Multiple Pursuer-Based Intercept via Forward Stochastic Reachability

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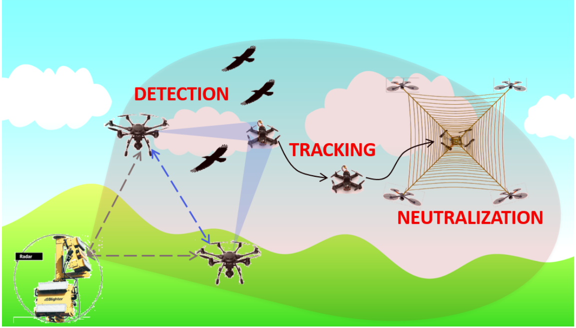
Motivation: Aerial Suppression of Airborne Platform



- Protect assets from an intruder UAV
  - Modeled as a (non-reactive) stochastically moving goal/threat
- Pursue the intruder with UAV teams
  - Stochastic reachability-based pursuit

$\boxed{\text{Maximize capture probability via a team of pursuer UAVs}}$

Related work

- **Multi-agent differential game-based pursuit**
  - Vidal, Shakernia, Kim, Shim, & Sastry (2002); Tomlin, Mitchell, Bayen, & Oishi(2003); Mitchell, Bayen, & Tomlin (2005); C. F. Chung, T. Furukawa, & A. H. Goktogan (2006); C. F. Chung, & A. H. Goktogan (2008); Huang, Zhang, Ding, Stipanovíc, & Tomlin (2011);
- **Stochastic reachability**
  - Abate, Prandini, Lygeros, & Sastry (2008); G. Hollinger, S. Singh, J. Djugash, & A. Kehagias (2009); Summers, & Lygeros (2011); Kariotoglou, Raimondo, Summers, & Lygeros (2011); Lesser, Oishi, & Erwin (2013); Vinod, HomChaudhuri, & Oishi (2017); N. Malone, K. Lesser, M. Oishi, & L. Tapia (2017);

System description

- Linearized UAV dynamics ($n = 12$)
  - Based on Asctec Hummingbird

- Goal UAV G

$$\boldsymbol{x}_G[t+1] = A\boldsymbol{x}_G[t] + BK(\boldsymbol{x}_G[t] - \overline{x}_a) + B\boldsymbol{w}[t]$$

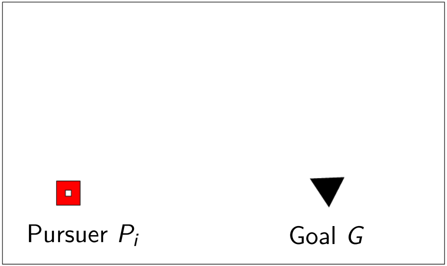
  - LQR to an asset location $\overline{x}_a$
  - Model uncertainty via $\boldsymbol{w} \sim \mathcal{N}(\overline{\mu}_{\boldsymbol{w}}, \Sigma_{\boldsymbol{w}})$
  - Uncontrolled stochastic dynamics
- N pursuers UAV P_i

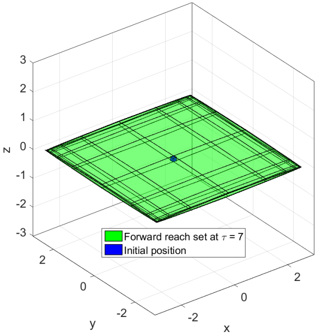
$$\overline{x}_{P_i}[t+1] = A\overline{x}_{P_i}[t] + B\overline{u}_{P_i}[t]$$

  - Bounded control authority $\overline{u}_{P_i} \in \mathcal{U}_P$
  - Controlled deterministic dynamics
  - Capture set for each P_i
- Known initial states for goal and pursuer UAVs

