Calculus Ch2.1: 5, 6, 10, 11, 23

(5) a) $\frac{s(2)-s(0)}{2-0} = \frac{72-0}{2} = 36$

b) $\frac{s(1.5)-s(0)}{1.5-0} = \frac{66}{1.5} = 44$

c) $\frac{s(1)-s(0)}{1-0} = \frac{52-0}{1} = 52$

d) $\frac{s(.5)-s(0)}{.5-0} = \frac{30}{.5} = 60$

(6) a) $\frac{s(2.5)-s(.5)}{2.5-.5} = \frac{150-46}{2} = 52$

b) $\frac{s(2)-s(.5)}{2-.5} = \frac{136-46}{1.5} = 60$

c) $\frac{s(1.5)-s(.5)}{1.5-.5} = \frac{114-46}{1} = 68$

d) $\frac{s(1)-s(.5)}{1-.5} = \frac{84-46}{.5} = 76$

(10) find the slope of several secant lines getting closer and closer to $x=a$ for x_2. The limit of the slopes of the secant lines is the slope of the tangent line.

(11) instantaneous velocity is the slope of tan line

(23)

t	
[0,1]	
[0,.5]	
[0,.1]	
[0,.01]	
[0,.001]	

avg. vel.
should approach 80

Ch2.2: 10, 19, 28, 33, 40, 47

(10a) use graphing calculator for table

(b) $\lim_{x\to 0}(1+x)^{1/x} \approx 1$

I think e-book has error, should be

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{1/x} \approx e$$

(19) $\lim_{x\to -1^-}(x^2+1) = (-1)^2+1 = 2$

$\lim_{x\to -1^+} 3 = 3$

$\lim_{x\to -1} f(x)$: DNE

(28) $\lim_{x\to 0}\frac{e^{2x}-2x-1}{x^2} = +\infty$

x=0 is vert. asym

$\frac{e^{2(-.1)}-2(-.1)-1}{(-.1)^2}$ $\quad (e^{2(.1)}-2(.1)-1)/(.1)^2$

+ 0 +

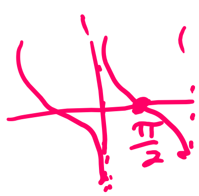
(33) $\lim_{x\to 3}\frac{x^2-9}{x-3} = \lim_{x\to 3} x+3 = 6$ (hole @ $x=3$)

(b) false b/c f(a) could make f(x) und but be a hole

(c) false. f(a) could be und which might be the limit (see "a")

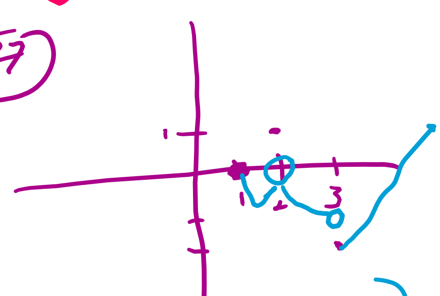
(d) $\lim_{x\to 0^+}\sqrt{x} = 0$ but left of 0, there is no graph

(e) true



(40) $\lim_{x\to 0}\frac{6^x-3^x}{x\ln 6} = .25$ (use calculator)

(47)



(answers vary)

Ch2.3: 2, 35, 43, 59, 61, 66, 67, 81, 87, 95

(26) $\lim_{t\to 3} \sqrt[3]{t^2-10} = \sqrt[3]{9-10} = -1$

(35) $\lim_{x\to 4} \frac{x^2-16}{4-x}$

$= \lim_{x\to 4} \frac{(x-4)(x+4)}{-1(x-4)}$

$= \lim_{x\to 4} \frac{x+4}{-1} = \frac{4+4}{-1} = -8$

(43) $\lim_{t\to 5} \frac{1}{t^2-4t-5} - \frac{1}{6(t-5)}$

$= \lim_{t\to 5} \frac{1}{(t-5)(t+1)} - \frac{1}{6(t-5)}$

$= \lim_{t\to 5} \frac{6(1) - 1(t+1)}{6(t-5)(t+1)}$ $\quad -t+5 = -1(t-5)$

$= \lim_{t\to 5} \frac{-1(t-5)}{6(t-5)(t+1)} = \frac{-1}{6(t+1)} = \frac{-1}{6(5+1)} = \frac{-1}{36}$ $\quad$ lim $t\to5$

