Discrete Mathematics
Lecture 1- Logics

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Compound statements. *"and"* $\wedge$ and *"or"* $\vee$

Using the words *"and"* and *"or"* one can form new statements from other statements. These new statements are said to be *compound statements*.

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Question

How does the truth of "and" and "or" compound statements depend on truth of its parts?

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Compound statements with Implication $\rightarrow$

Many mathematical statements are *implications*, that is, "p implies q" or "if p then q", where p is called *the hypothesis* and q is called *conclusion*.

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- If it is sunny tomorrow, then you may go swimming

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The implication $p \rightarrow q$ is false only when the hypothesis p is true and the conclusion q is false. In all other situations, it is true.

Truth table for $p \to q$ is the following.

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F	T	
T	F	
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Another compound statement: Double Implication $\leftrightarrow$

Another compound statement that we will use is the *double implication* $p \leftrightarrow q$ read "p if and only if q".
As the notations suggests, the statement $p \leftrightarrow q$ is simply a convenient way to express $p \to q$ and $q \to p$.

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Truth table for $\bar{p}$ is the following.

p	$\bar{p}$
F	T
T	

Negation of a statement: $\bar{p}$

The *negation* of a statement p is the statement that asserts that p is not true. We denote the negation of p by $\bar{p}$ and say “not p”.

Example

The negation of “x equals 4” is the statement “x does not equal to 4” or $x \neq 4$.

Truth table for $\bar{p}$ is the following.

p	$\bar{p}$
F	T
T	F

Complicated compound statements

Truth table for more complicated compound statements can be constructed using the truth tables we have seen so far.

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Example

Suppose we want the truth table for $p \to (\overline{p \vee q})$.

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Example

Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q			
F	F			
F	T			
T	F			
T	T			

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Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$		
F	F			
F	T			
T	F			
T	T			

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Example

Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$		
F	F	F		
F	T			
T	F			
T	T			

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Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$		
F	F	F		
F	T	T		
T	F			
T	T			

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Example

Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$		
F	F	F		
F	T	T		
T	F	T		
T	T			

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Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$		
F	F	F		
F	T	T		
T	F	T		
T	T	T		

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Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$	$\overline{p \vee q}$	
F	F	F		
F	T	T		
T	F	T		
T	T	T		

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Example

Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$	$\overline{p \vee q}$	
F	F	F	T	
F	T	T		
T	F	T		
T	T	T		

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Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$	$\overline{p \vee q}$	
F	F	F	T	
F	T	T	F	
T	F	T		
T	T	T		

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Example

Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$	$\overline{p \vee q}$	
F	F	F	T	
F	T	T	F	
T	F	T	F	
T	T	T		

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Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$	$\overline{p \vee q}$	
F	F	F	T	
F	T	T	F	
T	F	T	F	
T	T	T	F	

Complicated compound statements

Truth table for more complicated compound statements can be constructed using the truth tables we have seen so far.

Example

Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$	$\overline{p \vee q}$	$p \to (\overline{p \vee q})$
F	F	F	T	
F	T	T	F	
T	F	T	F	
T	T	T	F	

Complicated compound statements

Truth table for more complicated compound statements can be constructed using the truth tables we have seen so far.

Example

Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$	$\overline{p \vee q}$	$p \to (\overline{p \vee q})$
F	F	F	T	T
F	T	T	F	
T	F	T	F	
T	T	T	F	

Complicated compound statements

Truth table for more complicated compound statements can be constructed using the truth tables we have seen so far.

Example

Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$	$\overline{p \vee q}$	$p \to (\overline{p \vee q})$
F	F	F	T	T
F	T	T	F	T
T	F	T	F	
T	T	T	F	

Complicated compound statements

Truth table for more complicated compound statements can be constructed using the truth tables we have seen so far.

Example

Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$	$\overline{p \vee q}$	$p \to (\overline{p \vee q})$
F	F	F	T	T
F	T	T	F	T
T	F	T	F	F
T	T	T	F	

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Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$	$\overline{p \vee q}$	$p \to (\overline{p \vee q})$
F	F	F	T	T
F	T	T	F	T
T	F	T	F	F
T	T	T	F	F

Complicated compound statements

Truth table for more complicated compound statements can be constructed using the truth tables we have seen so far.

Example

Suppose we want the truth table for $p \to (\overline{p \vee q})$.

p	q	$p \vee q$	$\overline{p \vee q}$	$p \to (\overline{p \vee q})$
F	F	F	T	T
F	T	T	F	T
T	F	T	F	F
T	T	T	F	F

When three statements say p, q and r are involved, 8 rows are required in a truth table since it is necessary to consider the two possible truth values for r for each of the four possible truth values of p and q.

