

ALL PROBLEMS ARE EQUALLY WEIGHTED.

- (1) The response of a LTI causal system to the input $u_1(t) = u_s(t) u_s(2-t)$ is $y_1(t) = (1 - e^{-t}) u_s(t) u_s(2-t) + e^{-(t-2)} u_s(t-2)$. Find the response of the system to a new input $u_2(t) = (t-2) u_s(t-2) u_s(4-t)$ for $t \leq 4$. Note that $u_s(t)$ designates a unit step function.

- (2) Consider a LTI system $\dot{x} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} u$
 $y = \begin{bmatrix} 1 & -1 \end{bmatrix} x$

Let $u(t) = \begin{bmatrix} -3e^{-t} + 5e^{-2t} \\ 3e^{-2t} \end{bmatrix}$ for $t \geq 0$, and suppose that

$y(t) = -2e^{-t} + 6e^{-2t}$. If $\lim_{t \rightarrow \infty} x(t) = 0$, find $x(0)$.

- (3) Given $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_1(t-1) + |u(t)| \\ x_1(t) \end{bmatrix}$; $y(t) = \begin{bmatrix} x_1(t+1) + x_2(t) - u(t) \end{bmatrix}$

Find the appropriate description of the system among:

Linear	Time-Invariant	Causal	Lumped	Continuous-time
Nonlinear	Time-Variant	Noncausal	Distributed	Discrete-time

- (4) Find a state space representation (A, B, C, D) for the system described by

$$\begin{cases} \ddot{y}_1 + 2\dot{y}_1 + 3(y_1 - y_2) = 2u_1 + \dot{u}_2 \\ \ddot{y}_2 - 4(y_1 - y_2) = 2u_2 + 4\dot{u}_1 \end{cases} \quad \left| \begin{array}{l} \text{Hint: Use the} \\ \text{state diagram} \\ \text{approach.} \end{array} \right.$$