User-interface design for MIMO LTI human-automation systems through sensor placement

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Motivation

- Intuition not sufficient
- Design tools lack human factors perspective
  - Too little information $\Rightarrow$ nondeterminism
  - Too much information $\Rightarrow$ overwhelming
- Dynamics-driven interface design required

Main contributions

- Synthesis of user-interfaces for MIMO LTI systems
- Translation of human factors guidelines into constraints and cost function
- Sensor placement algorithms for optimal user-interfaces
  - Operation scenario
  - Measurement noise

Related work

Formal methods in user-interface design
Dix (1991); Sarter, Woods, & Billings (1999); Pritchett & Feary (2011); Bailleiul, Leonard, & Morgansen (2012); Bolton, Bass, & Siminicean (2013); Gelman, Feigh, & Rushby (2014) ;

User-interface analysis
Suzuki, Ushio, & Adachi (2006); Hyun, Park, Wang, & Girard (2010); Eskandari & Oishi (2011); Oishi (2014); Hammond, Eskandari, & Oishi (2015)

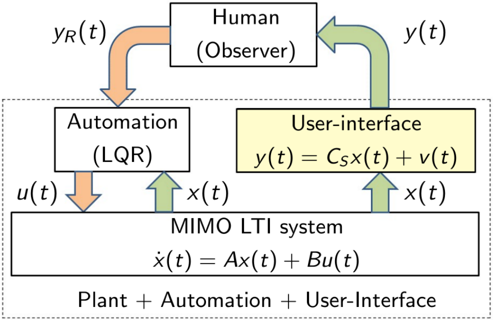
User-interface design
Degani & Heymann (2002); Oishi, Mitchell, Bayen, & Tomlin (2008)

Sensor placement
Van De Wal & De Jager (2001); Mourikis & Roumeliotis (2006); Rowaihy, Eswaran, Johnson, Verma, Bar-Noy, Brown, & La Porta (2007); Krause & Guestrin (2007); Joshi & Boyd (2009); Shamaiah, Banerjee, & Vikalo (2010); Summers, Cortesi & Lygeros (2016)

Outline

LTI MIMO system under "manual" control

- Under "manual" control:
  - Low-level control by the automation
  - High-level reference tracking by the human



$$x \in \mathbb{R}^n$$
$$u \in \mathbb{R}^m$$
$$y, y_R \in \mathbb{R}^p$$

- Measurement noise $v(t)$ with bounded energy V
- *Sensors*: $\mathscr{S} = \{s_1, \cdots, s_N\}$, with $s_i \in \mathbb{R}^n$
- *Sensor combination* $S \in 2^{\mathscr{S}}$ determine user-interface C_S

Optimization problem

Given a task, find $S^* \in 2^{\mathscr{S}}$ that:

1. Provides sufficient information to complete the specified task
2. Accounts for the trust of the user on automation
3. Minimizes an information objective

Optimization problem

$$\begin{array}{lll} \underset{S \in 2^{\mathscr{S}}}{\text{minimize}} & J(S) & \text{(information objective)} \\ \text{subject to} & \left\{ \begin{array}{l} S \in \mathscr{S}_{\mathrm{avail}} \subseteq 2^{\mathscr{S}} \\ S \in \mathscr{S}_{\mathrm{trust}} \subseteq 2^{\mathscr{S}} \end{array} \right. & \begin{array}{l} \text{(availability constraint)} \\ \text{(trust constraint)} \end{array} \end{array}$$

Outline

Modeling the task

- Tasks considered:
  - Depends on the current state
  - Uses information from $S_{\text{task}} \subseteq \mathscr{S}$
  - Safety/liveness specification
- Set of states for which task is feasible

$$\mathcal{F} = \{x \in \mathbb{R}^n : \mathcal{L}\left(C_{S_{\text{task}}} x\right) \geq 0\}$$

  - Sensors used $S_{\text{task}} \subseteq \mathscr{S}$
  - *Task matrix*: $C_{S_{\text{task}}} \in \mathbb{R}^{|S_{\text{task}}| \times n}$
  - *Task space*: $\mathbb{T} = \mathcal{R}^{\perp}(C_{S_{\text{task}}})$
  - $\mathcal{L} : \mathbb{T} \to \mathbb{R}$ (possibly nonlinear)

Example: Stop at intersection (Eventually $\mathcal{F}$ — liveness)

$$\mathcal{F} = \{(p, v, \cdots) \in \mathbb{R}^n : p = p_{\text{stop}} \wedge v = 0\}$$