Example

$A = (p \vee q) \leftrightarrow ((\bar{p} \wedge r) \rightarrow (q \wedge r))$

p	q	r						

Example

$A = (p \vee q) \leftrightarrow ((\bar{p} \wedge r) \rightarrow (q \wedge r))$

p	q	r	$\bar{p}$	$\bar{p} \wedge r$	$q \wedge r$	$(\bar{p} \wedge r) \rightarrow (q \wedge r)$	$p \vee q$	A
T	T	T						
T	F	T						
F	T	T						
F	F	T						
T	T	F						
T	F	F						
F	T	F						
F	F	F						

Example

$A = (p \vee q) \leftrightarrow ((\bar{p} \wedge r) \to (q \wedge r))$

p	q	r	$\bar{p}$	$\bar{p} \wedge r$	$q \wedge r$	$(\bar{p} \wedge r) \to (q \wedge r)$	$p \vee q$	A
T	T	T	F					
T	F	T	F					
F	T	T	T					
F	F	T	T					
T	T	F	F					
T	F	F	F					
F	T	F	T					
F	F	F	T					

Example

$A = (p \vee q) \leftrightarrow ((\bar{p} \wedge r) \to (q \wedge r))$

p	q	r	$\bar{p}$	$\bar{p} \wedge r$	$q \wedge r$	$(\bar{p} \wedge r) \to (q \wedge r)$	$p \vee q$	A
T	T	T	F	F				
T	F	T	F	F				
F	T	T	T	T				
F	F	T	T	T				
T	T	F	F	F				
T	F	F	F	F				
F	T	F	T	F				
F	F	F	T	F				

Example

$A = (p \vee q) \leftrightarrow ((\bar{p} \wedge r) \rightarrow (q \wedge r))$

p	q	r	$\bar{p}$	$\bar{p} \wedge r$	$q \wedge r$	$(\bar{p} \wedge r) \rightarrow (q \wedge r)$	$p \vee q$	A
T	T	T	F	F	T			
T	F	T	F	F	F			
F	T	T	T	T	T			
F	F	T	T	T	F			
T	T	F	F	F	F			
T	F	F	F	F	F			
F	T	F	T	F	F			
F	F	F	T	F	F			

Example

$A = (p \vee q) \leftrightarrow ((\bar{p} \wedge r) \to (q \wedge r))$

p	q	r	$\bar{p}$	$\bar{p} \wedge r$	$q \wedge r$	$(\bar{p} \wedge r) \to (q \wedge r)$	$p \vee q$	A
T	T	T	F	F	T	T		
T	F	T	F	F	F	T		
F	T	T	T	T	T	T		
F	F	T	T	T	F	F		
T	T	F	F	F	F	T		
T	F	F	F	F	F	T		
F	T	F	T	F	F	T		
F	F	F	T	F	F	T		

Example

$A = (p \vee q) \leftrightarrow ((\bar{p} \wedge r) \rightarrow (q \wedge r))$

p	q	r	$\bar{p}$	$\bar{p} \wedge r$	$q \wedge r$	$(\bar{p} \wedge r) \rightarrow (q \wedge r)$	$p \vee q$	A
T	T	T	F	F	T	T	T	
T	F	T	F	F	F	T	T	
F	T	T	T	T	T	T	T	
F	F	T	T	T	F	F	F	
T	T	F	F	F	F	T	T	
T	F	F	F	F	F	T	T	
F	T	F	T	F	F	T	T	
F	F	F	T	F	F	T	F	

Example

$A = (p \vee q) \leftrightarrow ((\bar{p} \wedge r) \to (q \wedge r))$

p	q	r	$\bar{p}$	$\bar{p} \wedge r$	$q \wedge r$	$(\bar{p} \wedge r) \to (q \wedge r)$	$p \vee q$	A
T	T	T	F	F	T	T	T	T
T	F	T	F	F	F	T	T	T
F	T	T	T	T	T	T	T	T
F	F	T	T	T	F	F	F	T
T	T	F	F	F	F	T	T	T
T	F	F	F	F	F	T	T	T
F	T	F	T	F	F	T	T	T
F	F	F	T	F	F	T	F	F

Logical Equivalence

Definition

Statements p and q are are said to be *logically equivalent* if they have identical truth tables.