(59) $\lim_{x\to 0} x\cos x = 0\cos 0 = 0$

(61) $\lim_{x\to 0} \frac{1-\cos x}{\cos^2 x - 3\cos x + 2} = \lim_{x\to 0} \frac{-1(\cos x - 1)}{(\cos x - 1)(\cos x - 2)}$

$= \lim_{x\to 0} \frac{-1}{\cos x - 2} = \frac{-1}{\cos 0 - 2} = \frac{-1}{1-2} = 1$

(66) $\lim_{x\to -1^-} \frac{x^2-1}{x+1} = \lim_{x\to -1^-} \frac{(x-1)(x+1)}{x+1} = -2$

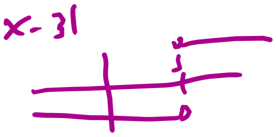
$\lim_{x\to -1^+} -2 = -2$ $\quad = \checkmark$

$\therefore \lim_{x\to -1} g(x) = -2$

(67) $\lim_{x\to 3} \frac{x-3}{x-3} = 1$

$\lim_{x\to 3} \frac{x-3}{-(x-3)} = -1$

$\lim_{x\to 3} \frac{x-3}{|x-3|} = \text{DNE}$



(81) See Sqz thm ex in class (similar pros)

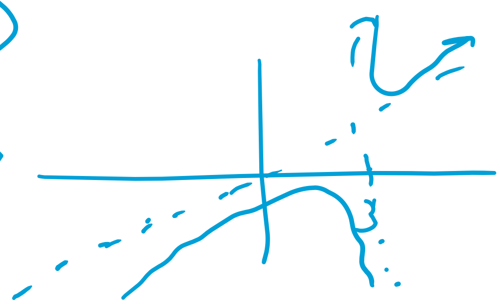
(87) $\frac{x^2-5x+6}{x-3} = \frac{(x-2)(x-3)}{x-3}$ so need $x-2 = a$ @ $x=3$

$3-2 = 1 = a$

(95) use calculator

Ch2.4: 16, 25, 29, 32, 39, 43, 49, 53

(16) ∞ (see graph)



(25) $\lim_{z \to 3^+} \frac{(z-1)(z-2)}{(z-3)} = \infty$

$\lim_{z \to 3^-} f(x) = -\infty$

$\lim_{z \to 3} f(x) = DNE$

(29) $\underbrace{\lim_{x \to 2^+} \frac{1}{\sqrt{x(x-2)}}}_{\text{reciprocal}} = \infty$ $\lim_{x \to 2^+} \sqrt{x(x-2)} = 0$

so it's like $\frac{1}{\text{limit of zero}}$ which means

$1 \div \text{tiny \#s} = \infty$

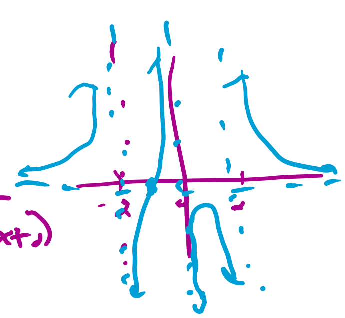
(32) $\frac{x-2}{x^5-4x^3} = \frac{x-2}{x^3(x-2)(x+2)} = \frac{1}{x^3(x+2)}$

$\lim_{x \to 0} \frac{1}{x^3(x+2)} = DNE$ $x=0$ vert. asym ($\pm\infty$ varying sides)

$\lim_{x \to 2} \frac{1}{x^3(x+2)} = \frac{1}{8(4)} = \frac{1}{32}$

$\lim_{x \to -2} \frac{1}{x^3(x+2)} = DNE$ $x = -2$ vert asym.