- $S_{\text{task}} = \{\mathbf{e}_1, \mathbf{e}_2\} \subseteq \mathscr{S}_{\text{car}}$
- $C_{S_{\text{task}}} = [\mathcal{I}_{2\times 2}\ \mathbf{0}]$
- $\mathbb{T} = \mathbb{R}^2$
- $\mathcal{L}(p, v) = -(p - p_{\text{stop}})^2 - v^2$

Eskandari & Oishi, IEEE ICSMC, 2011; Oishi, ACC, 2014

Interaction of the user with the user-interface

- User modeled as a special observer
  - Unreliable memory
  - Comprehends displayed information (output)
  - Understand higher derivatives of output
- Task space $\mathbb{T}$ must be user-observable and user-predictable
  - Reconstruct current information relevant for task
  - Predict evolution of information relevant for task
  - *User-interface available*: Information displayed is sufficient for task completion

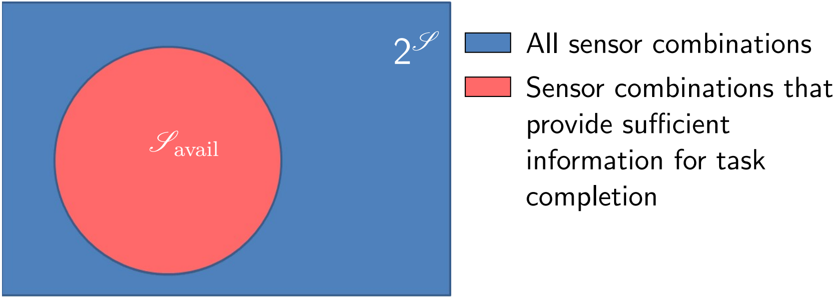
$$\mathscr{S}_{\mathrm{avail}} = \{S \in 2^{\mathscr{S}} : S_{\mathrm{task}} \in \mathcal{X}_O(S)\}$$

where $\mathcal{X}_O(S)$ is the user-observable (and user-predictable) space

Eskandari & Oishi, IEEE ICSMC, 2011; Oishi, ACC, 2014

- $\gamma(S)$ — dimension of $\mathcal{X}_O(S) = \mathcal{R}^\perp(T(S))$
  - Amount of information processed by user
  - Computed using the Markov parameters
  - For SISO systems, $\gamma(S)$ is relative degree
  - $\gamma(S)$ is monotone increasing $\Rightarrow$ Compute $\gamma_{\mathrm{min}}, \gamma_{\mathrm{max}}$ easily
- Equivalent representation of $\mathscr{S}_{\mathrm{avail}}$ using $\gamma(S)$

$$\mathscr{S}_{\mathrm{avail}} = \{S \in 2^{\mathscr{S}} : \gamma(S \cup S_{\mathrm{task}}) = \gamma(S)\}$$



Outline

Trust constraint and objective

$$\begin{array}{lll} \underset{S \in 2^{\mathscr{S}}}{\text{minimize}} & J(S) & \text{(information objective)} \\ \text{subject to} & \begin{cases} S \in \mathscr{S}_{\text{avail}} \\ S \in \mathscr{S}_{\text{trust}} \end{cases} & \begin{array}{l} \text{(availability constraint)} \\ \text{(trust constraint)} \end{array} \end{array}$$

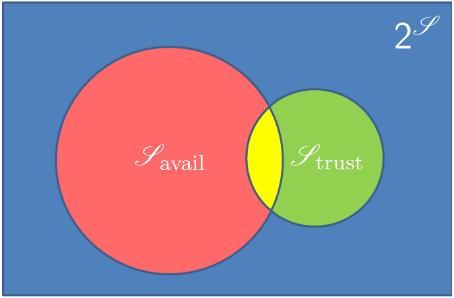
- Operation scenario $\Rightarrow$ trust constraint
- Measurement noise $\Rightarrow$ information objective

Operation Scenario	Measurements	
	Noise-free	Noisy
Nominal	Case 1	Case 3
Off-Nominal	Case 2	Case 4

Case 1: Nominal operation, noise-free measurements

Problem 1a.

$$\begin{array}{ll} \textit{minimize} & |S| \\ \textit{subject to} & \left\{ \begin{array}{rcll} S & \in & 2^{\mathscr{S}} & \\ \gamma(S) & = & \gamma(S \cup S_{\text{task}}) & (\mathscr{S}_{\text{avail}}) \\ \gamma(S) & = & \gamma_{\text{min}} & (\mathscr{S}_{\text{trust}}) \end{array} \right. \end{array}$$