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Example

$p \to q$ and $\bar{q} \to \bar{p}$ are logically equivalent.

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Example

$p \to q$ and $\bar{q} \to \bar{p}$ are logically equivalent.

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

p	q	$\bar{q}$	$\bar{p}$	$\bar{q} \to \bar{p}$
F	F	T	T	T
F	T	F	T	T
T	F	T	F	F
T	T	F	F	T

Tautology and Contradiction

Definition

A compound statement that is always true, regardless of the truth values assigned to its variables, is a *tautology*. A compound statement that is always false is a *contradiction*.

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- $p \vee \bar{p}$ is a tautology.
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A compound statement that is always true, regardless of the truth values assigned to its variables, is a *tautology*. A compound statement that is always false is a *contradiction*.

Examples

- $p \vee \bar{p}$ is a tautology.
- $p \wedge \bar{p}$ is a contradiction.

p	$\bar{p}$	$p \vee \bar{p}$
F	T	T
T	F	T

p	$\bar{p}$	$p \wedge \bar{p}$
F	T	F
T	F	F

Example

Show that $(p \land q) \to (p \lor q)$ is a tautology.

Example

Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Proof:

Example

Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

Proof:
Suppose that $(p \wedge q) \to (p \vee q)$ is not a tautology.

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Then for some p and q our statement has the false value.

Example

Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Proof:
Suppose that $(p \wedge q) \rightarrow (p \vee q)$ is not a tautology.

Then for some p and q our statement has the false value.

By the truth table of implication $(p \wedge q) \rightarrow (p \vee q)$ is false only when

$p \wedge q$ is true and $p \vee q$ is false.

Example

Show that $(p \land q) \to (p \lor q)$ is a tautology.

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$p \land q$ is true and $p \lor q$ is false.

$p \land q$ is true if and only if both p and q are true.

Example

Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

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Suppose that $(p \wedge q) \to (p \vee q)$ is not a tautology.

Then for some p and q our statement has the false value.

By the truth table of implication $(p \wedge q) \to (p \vee q)$ is false only when

$p \wedge q$ is true and $p \vee q$ is false.

$p \wedge q$ is true if and only if both p and q are true.

On the other hand, $p \vee q$ is false if and only if both p and q is false.

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However, it is impossible. So we can not suppose that $(p \wedge q) \rightarrow (p \vee q)$ is not a tautology.

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On the other hand, $p \vee q$ is false if and only if both p and q is false.

However, it is impossible. So we can not suppose that $(p \wedge q) \to (p \vee q)$ is not a tautology.

Therefore, $(p \wedge q) \to (p \vee q)$ is a tautology. $\square$

Example

Show that $(\bar{p} \wedge q) \wedge (p \vee \bar{q})$ is a contradiction.

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Suppose that $(\bar{p} \wedge q) \wedge (p \vee \bar{q})$ is not a contradiction.

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By by the truth table of “And” $(\bar{p} \wedge q) \wedge (p \vee \bar{q})$ is true only when

both $\bar{p} \wedge q$ and $p \vee \bar{q}$ are true.

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$\bar{p} \wedge q$ is true if and only if p is false and q is true.

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Show that $(\bar{p} \wedge q) \wedge (p \vee \bar{q})$ is a contradiction.

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Suppose that $(\bar{p} \wedge q) \wedge (p \vee \bar{q})$ is not a contradiction.

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$\bar{p} \wedge q$ is true if and only if p is false and q is true.

Then $p \vee \bar{q}$ will be false, which is impossible.

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Show that $(\bar{p} \wedge q) \wedge (p \vee \bar{q})$ is a contradiction.

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$\bar{p} \wedge q$ is true if and only if p is false and q is true.

Then $p \vee \bar{q}$ will be false, which is impossible.

So, we can not suppose that $(\bar{p} \wedge q) \wedge (p \vee \bar{q})$ is not a contradiction.

Therefore, $(\bar{p} \wedge q) \wedge (p \vee \bar{q})$ is a contradiction. $\square$

The Algebra of Propositions and $\Leftrightarrow$

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Let $\mathcal{A}$ and $\mathcal{B}$ be some statments.

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If $\mathcal{A}$ and $\mathcal{B}$ are logically equivalent, then $\mathcal{A} \leftrightarrow \mathcal{B}$ is a tautology.

The Algebra of Propositions and $\Leftrightarrow$

Let $\mathcal{A}$ and $\mathcal{B}$ be some statments.

