

# IBM Quantum Challenge 2020

## Hint for Learning Exercise I-B

### Practical method

You can easily experiment by rewriting the code used in cell 22. Let's extend the size of the database register, the oracle (use any number up to  $2^7 - 1$ ), and the diffusion. If you want to search for numbers that include 0, such as 0101111, use the X gate on the bits with 0s (in the example, 0th and 2nd) before and after the mct gate (extended CX gate) to flip and return them.

### Theoretical aspects

For those who want to think a little more theoretically, here are some hint formulas. When the iteration is repeated  $t$  times, the probability of getting answer is as follows.

$$|\langle w | (U_s U_w)^t | \Psi_0 \rangle|^2 = \sin^2((2t + 1)\theta)$$

Therefore, the probability of finding a solution is high when  $(2t + 1)\theta$  is close to a right angle ( $\frac{\pi}{2}$ ). The following approximation can be made for a sufficiently large  $N$ .

$$\theta \sim \tan \theta$$

Then we got,

$$\tan \theta = \frac{1}{\sqrt{N-1}}$$

Also, we can approximate as shown for large  $N$ .

$$\frac{1}{\sqrt{N-1}} \sim \frac{1}{\sqrt{N}}$$

Therefore, we obtain the following constraints of  $t$  for a sufficiently large  $N$ .

$$(2t + 1) \frac{1}{\sqrt{N}} = \frac{\pi}{2}$$

Transforming the formula gives:

$$t = \frac{1}{2} \left( \frac{\pi}{2} \sqrt{N} - 1 \right) = \frac{\pi}{4} \sqrt{N} - \frac{1}{2}$$

By implementing the algorithm paying attention to the number of iterations in this way, a clearer solution can be obtained.