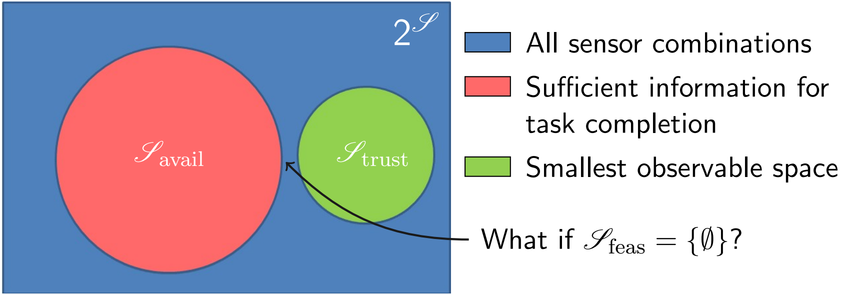


- All sensor combinations
- Sufficient information for task completion
- Smallest observable space
- Feasible solution space $\mathscr{S}_{\text{feas}}$ contains S_{task}

Case 1: Nominal operation, noise-free measurements

Problem 1a.

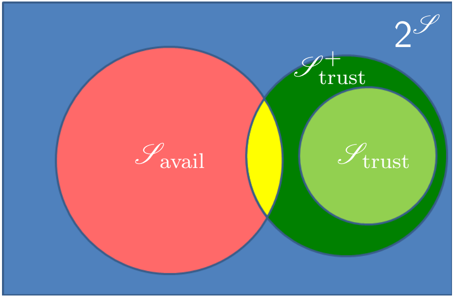
$$\begin{array}{rl} \textit{minimize} & |S| \\ \textit{subject to} & \left\{ \begin{array}{rcll} S & \in & 2^{\mathscr{S}} & \\ \gamma(S) & = & \gamma(S \cup S_{\text{task}}) & (\mathscr{S}_{\text{avail}}) \\ \gamma(S) & = & \gamma_{\min} & (\mathscr{S}_{\text{trust}}) \end{array} \right. \end{array}$$



Case 1: Nominal operation, noise-free measurements

Problem 1b (Relaxed Problem 1a when $\mathscr{S}_{\text{feas}} = \{\emptyset\}$).

$$\begin{array}{rl} \textit{minimize} & |S| \\ \textit{subject to} & \left\{ \begin{array}{rcll} S & \in & 2^{\mathscr{S}} & \\ \gamma(S) & = & \gamma(S \cup S_{\text{task}}) & (\mathscr{S}_{\text{avail}}) \\ \gamma(S) & \leq & \gamma(S_{\text{task}}) & (\mathscr{S}^{+}_{\text{trust}}) \end{array} \right. \end{array}$$

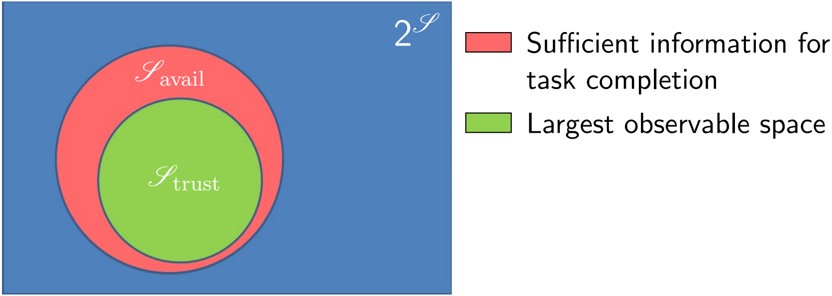


- Observable space at most as large as when using S_{task}
- Relaxed feasible solution space $\mathscr{S}^{+}_{\text{feas}}$ guaranteed to be non-empty

Case 2: Off-Nominal operation, noise-free measurements

Problem 2a.

