Ch2 Review Exercises Solutions

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a. $f(-1) = 1$

b. $\lim_{x \to -1^-} f(x) = 3.$

c. $\lim_{x \to -1^+} f(x) = 1.$

d. $\lim_{x \to -1} f(x)$ does not exist.

e. $f(1) = 5.$

f. $\lim_{x \to 1} f(x) = 5.$

g. $\lim_{x \to 2} f(x) = 4.$

h. $\lim_{x \to 3^-} f(x) = 3.$

i. $\lim_{x \to 3^+} f(x) = 5.$

j. $\lim_{x \to 3} f(x)$ does not exist.

10 $\lim_{x \to 1000} 18\pi^2 = 18\pi^2.$

11 $\lim_{x \to 1} \sqrt{5x+6} = \sqrt{11}.$

15 $$\lim_{x \to 1} \frac{x^3 - 7x^2 + 12x}{4 - x} = \frac{1 - 7 + 12}{4 - 1} = \frac{6}{3} = 2.$$

16 $$\lim_{x \to 4} \frac{x^3 - 7x^2 + 12x}{4 - x} = \lim_{x \to 4} \frac{x(x-3)(x-4)}{4 - x} = \lim_{x \to 4} x(3 - x) = -4.$$

17 $$\lim_{x \to 1} \frac{1 - x^2}{x^2 - 8x + 7} = \lim_{x \to 1} \frac{(1-x)(1+x)}{(x-7)(x-1)} = \lim_{x \to 1} \frac{-(x+1)}{x - 7} = \frac{1}{3}.$$

28 $$\lim_{x \to -5^+} \frac{x - 5}{x + 5} = -\infty.$$

29 $$\lim_{x \to 3^-} \frac{x - 4}{x^2 - 3x} = \lim_{x \to 3^-} \frac{x - 4}{x(x-3)} = \infty.$$

40 $\lim_{z\to\infty}\left(e^{-2z}+\frac{2}{z}\right)=0+0=0.$

41 $\lim_{x\to\infty}(3\tan^{-1}x+2)=\frac{3\pi}{2}+2.$

42 $\lim_{x\to-\infty}(-3x^3+5)=\infty.$

43 For $x<1$, $|x-1|+x=-(x-1)+x=1$. Therefore $\lim_{x\to-\infty}(|x-1|+x)=\lim_{x\to-\infty}1=1$.
For $x>1$, $|x-1|+x=x-1+x=2x-1$. Therefore $\lim_{x\to\infty}(|x-1|+x)=\lim_{x\to\infty}2x-1=\infty$.

44 For $x<2$, $|x-2|+x=-(x-2)+x=2$. Therefore $\lim_{x\to-\infty}\frac{|x-2|+x}{x}=\lim_{x\to-\infty}\frac{2}{x}=0$.
For $x>2$, $|x-2|+x=x-2+x=2x-2$. Therefore $\lim_{x\to\infty}\frac{|x-2|+x}{x}=\lim_{x\to\infty}\frac{2x-2}{x}=2$.

45 $\lim_{w\to\infty}\frac{\ln w^2}{\ln w^3+1}=\lim_{w\to\infty}\frac{2\ln w}{(3\ln w+1)}\cdot\frac{1/\ln w}{1/\ln w}=\lim_{w\to\infty}\frac{2}{3+(1/\ln w)}=\frac{2}{3}.$

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50 Recall that $-1\le\cos t\le 1$, and that $e^{3t}>0$ for all t. Thus $-\frac{1}{e^{3t}}\le\frac{\cos t}{e^{3t}}\le\frac{1}{e^{3t}}$. Because $\lim_{t\to\infty}\frac{1}{e^{3t}}=\lim_{t\to\infty}-\frac{1}{e^{3t}}=0$, we can conclude $\lim_{t\to\infty}\frac{\cos t}{e^{3t}}=0$ by the Squeeze Theorem.

71 The function f is not continuous at 5 because $f(5)$ is not defined.

72 g is discontinuous at 4 because $\lim_{x\to 4}g(x)=\lim_{x\to 4}\frac{(x+4)(x-4)}{x-4}=8\ne g(4)$.

73 Observe that $h(5)=-2(5)+14=4$. Because $\lim_{x\to 5^-}h(x)=\lim_{x\to 5^-}(-2x+14)=4$ and $\lim_{x\to 5^+}h(x)=\lim_{x\to 5^+}\sqrt{x^2-9}=\sqrt{25-9}=4$, we have $\lim_{x\to 5}h(x)=4$. Thus f is continuous at $x=5$.

74 Observe that $g(2)=-2$ and $\lim_{x\to 2}g(x)=\lim_{x\to 2}\frac{x^3-5x^2+6x}{x-2}=\lim_{x\to 2}\frac{x(x-2)(x-3)}{x-2}=\lim_{x\to 2}x(x-3)=-2$. Therefore g is continuous at $x=2$.

79 In order for g to be left continuous at 1, it is necessary that $\lim_{x\to 1^-}g(x)=g(1)$, which means that $a=3$. In order for g to be right continuous at 1, it is necessary that $\lim_{x\to 1^+}g(x)=g(1)$, which means that $a+b=3+b=3$, so $b=0$.

82

a. Consider the function $f(x) = x^5 + 7x + 5$. f is continuous everywhere, and $f(-1) = -3 < 0$ while $f(0) = 5 > 0$. Therefore, 0 is an intermediate value between $f(-1)$ and $f(0)$. By the Intermediate Value Theorem, there must a number c between 0 and 1 so that $f(c) = 0$.

b. Using a computer algebra system, one can find that $c \approx -0.691671$ is a root.

83

a. Rewrite The equation as $x - \cos x = 0$ and let $f(x) = x - \cos x$. Because x and $\cos x$ are continuous on the given interval, so is f. Because $f(0) = -1 < 0$ and $f(\pi/2) = \pi/2 > 0$, it follows from the Intermediate Value Theorem that the equation has a solution on $(0, \pi/2)$.

b. Using a computer algebra system, one can find that $c \approx 0.739085$ is a root.