

1. The decomposition is NOT lossless. From the decomposed tables, we see that the shared attributes are only A. Therefore, the decomposition is lossless if A is a key of one of the decomposed tables. However, after checking the FDs, we see that A is not a key of either of the tables, so it is not lossless.
2.  $A \rightarrow B, C \rightarrow B, AC \rightarrow B, BC \rightarrow A$
3.
  - a. If there is a FD  $\text{dept} \rightarrow \text{sid}$ , then there is a one-to-one relationship between Student and Class because this FD means that in each class, there is only one student
  - b. If there is a FD  $\text{sid} \rightarrow \text{dept}$ , cnum, then there is a many-to-one relationship between Student and Class because this FD means that each student can only take one class, but does not limit how many students can take a particular class.
4.
  - a. Yes. The closure of  $\{E\}$  is R, so E is a key. Specifically, the FDs used in order are  $E \rightarrow A, A \rightarrow BC, B \rightarrow D$ .
  - b. No. The closure of  $\{BC\}$  is only  $\{BCD\}$ , so BC is not a key.
5. No, because there exist non-trivial FDs  $X \rightarrow Y$ , where X does not contain a key.  
After normalizing to BCNF, we get:  $R_1(A, B, C), R_2(B, D), R_3(C, E)$ , and  $R_4(A, F)$ .
6.
  - (a, b1, c1, d2)
  - (a, b2, c2, d1)
  - (a, b1, c1, d3)
  - (a, b3, c3, d1)
  - (a, b2, c2, d3)
  - (a, b3, c3, d2)
7. No, because it is not even in BCNF, so it cannot be in 4NF.  
After normalizing to 4NF, we get  $R_1(A, B), R_2(A, E), R_3(A, C)$ , and  $R_4(A, D, F)$ .