Mahony, Kumar, & Corke, IEEE Rob. Auto. Mag., 2012

Capture of stochastic target via convex optimization



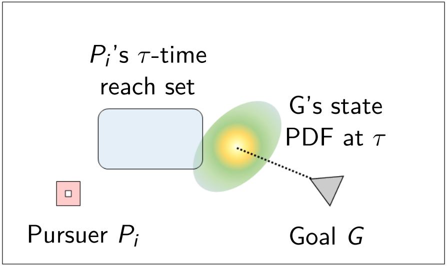


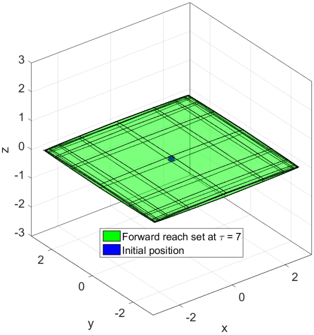
Single Pursuer Problem :

$$\begin{cases} \underset{\tau,\ \overline{x}_{P_i}[\tau]}{\text{maximize}} & \mathrm{CatchPr}_{\overline{x}_{P_i}}(\tau, \overline{x}_{P_i}[\tau]; \overline{x}_0) \\ \text{subject to} & \begin{cases} \tau \in \mathbb{Z}_{[1,N]} \\ \overline{x}_{P_i}[\tau] \in \mathrm{Reach}_{P_i}(\tau; \overline{x}_{P_i}[0]) \end{cases} \end{cases}$$

Vinod, HomChaudhuri, and Oishi, HSCC 2017

Capture of stochastic target via convex optimization





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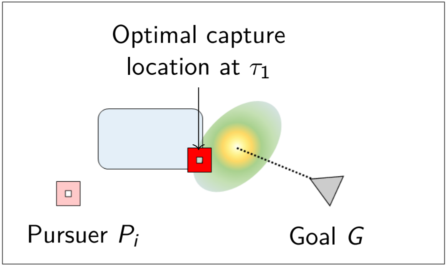

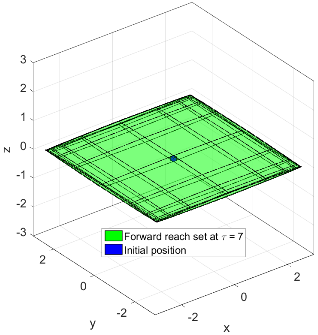

Single Pursuer Problem :

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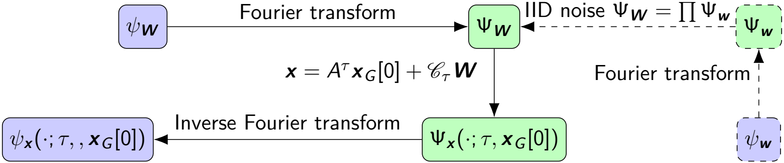
Vinod, HomChaudhuri, and Oishi, HSCC 2017

Forward reachability for goal and pursuer UAVs

- Catch probability

$$\mathrm{CatchPr}_{\overline{x}_{P_i}}(\tau, \overline{x}_{P_i}[\tau]; \boldsymbol{x}_G[0]) = \int_{\mathrm{CatchSet}(\overline{x}_{P_i}[\tau])} \psi_{\boldsymbol{x}_G}(\overline{y}; \tau, \boldsymbol{x}_G[0]) d\overline{y}$$



$$\Psi_{\boldsymbol{w}}(\overline{\alpha}) = \mathbb{E}_{\boldsymbol{w}}\left[\exp\left(j\overline{\alpha}^\top \boldsymbol{w}\right)\right] = \int_{\mathbb{R}^p} e^{j\overline{\alpha}^\top \overline{z}} \psi_{\boldsymbol{w}}(\overline{z}) d\overline{z}$$

$$\psi_{\boldsymbol{w}}(\overline{z}) = \left(\frac{1}{2\pi}\right)^p \int_{\mathbb{R}^p} e^{-j\overline{\alpha}^\top \overline{z}} \Psi_{\boldsymbol{w}}(\overline{\alpha}) d\overline{\alpha}$$