(49) $\frac{x+1}{x(x-2)(x+2)}$



(39) $\lim_{\theta \to 0^+} \csc\theta = \lim_{x \to 0^+} \frac{1}{\sin\theta} = +\infty$

Ch2.5: 2, 16, 25, 55, 64, 65

(2) $\lim_{x\to\infty} f(x) = -3$

$\lim_{x\to-\infty} f(x) = 4$

(16) $\lim_{x\to-\infty} g(x) = 3$ $\lim_{x\to 2^-} g(x) = +\infty$

$\lim_{x\to\infty} g(x) = -1$ $\lim_{x\to 2^+} g(x) = -\infty$

(25) $\lim_{x\to\infty} \frac{14x^3+3x^2-2x}{21x^3+x^2+2x+1}$

$= \lim_{x\to\infty} \frac{\frac{14x^3}{x^3}+\frac{3x^2}{x^3}-\frac{2x}{x^3}}{\frac{21x^3}{x^3}+\frac{x^2}{x^3}+\frac{2x}{x^3}+\frac{1}{x^3}}$ ÷ by degree of denominator

$= \lim_{x\to\infty} \frac{14+\frac{3}{x}-\frac{2}{x^2}}{21+\frac{1}{x}+\frac{2}{x^2}+\frac{1}{x^3}}$ as $x\to\infty$, the fracs $\to 0$

$= \frac{14}{21} = \frac{2}{3}$

(55) $f(x) = \frac{4x^3+4x^2+7x+4}{x^2+1}$ ← won't $=0$, ∴ no vert. asym. (x^2+1 is always positive)

$4x+4 = y$ slant asym.

$$\begin{array}{r|l} & 4x+4 \\ \hline x^2+1 & 4x^3+4x^2+7x+4 \\ & -(4x^3+\quad\quad 4x) \\ \hline & 4x^2-3x+4 \\ & -(4x^2+\quad\quad 4) \\ \hline & -3x \end{array}$$

(64) $\lim_{t\to\infty} \frac{2500}{\underbrace{t+1}_{\to\infty}} = 0$

(65) $\lim_{t\to\infty} \frac{3500t}{t+1} = 3500$

$\frac{3500t/t}{t/t+1/t} = \frac{3500}{1+1/t}$

Ch2.6: 6, 16, 18, 24, 64, 102

(6) $x = 1$ $\quad \lim_{x\to 1} f(x) = 3$ but $f(1) = 4$ $\quad \lim_{x\to 1} f(x) \neq f(1)$

$x = 2$ $\quad \lim_{x\to 2} f(x) =$ DNE b/c $\lim_{x\to 2^-} f \neq \lim_{x\to 2^+} f$

$x = 3$ b/c $\lim_{x\to 3} f(x) = 4$ but $f(3)$ is und.

(16) $(0,15)$ $(15,30)$ $(30,45)$ $(45,60)$ (separate intervals)

(18) $\lim_{x\to 5} \frac{2x^2+3x+1}{x^2+5x} = \frac{2(25)+3(5)+1}{25+25} = \frac{66}{50}$

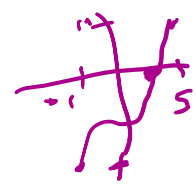
$f(5)$ is defined & $\lim_{x\to 5} f(x) = f(5)$

(24) $\lim_{x\to -1} \frac{x^2+x}{x+1} = \lim_{x\to -1} \frac{x(x+1)}{x+1} = \lim_{x\to -1} x = -1$

$f(-1) = 2$

$\lim_{x\to -1} f(x) \neq f(-1)$ $\quad \therefore$ not continuous

(64) $f(x) = x^3 - 5x^2 + 2x + 1 = 0$



$f(-1) = (-1)^3 - 5(-1)^2 + 2(-1) + 1 = -1 - 5 - 2 + 1 = -7$

$f(5) = 5^3 - 5(5)^2 + 2(5) + 1 = 125 - 125 + 10 + 1 = 11$

$f(-1) < 0$, $f(5) > 0$, $\therefore \exists$ a value in $(-1,5)$

$\ni$ $f(x) = 0$ since $f(x)$ is continuous