If $\mathcal{A}$ and $\mathcal{B}$ are logically equivalent, then $\mathcal{A} \leftrightarrow \mathcal{B}$ is a tautology.

We use the notation $\mathcal{A} \Leftrightarrow \mathcal{B}$ to denote the fact that $\mathcal{A}$ and $\mathcal{B}$ are logically equivalent.

Some Basic Logical Equivalences

Some Basic Logical Equivalences

1. Idempotence:
   - $p \vee p \Leftrightarrow p$
   - $p \wedge p \Leftrightarrow p$
2. Commutativity:
   - $(p \vee q) \Leftrightarrow (q \vee p)$
   - $(p \wedge q) \Leftrightarrow (q \wedge p)$
3. Associativity:
   - $((p \vee q) \vee r) \Leftrightarrow (p \vee (q \vee r))$
   - $((p \wedge q) \wedge r) \Leftrightarrow (p \wedge (q \wedge r))$
4. Distributivity:
   - $(p \vee (q \wedge r)) \Leftrightarrow ((p \vee q) \wedge (p \vee r))$
   - $(p \wedge (q \vee r)) \Leftrightarrow ((p \wedge q) \vee (p \wedge r))$
5. Double Negation:
   - $\overline{\overline{p}} \Leftrightarrow p$
6. De Morgan's Laws:
   - $\overline{p \vee q} \Leftrightarrow \overline{p} \wedge \overline{q}$
   - $\overline{p \wedge q} \Leftrightarrow \overline{p} \vee \overline{q}$

$\mathbf{1}$ and $\mathbf{0}$

By $\mathbf{1}$ we denote a tautology and by $\mathbf{0}$ a contradiction.
Then we add the following properties to our list:

7.
- $(p \vee \mathbf{1}) \Leftrightarrow \mathbf{1}$
- $(p \wedge \mathbf{1}) \Leftrightarrow p$

8.
- $(p \vee \mathbf{0}) \Leftrightarrow p$
- $(p \wedge \mathbf{0}) \Leftrightarrow \mathbf{0}$

9.
- $(p \vee \overline{p}) \Leftrightarrow \mathbf{1}$
- $(p \wedge \overline{p}) \Leftrightarrow \mathbf{0}$

10.
- $\overline{\mathbf{1}} \Leftrightarrow \mathbf{0}$
- $\overline{\mathbf{0}} \Leftrightarrow \mathbf{1}$

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By **1** we denote a tautology and by **0** a contradiction.
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9.

- $(p \vee \overline{p}) \Leftrightarrow \mathbf{1}$
- $(p \wedge \overline{p}) \Leftrightarrow \mathbf{0}$

10.

- $\overline{\mathbf{1}} \Leftrightarrow \mathbf{0}$
- $\overline{\mathbf{0}} \Leftrightarrow \mathbf{1}$

We add three more properties

11. $(p \to q) \Leftrightarrow (\overline{q} \to \overline{p})$

12. $(p \to q) \Leftrightarrow (\overline{p} \vee q)$

13. $(p \leftrightarrow q) \Leftrightarrow ((p \to q) \wedge (q \to p))$

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- $(p \vee \mathbf{0}) \Leftrightarrow p$
- $(p \wedge \mathbf{0}) \Leftrightarrow \mathbf{0}$

9.

- $(p \vee \overline{p}) \Leftrightarrow \mathbf{1}$
- $(p \wedge \overline{p}) \Leftrightarrow \mathbf{0}$

10.

- $\overline{\mathbf{1}} \Leftrightarrow \mathbf{0}$
- $\overline{\mathbf{0}} \Leftrightarrow \mathbf{1}$

We add three more properties

11. $(p \to q) \Leftrightarrow (\overline{q} \to \overline{p})$

12. $(p \to q) \Leftrightarrow (\overline{p} \vee q)$

13. $(p \leftrightarrow q) \Leftrightarrow ((p \to q) \wedge (q \to p))$

Proof of all these properties: Just construct the truth table of each statement and then apply to them the definition of logically equivalent statements $\square$

Example

Show that $\overline{p} \to (p \to q)$ is tautology.