Vinod, HomChaudhuri, and Oishi, HSCC 2017

Forward reachability for goal and pursuer UAVs

- Log-concave (over $\overline{x}_{P_i}[\tau]$) τ-time catch probability

$$\mathrm{CatchPr}_{\overline{x}_{P_i}}(\tau, \overline{x}_{P_i}[\tau]; \boldsymbol{x}_G[0]) = \int_{\mathrm{CatchSet}(\overline{x}_{P_i}[\tau])} \psi_{\boldsymbol{x}_G}(\overline{y}; \tau, \boldsymbol{x}_G[0]) d\overline{y}$$

- Goal UAV G's state is Gaussian ($\overline{x}_0 = \boldsymbol{x}_G[0]$)

$$\boldsymbol{x}_G[\tau; \overline{x}_0] \sim \mathcal{N}(\overline{\mu}[\tau], \Sigma[\tau])$$

- Deterministic forward reach set for pursuer P_i

$$\begin{aligned}\mathrm{Reach}_{P_i}(\tau; \bar{x}_{P_i}[0]) &= \{\bar{y} \in \mathcal{X} | \exists \bar{\pi}_\tau \in \mathcal{U}_P^\tau \text{ s.t. } \bar{x}_{P_i}[\tau] = \bar{y}\} \\ &= \{A^\tau \bar{x}_{P_i}[0]\} \oplus \mathscr{C}_\tau \mathcal{U}_P^\tau\end{aligned}$$

Single Pursuer Problem-τ : $\begin{cases} \underset{\overline{x}_{P_i}[\tau]}{\text{minimize}} & -\log\left(\mathrm{CatchPr}_{\overline{x}_{P_i}}(\tau, \overline{x}_{P_i}[\tau]; \overline{x}_0)\right) \\ \text{subject to} & \overline{x}_{P_i}[\tau] \in \mathrm{Reach}_{P_i}(\tau; \overline{x}_{P_i}[0]) \end{cases}$

Single Pursuer Problem-τ is convex!

Vinod, HomChaudhuri, and Oishi, HSCC 2017

Problem statements

Q1 Maximize the team capture probability over a finite time horizon

Q2 Perform experimental validation of forward stochastic reachability-based interception

Extension to multiple pursuers w/ convexity preserved

- For $\overline{X}_P[\tau] = \left[(\overline{x}_{P_1}[\tau])^\top, (\overline{x}_{P_2}[\tau])^\top, \ldots, (\overline{x}_{P_N}[\tau])^\top\right]^\top$,

$$\mathrm{TeamCatchPr}(\tau, \overline{X}_P; \boldsymbol{x}_G[0]) = \max_{P_i}[\mathrm{CatchPr}_{\overline{x}_{P_i}}(\tau, \overline{x}_{P_i}[\tau]; \boldsymbol{x}_G[0])]$$

$$\text{Multiple Pursuer Problem :} \left\{ \begin{array}{ll} \underset{\tau,\ \overline{X}_P[\tau]}{\text{maximize}} & \mathrm{TeamCatchPr}(\tau, \overline{X}_P[\tau]; \overline{x}_0) \\ \text{subject to} & \left\{ \begin{array}{r} \tau \in \mathbb{Z}_{[1,N]} \\ \overline{X}_P[\tau] \text{ from dynamics} \end{array} \right. \end{array} \right.$$

