Practice on Discrete Math.

Classroom Assignments of Lecture 1.

Section 1 : 1 (a, c), 2 (a, b), 7 (a, b)
8, 10.

Section 2 : 5 (a, b, c, d), 9 (a, b, c)
10 (a, b, c, e).

Section 1.

1. Construct a truth table.

(a) $p \wedge ((\neg q) \vee p)$

Solution.:

p	q	$\neg q$	$(\neg q) \vee p$	$p \wedge ((\neg q) \vee p)$
F	F	T	T	F
F	T	F	F	F
T	F	T	T	T
T	T	F	T	T

//

(c) $\neg(p \wedge (q \vee p)) \leftrightarrow p$

p	q	$q \vee p$	$p \wedge (q \vee p)$	$\neg(p \wedge (q \vee p))$	$\neg(p \wedge (q \vee p)) \leftrightarrow p$
F	F	F	F	T	F
F	T	T	F	T	F
T	F	T	T	F	F
T	T	T	T	F	F

We observe that $\neg(p \wedge (q \vee p)) \leftrightarrow p$ is a contradiction. //

2 (a) If $p \to q$ is false, determine the truth value of
$(p \wedge (\neg q)) \vee ((\neg p) \to q)$.

Solution.

If $p \to q$ is false then from the truth table of $p \to q$ we have that p is true and q is false. Now we substitute these values into our the compound statement:

$$(T \wedge (\neg F)) \vee ((\neg T) \to F) = T$$

Since $T \wedge (\neg F) = T \wedge T = T$.

Then "or" statement is true.

The answer is "True" //

2(b) Is it possible to answer 2(a) if $p \to q$ is true instead of false? Why or Why not?

Answer : No.

Solution.

There are 3 possibilities when $p \to q$ is true.

p	q	$p \to q$
F	F	T
F	T	T
T	T	T

① Suppose p and q are false.

Then

$$(F \wedge (\neg F)) \vee ((\neg F) \to F)$$
$$= (F \wedge T) \vee (T \to F)$$
$$= F \vee F = F$$

② Suppose p is False, q is True.

$$(F \wedge (\neg T)) \vee ((\neg F) \to T) = (F \wedge F) \vee (T \to T)$$
$$= F \vee T = T.$$

As we see, there is no unique answer.

7(a) Show that $[(p \to q) \wedge (q \to r)] \to (p \to r)$.

is a tautology.

<u>Proof</u> : There are 2 ways of showing that It is a tautology.

① By constructing truth table.

② By properties of logical operations

<u>1st method</u> :

p	q	r	$p \to q$	$q \to r$	$p \to r$	A^{*}	B^{**}
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
T	F	T	F	T	T	F	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

$A = (p \to q) \wedge (q \to r)$

$B = [(p \to q) \wedge (q \to r)] \to (p \to r)$

tautology.

//

2nd method:

Suppose that $[(p\to q)\wedge(q\to r)]\to(p\to r)$ is not a tautology.
Then for some p, q, r, our statement is false. It is false only if

$$\begin{cases} (p\to q)\wedge(q\to r) & \text{is True} \\ p\to r & \text{is False} \end{cases} \iff$$

$$\begin{cases} p\to q & \text{is True} \\ q\to r & \text{is True} \\ p & \text{is True} \\ r & \text{is False} \end{cases} \iff \begin{cases} T\to q & \text{is True} \\ q\to F & \text{is True} \end{cases}$$

$$\iff \begin{cases} p & \text{is true} \\ r & \text{is true} \end{cases}$$

$$\iff \begin{cases} q & \text{is True} \\ q & \text{is False} \end{cases}$$

It is impossible

So we cannot suppose that our statement is not a tautology.
Hence It is a tautology. //

8. Show that the statement

$$[p \vee ((\neg r) \Rightarrow (\neg s))] \vee [(s \Rightarrow ((\neg t) \vee p)) \vee ((\neg p) \Rightarrow r)]$$

is neither a tautology nor a contradiction.

<u>Proof</u> :

It is not a contradiction, because when p is true, we have a true statement.

It is not a tautology, because when

p, r, q – False

s, t – True

we have a false statement //.

10. (a)

Show that the statement $p \to (q \to r)$ is not logically equivalent to the statement $(p \to q) \to r$.

Solution.

To be logically equivalent, the statements ~~must~~ must have the same truth tables.

However,

if p, r - False and q - True

$p \to (q \to r)$ is true

$(p \to q) \to r$ is false.

So, they are not logically equivalent //

What can you conclude from 10(a) about compound statement

$$[p \to (q \to r)] \leftrightarrow [(p \to q) \to r]?$$

Answer: It is not a tautology

Moreover, It is not a contradiction

since p - True, q - True, r - True

the statement is true. //

(!)