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Proof. By Property 12 we have

$$\overline{p} \to (p \to q) \Leftrightarrow \overline{p} \to (\overline{p} \vee q)$$

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$$\overline{p} \to (p \to q) \Leftrightarrow \overline{p} \to (\overline{p} \vee q)$$

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$$\Leftrightarrow \overline{\overline{p}} \vee (\overline{p} \vee q) \Leftrightarrow p \vee (\overline{p} \vee q)$$

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$$\Leftrightarrow \overline{\overline{p}} \vee (\overline{p} \vee q) \Leftrightarrow p \vee (\overline{p} \vee q)$$

$$\Leftrightarrow (p \vee \overline{p}) \vee q$$

Example

Show that $\overline{p} \to (p \to q)$ is tautology.

Proof. By Property 12 we have

$$\overline{p} \to (p \to q) \Leftrightarrow \overline{p} \to (\overline{p} \vee q)$$

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So, the given statement is logically equivalent simply to $\overline{p}$.

Disjunctive normal form (DNF)

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Definition (minterm)

Let $n \geq 1$ be an integer and let $x_1, x_2, \ldots, x_n$ be variables. A *minterm* based on these variables is a compound statement of the form

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Definition (Disjunctive normal form)

A compound statement in $x_1, x_2, \ldots, x_n$ is said to be in *disjunctive normal form* or just *(DNF)* if it looks like

$$y_1 \vee y_2 \vee \cdots \vee y_m$$

where the statements $y_1, y_2, \ldots, y_m$ are different minterms.

Examples

- $x_1 \wedge \overline{x_2} \wedge \overline{x_3}$ is a minterm on variables x_1, x_2, x_3.
- $(x_1 \wedge x_2 \wedge x_3) \vee (x_1 \wedge \overline{x_2} \wedge \overline{x_3})$ is a DNF.
- $(p \wedge q) \vee (\overline{p} \wedge \overline{q})$ is in disjunctive normal form on variables p, q.
- $(p \wedge q) \vee (\overline{p} \wedge \overline{q}) \vee (p \wedge q)$ is not a DNF (because two minterms are the same).
- $p \wedge (q \vee r)$ is not a minterm (because it involes the symbol $\vee$).
- $((p \wedge q) \vee r) \wedge ((p \wedge q) \vee \overline{q})$ is not a DNF, one reason being that the minterms $(p \wedge q) \vee r$ and $(p \wedge q) \vee \overline{q}$ involve the symbol $\vee$.

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In fact, we can construct DNF of any (compound) statement which is not a contradiction. There are two main methods to do that. The first method by logical equivalences and the second method through truth table. Below we will demonstrate both of them.

Method 1 (By Logical Equivalences)

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Proposition

$x \Leftrightarrow (x \wedge y) \vee (x \wedge \overline{y})$ for any statements x and y.

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Proof

$x \Leftrightarrow (x \wedge \mathbf{1})$

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This proposition is simple but important in finding DNF.

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Express $p \to (q \wedge r)$ in disjunctive normal form.

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Solution.

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Method 2 (By Truth Table)

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Example

We construct a truth table for $p \to (q \wedge r)$

p	q	r	$q \wedge r$	$p \to (q \wedge r)$
T	T	T	T	T
T	F	T	F	F
F	T	T	T	T
F	F	T	F	T
T	T	F	F	F
T	F	F	F	F
F	T	F	F	T
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Now focus on the rows for which the statement is true.

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Now focus on the rows for which the statement is true. Each of these will contribute a minterm to our answer.

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Now focus on the rows for which the statement is true. Each of these will contribute a minterm to our answer.

Row 1 gives the minterm $p \wedge q \wedge r$.
Row 3 row gives the minterm $\overline{p} \wedge q \wedge r$.
Row 4 gives the minterm $\overline{p} \wedge \overline{q} \wedge r$.
Row 7 gives the minterm $\overline{p} \wedge q \wedge \overline{r}$.
Row 8 gives the minterm $\overline{p} \wedge \overline{q} \wedge \overline{r}$.

Then we join all the minterms with disjunction $\vee$ as follows

$$(\overline{p} \wedge q \wedge r) \vee (\overline{p} \wedge q \wedge \overline{r}) \vee (\overline{p} \wedge \overline{q} \wedge r) \vee (\overline{p} \wedge \overline{q} \wedge \overline{r}) \vee (p \wedge q \wedge r).$$

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Observations:

- We see that the obtained expression is a DNF on variables p, q and r.
- We note that it is logically equivalent to $p \to (q \wedge r)$. (Why?)

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Hence,

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Hence,

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is a DNF of $p \to (q \wedge r)$ $\square$

We leave it to you to decide for yourself which method you prefer.

The End of Lecture 1