄*_R)
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III. Simulation Results: Threat Capture

We simulate a scenario with $N = 3$ pursuers, with $\boldsymbol{w} \sim \mathcal{N}(0, \Sigma_w)$. Fig. 2 shows the forward reachable sets of each pursuer as well as the threat UAS's possible trajectories (captured visually by the mean threat position), used to compute the optimal capture locations for each pursuer.

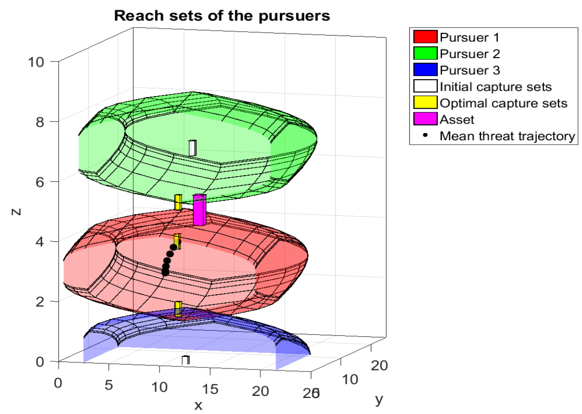

Fig. 2: Mean threat trajectory, reachable sets of the pursuer UAS P_i (green, red, and blue), and optimal capture probability locations of each pursuers (yellow).

IV. Experimental Platform

Our first experiments will be conducted in a 20'x16'x10' indoor test facility (Fig. 3) at the University of New Mexico, using small scale multirotor aircraft (Asctec Hummingbird and Lumenier QAV250 Mini frame based quadrotors). A high-fidelity, real-time, Vicon motion capture system provides vehicle positions. The onboard computers use ROS to enable for faster computation, and hence improved flight stability. A ground-based workstation calculates the threat UAS' FSRPD using MATLAB, as well as way points for the pursuer UASs, to generate maximum capture probabilities. These way points are sent to the quadrotors via rosbridge, by using MATLAB's Robotic Systems Toolbox to interact with ROS on the workstation.

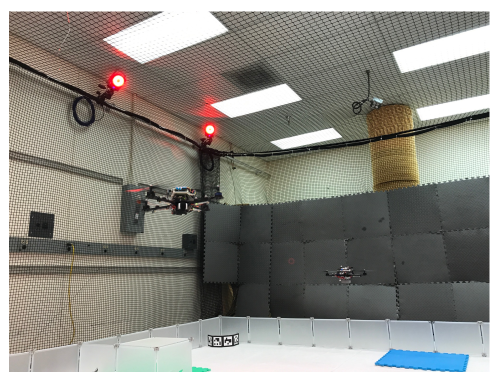

Fig. 3: UAS testbed at the University of New Mexico.

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