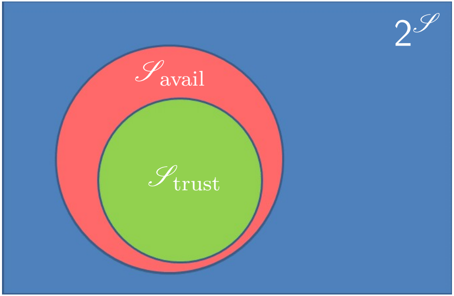
$$\begin{array}{rl} \textit{minimize} & |S| \\ \textit{subject to} & \left\{ \begin{array}{rcll} S & \in & 2^{\mathscr{S}} & \\ \gamma(S) & = & \gamma(S \cup S_{\text{task}}) & (\mathscr{S}_{\text{avail}}) \\ \gamma(S) & = & \gamma_{\max} & (\mathscr{S}_{\text{trust}}) \end{array} \right. \end{array}$$



Case 2: Off-Nominal operation, noise-free measurements

Problem 2a.

$$
\begin{aligned}
\textit{minimize} \quad & |S| \\
\textit{subject to} \quad & \begin{cases} S \in 2^{\mathscr{S}} \\ \gamma(S) = \gamma(S \cup S_{\mathrm{task}}) & (\mathscr{S}_{\mathrm{avail}}) \\ \gamma(S) = \gamma_{\mathrm{max}} & (\mathscr{S}_{\mathrm{trust}}) \end{cases}
\end{aligned}
$$



- Sufficient information for task completion
- Largest observable space

$\mathscr{S}_{\mathrm{trust}}$ is independent of S_{task}

$\Downarrow$

Task-agnostic design!

Case 2: Off-Nominal operation, noise-free measurements

Problem 2b (Equivalent set-covering problem).

From a collection of subspaces $\mathcal{R}^{\perp}(T(S)) \subseteq \mathbb{R}^{\gamma_{\mathsf{max}}}$ *where* $S \in 2^{\mathscr{S}}$*, find the minimum cardinality set* S^* *such that* $\mathcal{R}^{\perp}(T(S^*)) = \mathbb{R}^{\gamma_{\mathsf{max}}}$

- Problem 2b is NP-hard
- $\mathrm{OffGreedy}()$: Best polynomial time approximation algorithm

Vazirani, 2013

Case 2: Off-Nominal operation, noise-free measurements

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- $\mathrm{OffGreedy}()$: Best polynomial time approximation algorithm

$$\boxed{\begin{array}{c} S^{+} = \mathrm{OffGreedy}(\gamma(\cdot), \mathscr{S}) \\ \gamma(S^{+}) = \gamma(S^{*}) = \gamma_{\max} \\ |S^{*}| \leq |S^{+}| \leq |S^{*}|(\ln |\mathscr{S}| + 1) \end{array}}$$

Vazirani, 2013

Case 2: Off-Nominal operation, noise-free measurements

Problem 2b (Equivalent set-covering problem).

From a collection of subspaces $\mathcal{R}^{\perp}(T(S)) \subseteq \mathbb{R}^{\gamma_{\max}}$ where $S \in 2^{\mathscr{S}}$, find the minimum cardinality set S^* such that $\mathcal{R}^{\perp}(T(S^*)) = \mathbb{R}^{\gamma_{\max}}$

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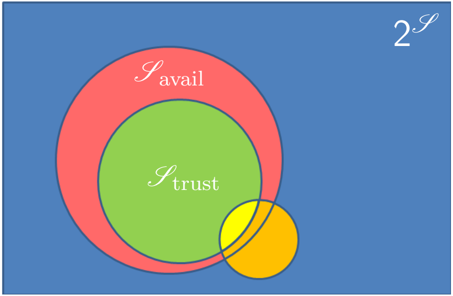


$$S^+ = \mathrm{OffGreedy}(\gamma(\cdot), \mathscr{S})$$
$$\gamma(S^+) = \gamma(S^*) = \gamma_{\max}$$
$$|S^*| \leq |S^+| \leq |S^*|(\ln|\mathscr{S}| + 1)$$

Largest observable space

Sensor combinations with cardinality $\leq |S^+|$

Restricted feasible solution space

Vazirani, 2013

User-interface design with noisy measurements

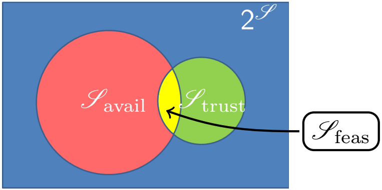
- Noisy measurements $\Rightarrow$ User has access to unreliable data
- User-interface availability and trust constraints remain same
- Uncertainty ellipse $\mathcal{E}_{\mathrm{err}}(S) = \{e \in \mathbb{R}^n : e^\top \frac{W_S}{V^2} e \leq 1\}$ where
  - W_S — observability gramian associated with C_S
  - e — error in estimation of observable initial state $\xi(0)$
- MMSE operation $\Rightarrow$ Present information with least $\mathrm{Vol}(\mathcal{E}_{\mathrm{err}})$

$$J(S) = \log(\mathrm{Vol}(\mathcal{E}_{\mathrm{err}}(S)))$$

Case 3 & 4: Operation with noisy measurements

Problem 3a (Nominal operation).