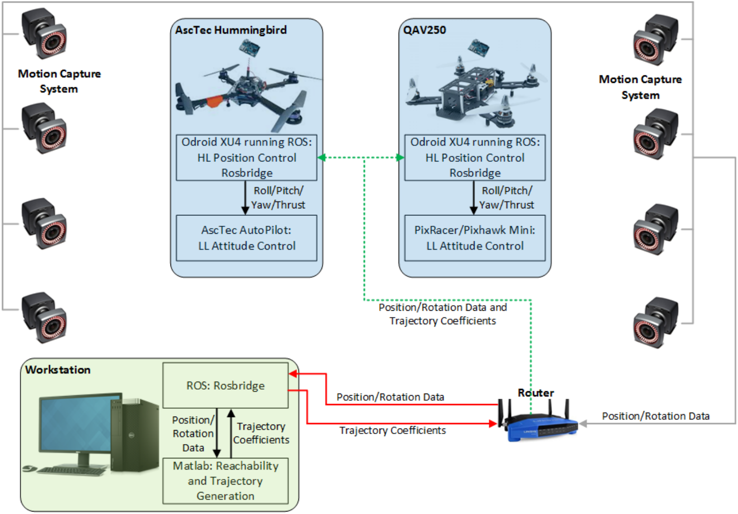
$$\text{Single Pursuer Problem-}i\text{ :} \left\{ \begin{array}{ll} \underset{\tau,\ \overline{x}_{P_i}[\tau]}{\text{maximize}} & \mathrm{CatchPr}_{\overline{x}_{P_i}}(\tau, \overline{x}_{P_i}[\tau]; \overline{x}_0) \\ \text{subject to} & \left\{ \begin{array}{r} \tau \in \mathbb{Z}_{[1,N]} \\ \overline{x}_{P_i}[\tau] \in \mathrm{Reach}_{P_i}(\tau; \overline{x}_{P_i}[0]) \end{array} \right. \end{array} \right.$$

$$\text{Single Pursuer Problem-}(\tau, i)\text{ :} \left\{ \begin{array}{ll} \underset{\overline{x}_{P_i}[\tau]}{\text{minimize}} & -\log\left(\mathrm{CatchPr}_{\overline{x}_{P_i}}(\tau, \overline{x}_{P_i}[\tau]; \overline{x}_0)\right) \\ \text{subject to} & \overline{x}_{P_i}[\tau] \in \mathrm{Reach}_{P_i}(\tau; \overline{x}_{P_i}[0]) \end{array} \right.$$

Single Pursuer Problem-(τ, i) is convex!

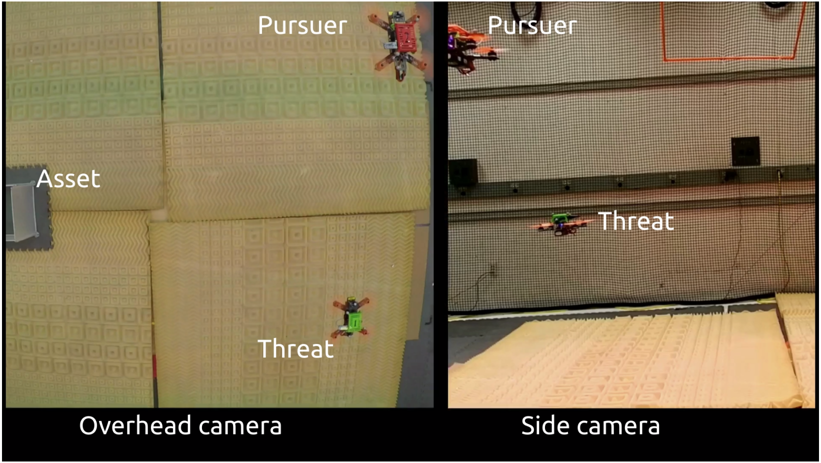
- Enforce coordination via other definitions for TeamCatchPr

Experimental Setup



Multi-Agent, Robotics, Heterogeneous Systems (MARHES) Lab (http://marhes.unm.edu/)