Section 2

5 (a).

$$[(p \vee q) \wedge (\neg p)] \Longleftrightarrow [(\neg p) \wedge q]$$

Solution.

distributivity

$$[(p \vee q) \wedge (\neg p)] \Longleftrightarrow \underline{(p \wedge (\neg p)) \vee (q \wedge (\neg p))}$$

$$\Longleftrightarrow 0 \vee (q \wedge (\neg p)) \Longleftrightarrow q \wedge (\neg p) \Longleftrightarrow$$

$$\Longleftrightarrow (\neg p) \wedge q. //$$

5(b)

$$[p \rightarrow (q \rightarrow r)] \Longleftrightarrow [(p \wedge (\neg r)) \rightarrow (\neg q)]$$

LHS: $p \rightarrow (q \rightarrow r) \Longleftrightarrow p \rightarrow ((\neg q) \vee r) \Longleftrightarrow$

$\Longleftrightarrow (\neg p) \vee ((\neg q) \vee r) \Longleftrightarrow \underline{(\neg p) \vee (\neg q) \vee r}.$

<u>RHS:</u> $(p \wedge (\neg r)) \rightarrow (\neg q) \Longleftrightarrow$

$\Longleftrightarrow \neg(p \wedge (\neg r)) \vee (\neg q) \Longleftrightarrow ((\neg p) \vee r) \vee (\neg q)$

$\Longleftrightarrow (\neg p) \vee r \vee (\neg q) \Longleftrightarrow \underline{(\neg p) \vee (\neg q) \vee r}.$

//

5(c).

$$[\neg(p \leftrightarrow q)] \Longleftrightarrow [p \leftrightarrow (\neg q)]$$

LHS: $\neg(p \leftrightarrow q) \Longleftrightarrow \neg((p \to q) \wedge (q \to p))$

$\Longleftrightarrow \neg[((\neg p) \vee q) \wedge ((\neg q) \vee p)] \Longleftrightarrow$

$\Longleftrightarrow (\neg((\neg p) \vee q)) \vee (\neg((\neg q) \vee p)) \Longleftrightarrow$

$\Longleftrightarrow (p \wedge (\neg q)) \vee (q \wedge (\neg p)) \Longleftrightarrow$

$\Longleftrightarrow \underline{(p \wedge (\neg q)) \vee ((\neg p) \wedge q)}$;

RHS: $[p \leftrightarrow (\neg q)] \Longleftrightarrow$

$\Longleftrightarrow (p \to (\neg q)) \wedge ((\neg q) \to p) \Longleftrightarrow$

$\Longleftrightarrow ((\neg p) \vee (\neg q)) \wedge (q \vee p) \Longleftrightarrow$

$\Longleftrightarrow ((\neg p) \wedge q)) \vee ((\neg p) \wedge p) \vee ((\neg q) \wedge q) \vee$

$\vee ((\neg q) \wedge p) \Longleftrightarrow$

$\Longleftrightarrow ((\neg p) \wedge q) \vee 0 \vee 0 \vee ((\neg q) \wedge p)$

$\Longleftrightarrow \underline{((\neg p) \wedge q) \vee (p \wedge (\neg q))}$;

5(d)

$$\neg[(p \Leftrightarrow q) \vee (p \wedge (\neg q))] \Longleftrightarrow$$

$$(p \Leftrightarrow (\neg q)) \wedge ((\neg p) \vee q)$$

Solution.

$$\neg[(p \Leftrightarrow q) \vee (p \wedge (\neg q))] \Longleftrightarrow$$

$$\Leftrightarrow (\neg(p \Leftrightarrow q)) \wedge (\neg(p \wedge (\neg q))) \Longleftrightarrow \quad \text{by } 5(c)$$

$$\Leftrightarrow (p \Leftrightarrow (\neg q)) \wedge ((\neg p) \vee q) \;//$$

9(a).

$(p \vee q) \wedge ((\neg p) \vee (\neg q))$

<u>Answer</u> not DNF. //

9(b) $(p \wedge q) \vee ((\neg p) \wedge (\neg q))$

<u>Answer</u>: It is a DNF. //

9(c) $p \vee ((\neg p) \wedge q)$

<u>Answer</u>: It is not a DNF //

10.(a)

$p \wedge q$ DNF.//

10 (b) $(p \wedge q) \vee (\neg((\neg p) \vee q))$

Solution

$(p \wedge q) \vee (\neg((\neg p) \vee q)) \Longleftrightarrow$

$\Longleftrightarrow (p \wedge q) \vee (p \wedge (\neg q))$ DNF//

10.(c) $p \rightarrow q$.

Solution

p	q	$p \rightarrow q$	
F	F	T	$\leftarrow$
F	T	T	$\leftarrow$
T	F	F	
T	T	T	$\leftarrow$

DNF:

$((\neg p) \wedge (\neg q)) \vee$
$((\neg p) \wedge q) \vee$
$(p \wedge q)$;

//

10(d)

$$(p \vee q) \wedge ((\neg p) \vee (\neg q))$$

$\iff$

$\iff (p \wedge (\neg p)) \vee (p \wedge (\neg q)) \vee ((\neg p) \wedge q) \vee (q \wedge (\neg q)) \iff$

$\iff 0 \vee (p \wedge (\neg q)) \vee ((\neg p) \wedge q) \vee 0 \iff$

$\iff (p \wedge (\neg q)) \vee ((\neg p) \wedge q)$, DNF //