$$\begin{array}{rl} \textit{minimize} & \log(\mathrm{Vol}(\mathcal{E}_{\mathrm{err}}(S))) \\ \textit{subject to} & S \in \mathscr{S}_{\mathrm{feas}} \end{array}$$



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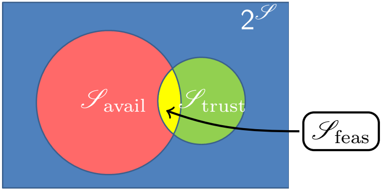
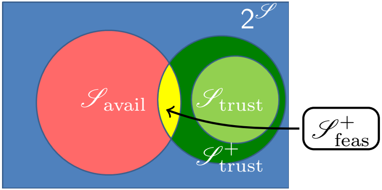


Problem 3b (Relax for $\mathscr{S}_{\mathrm{feas}} = \{\emptyset\}$).

$$\begin{array}{rl} \textit{minimize} & \log(\mathrm{Vol}(\mathcal{E}_{\mathrm{err}}(S))) \\ \textit{subject to} & S \in \mathscr{S}^{+}_{\mathrm{feas}} \end{array}$$



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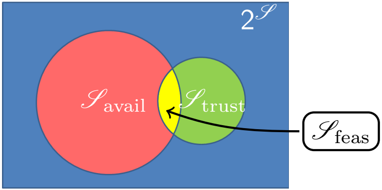
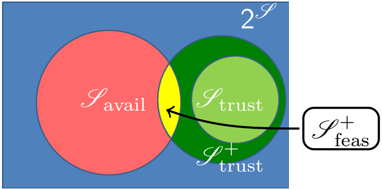


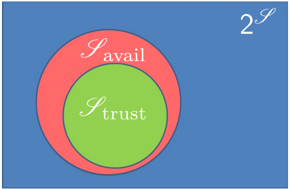
Problem 3b (Relax for $\mathscr{S}_{\mathrm{feas}} = \{\emptyset\}$).

$$\begin{array}{rl} \textit{minimize} & \log(\mathrm{Vol}(\mathcal{E}_{\mathrm{err}}(S))) \\ \textit{subject to} & S \in \mathscr{S}_{\mathrm{feas}}^{+} \end{array}$$



Problem 4 (Off-nominal operation).

$$\begin{array}{rl} \textit{minimize} & \log(\mathrm{Vol}(\mathcal{E}_{\mathrm{err}}(S))) \\ \textit{subject to} & S \in \mathscr{S}_{\mathrm{trust}} \end{array}$$



Summary of design solutions and computational complexity

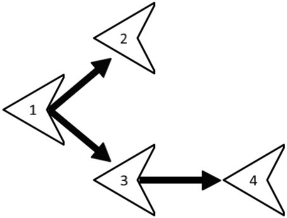
<table>
<tr><th></th><th colspan="2">Noise-free</th><th>Noisy</th></tr>
<tr><td rowspan="2">Nominal</td><td rowspan="2">$|S^\ast| \leq |S_{\mathrm{task}}|$</td><td>$S^\ast \in \mathscr{S}_{\mathrm{feas}}$ if $\mathscr{S}_{\mathrm{feas}} \neq \{\emptyset\}$</td><td>$S^\ast \in \mathscr{S}_{\mathrm{feas}}$ if $\mathscr{S}_{\mathrm{feas}} \neq \{\emptyset\}$</td></tr>
<tr><td>$S^\ast \in \mathscr{S}_{\mathrm{feas}}^{+}$ if $\mathscr{S}_{\mathrm{feas}} = \{\emptyset\}$</td><td>$S^\ast \in \mathscr{S}_{\mathrm{feas}}^{+}$ if $\mathscr{S}_{\mathrm{feas}} = \{\emptyset\}$</td></tr>
<tr><td rowspan="2">Off-Nominal</td><td rowspan="2">$|S^\ast| \leq |S^{+}|$</td><td>Sub-optimal S^{+} in $\mathcal{O}(|\mathscr{S}|^2)$</td><td rowspan="2">$S^\ast \in \mathscr{S}_{\mathrm{trust}}$</td></tr>
<tr><td>$S^\ast \in \mathscr{S}_{\mathrm{trust}}$</td></tr>
</table>

- Off-nominal noise-free suboptimal $\rightarrow$ Quadratic in $|\mathscr{S}|$.
- Noise-free cases $\rightarrow$ scalable designs (polynomial in $|\mathscr{S}|$).
- Noisy cases $\rightarrow$ need not scale well with $|\mathscr{S}|$.

