<u>**Assignment - 2 (CS F316)**</u> Name: **Abhay Kamble(2019A7PS0128G)**

1. The examples for a balanced and a constant function for size two are -
   a. For two-bit length functions, it is as follows –
      The two-bit balanced function is as follows: $f(x_0,x_1) = x_0 \oplus x_1$
      Since, $f(0,0)=0$; $f(0,1) = 1$; $f(1,0)=1$; $f(1,1)=0$
      Thus the corresponding phase oracle of this two-bit oracle can be given as
      $$U_f|x_1,x_0\rangle = (-1)^{f(x1,x0)}|x\rangle$$
      By taking the example state we can verify this, like - $|\psi_0\rangle = |00\rangle_{01} \otimes |1\rangle_2$
      Then after applying the Hadamard gate we get -
      $$|\psi_1\rangle = ½(|00\rangle+|01\rangle+|10\rangle+|11\rangle)_{01} \otimes 1/\sqrt{2}(|0\rangle - |1\rangle)_2$$
      Then the oracle function can be implemented as $Q_f = CX_{02}CX_{12}$ and then simplifying and applying hadamard on the first register we get -
      $$|\psi_f\rangle = |1\rangle_0 \otimes |1\rangle_1 \otimes (|0\rangle - |1\rangle)_2$$
      Then the measurement of the first two qubits will give the value 11, indicating a balanced function.

      The two-bit constant function is as follows: $f(x_0,x_1) = 00$
      Since $f(0,0) = f(0,1) = f(1,0) = f(1,1) = 00$
      Thus the corresponding can be verified as follows:
      Taking the example state we can verify this, like - $|\psi_0\rangle = |00\rangle_{01} \otimes |1\rangle_2$
      Then after applying the Hadamard gate we get –
      $$|\psi_1\rangle = ½(|00\rangle+|01\rangle+|10\rangle+|11\rangle)_{01} \otimes 1/\sqrt{2}(|0\rangle - |1\rangle)_2$$
      Then we apply the oracle as $Q_f = I_{0,1,2}$ and then after simplifying and applying the hadamard gate on the first register we get - $|\psi_f\rangle = |0\rangle_0 \otimes |0\rangle_1 \otimes (|0\rangle - |1\rangle)_2$
      Then the measurement of the two qubits gives the value 00, indicating a constant function.

   b. For four-bit length functions, it is as follows: $f(x_{0,1,2,3}) = 0$ or 1
      Then the corresponding can be verified as – initial would be $|\psi_0\rangle = |0000\rangle \otimes |1\rangle$
      Then after applying the Hadamard gate we get –
      $$|\psi_1\rangle = 1/\sqrt{2^5}\Sigma(x = 2^4\text{-}1)(-1)^{f(x)}|x\rangle(|0\rangle - |1\rangle)$$
      Then we apply the oracle as $Q_f = I_{0,1,2,3,4}$ and then after simplifying and applying the hadamard gate on the first register we get -
      $$|\psi_f\rangle = |0\rangle_0 \otimes |0\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_3(|0\rangle - |1\rangle)_4$$
      Then the measurement of the two qubits gives the value 0000, indicating a constant function.

      For balanced function, the function that can be defined is as follows where –
      $f(0,0,0,0) = 0$; $f(0,0,1,0) = 0$; $f(0,0,0,1) = 0$; $f(0,0,1,1) = 0$
      $f(1,0,0,0) = 1$; $f(1,0,1,0) = 1$; $f(1,0,0,1) = 1$; $f(1,0,1,1) = 1$
      $f(0,1,0,0) = 1$; $f(0,1,1,0) =1$; $f(0,1,0,1) = 1$; $f(0,1,1,1) = 1$
      $f(1,1,0,0) = 0$; $f(1,1,1,0) = 0$; $f(1,1,0,1) = 0$; $f(1,1,1,1) = 0$
      So, analyzing and solving the function we can solve it as -
      Let the initial state be $|\psi_0\rangle = |0000\rangle \otimes |1\rangle$

Then after applying the Hadamard gate we get –

$$|\psi_1\rangle = 1/\sqrt{2^5}\Sigma(x = 2^4\text{-}1)(-1)^{f(x)}|x\rangle(|0\rangle - |1\rangle)$$

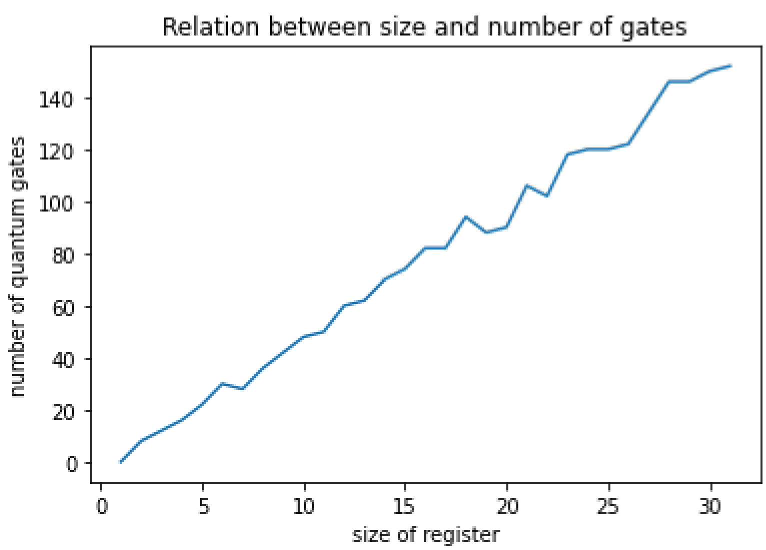
Then we apply the oracle as $Q_f = Q_f = CX_{04}CX_{14}CX_{24}CX_{34}$ and then after simplifying and applying the hadamard gate on the first register we get -