Robustness to model uncertainty

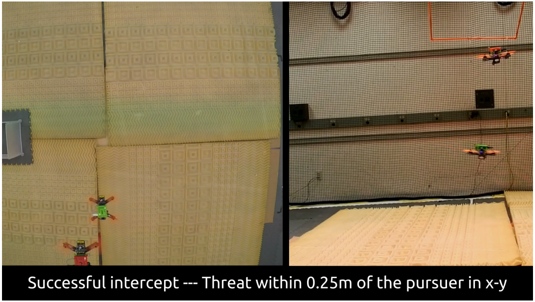


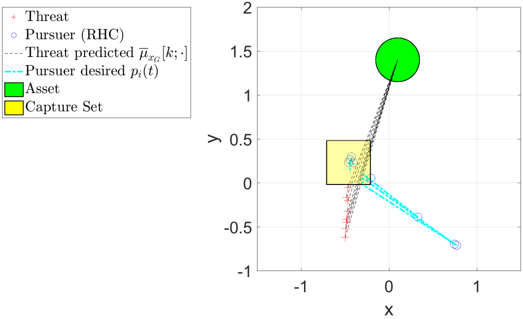
- Mismatch in dynamics (human-controlled)
- Robustness due to receding horizon control

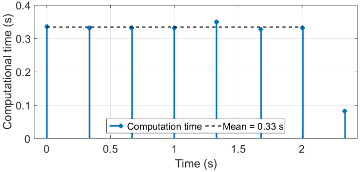
Robustness to model uncertainty

https://www.youtube.com/watch?v=eFGg7U7gEQw

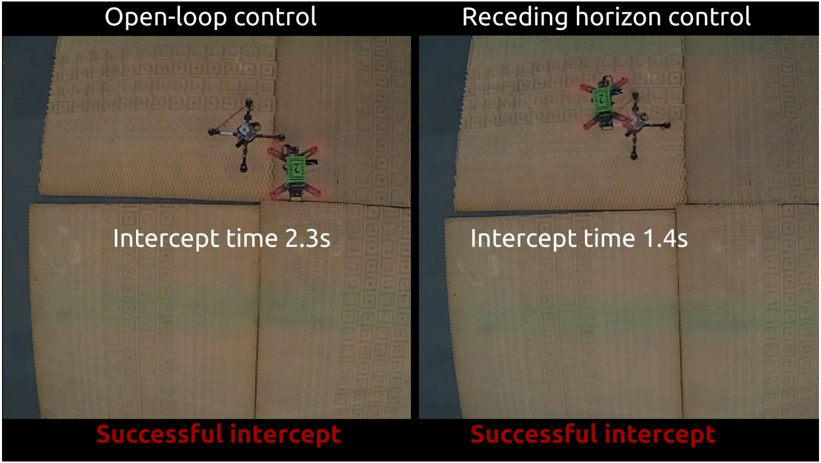
Robustness to model uncertainty







Open-loop vs receding horizon control

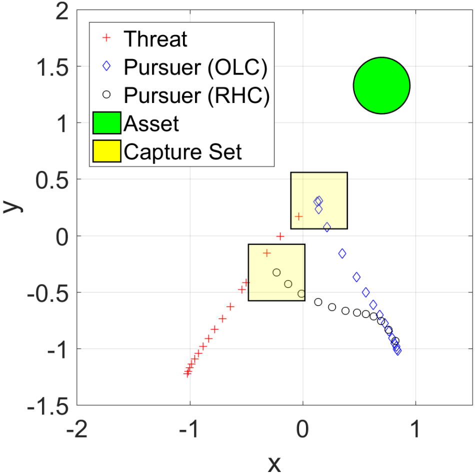


- Threat UAV follows the LQR-based stochastic dynamics
- Receding horizon control $>$ open-loop control

Open-loop vs receding horizon control

https://www.youtube.com/watch?v=H0BZrk9Goxg

Open-loop vs receding horizon control



Summary, future work, and acknowledgments

Summary

- Stochastic reachability-based pursuit w/ multiple pursuers
- Experimental validation of forward stochastic reachability

Future work

- Experiments with multiple pursuers
- Enforcing more coordination
- Enforcing keep-out regions

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- NSF CNS-1329878