Outline

Dynamics characterization

- Remotely controlled fleet:
  - Four homogeneous UAVs
  - Leader-follower formation control
- UAV model:
  - Moves only in the horizontal direction
  - Double integrator model



- Hierarchical control scheme for automation:
  - Inner control loop maintains formation h
  - Outer control loop tracks user-specified trajectory

$$\dot{x}(t) = I_4 \otimes A_v x(t) + \Gamma_L \otimes B_v F(x(t) - h) + \Gamma_F \otimes B_v z(t)$$

Lafferriere, Williams, Caughman, & Veerman, Systems & Control Letters, 2005

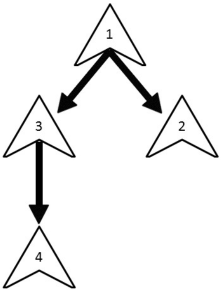
Problem formulation

- $\mathscr{S} = \{p_1, v_1, p_2, \ldots, v_4\}$ with $p_i = \mathbf{e}_{2i-1},\ v_i = \mathbf{e}_{2i},\ \mathbf{e}_j \in \mathbb{R}^8$
- *Problem*: User-interface synthesis for the following tasks:
  1. *Waypoint tracking*: Leader moves to a specific waypoint (or set of waypoints) with $S_{\mathrm{waypoint}} = \{p_1\}$, and
  2. *Trajectory tracking*: Leader tracks a time-varying profile of position and speed with $S_{\mathrm{trajectory}} = \{p_1, v_1\}$
- $\gamma_{\mathsf{max}} = \gamma(\mathscr{S}) = n = 8$

Results

- Nominal: $\mathscr{S}^{+}_{\mathrm{feas}} = \{\{p_1\}, \{p_1, v_1\}\}$ for both S_{task}

Case	S_{waypoint} $S_{\mathrm{task}} = \{p_1\}$	$S_{\mathrm{trajectory}}$ $S_{\mathrm{task}} = \{p_1, v_1\}$
Nominal No-Noise	$\{p_1\}$	$\{p_1\}$
Nominal Noise	$\{p_1, v_1\}$	$\{p_1, v_1\}$

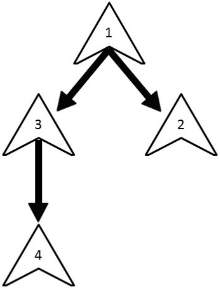


- Nominal $\leftarrow$ Matches intuition-based interface; simplified by dynamics

Results

- Nominal: $\mathscr{S}^+_{\text{feas}} = \{\{p_1\}, \{p_1, v_1\}\}$ for both S_{task}
- Off-nominal: $S^+ = \{p_1, p_2, p_4\}$.

Case	S_{waypoint} $S_{\text{task}} = \{p_1\}$	$S_{\text{trajectory}}$ $S_{\text{task}} = \{p_1, v_1\}$
Nominal No-Noise	$\{p_1\}$	$\{p_1\}$
Nominal Noise	$\{p_1, v_1\}$	$\{p_1, v_1\}$
Off-Nominal No-Noise	$\{p_1, p_2, p_4\}$	$\{p_1, p_2, p_4\}$
Off-Nominal Noise	$\{v_1, p_2, p_4\}$	$\{v_1, p_2, p_4\}$



- Nominal $\leftarrow$ Matches intuition-based interface; simplified by dynamics
- Off-nominal $\leftarrow$ Observability guaranteed interface

Outline

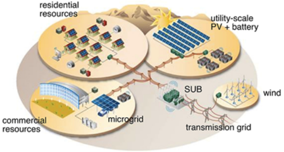
Summary & future work

Summary

- User-interface design problem: combinatorial optimization problems + human factors guidelines
- Scalable solutions for the noise-free cases and characterized the solution space for the noisy cases
- Demonstrated the user-interface synthesis on an example

Future work

- Improvement of suggested algorithms in terms of scalability
- Extensions to hybrid LTI systems, e.g, power systems



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- CMMI-1335038