$$|\psi_f\rangle = |1\rangle_0 \otimes |1\rangle_1 \otimes |1\rangle_2 \otimes |1\rangle_3(|0\rangle - |1\rangle)_4$$

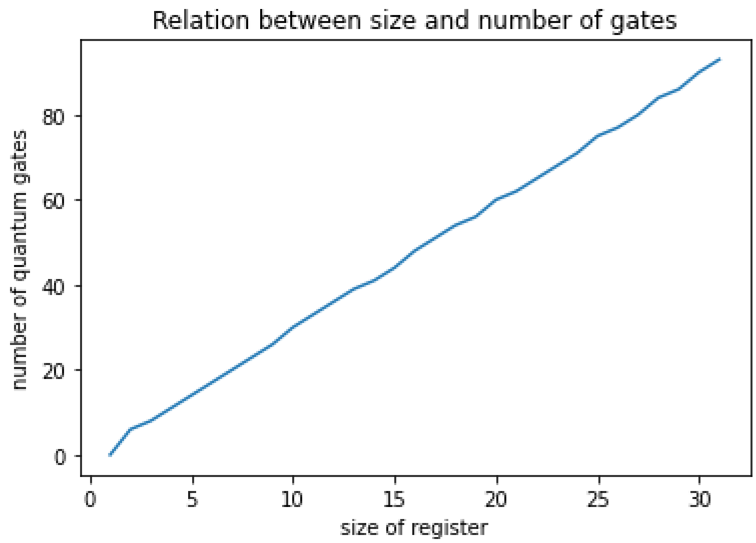
Then the measurement of the first two qubits will give the value 1111, indicating a balanced function.

2. Code of balanced and constant function of size two and four input register, and the code for generalized ‘n’ size input is provided in the Jupyter notebook attached in .zip file.

3. For the increase in ‘n’ i.e. the size of the input register, the amount of resources used increases almost linearly. This may be mainly due to the fact that more elements are used in the quantum circuit as the size of the circuit/input increases. The number of hadamard and the cx (CNOT) gates being used increases linearly with every new circuit and hence the overall sum of the number of gates increases almost linearly. But, it also depends on the type of function being assessed, as the constant functions rarely need more quantum gates, as there are no cx gates being used. The linear dependence of number of quantum gates on the size of input can be seen more clearly in the case of the constant function.

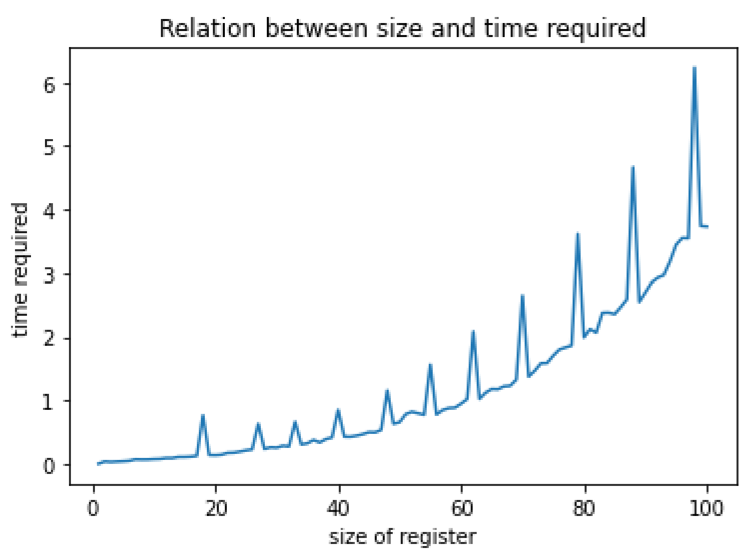


Balanced Function



Constant Function

4. The amount of time required to do the computation increases because the number of elements being used increases. This results in more depth of the circuit and also as the size increases the amount of time required increases. This means that for smaller sizes the time required is low, cuz the amount of information is less, while when the information is more/large the computer requires more time to handle the data.



(time required in seconds)

5. The power of the quantum approach to solve the problem versus the classical approach is that the quantum approach is exponentially faster than any possible deterministic classical algorithm for the same. For a conventional classical deterministic algorithm where n is the number of bits, $2^{n-1} + 1$ evaluations are needed in the worst case.But, the Deutsch-Jozsa algorithm finds an answer that is always correct with a single evaluation of the function.