Lab 3 - Exercise 4

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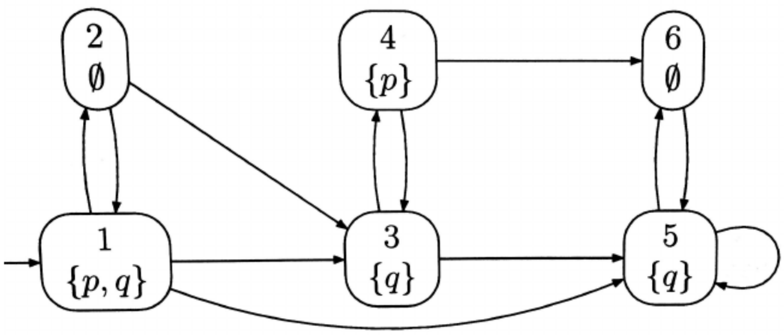
1 Exercise 6

For the Labelled Transition System below, compute the following sets of states:



1. $[\![EFp]\!]$
2. $[\![AFq]\!]$
3. $[\![\varphi]\!]$, where $\varphi = E(qU(p \wedge \neg q)$
4. $[\![\varphi]\!]$, where $\varphi = EGq \vee (EGp \wedge EFq)$

2 Solving of exercise 6

Elementary temporal modalities that are present in the most temporal logics inlude the operators:
$F(\Diamond)$ **"eventually"** (eventually in the future)
$G(\Box)$ **"always"** (now and forever in the future)

Definition(Semantics of CTL and CTL*)

Let $\pi = s_0, s_1, ...$ be a path and φ a $CTL*$ formula. If π_i is the suffix of π starting from position i,

- $\pi, i \models \top$
- $\pi, i \models p$ iff $p \in L(s_i)$
- $\pi, i \models \neg\varphi$ iff $\pi, i \not\models \varphi$

- $\pi, i \models \varphi_1 \vee \varphi_2$ iff $\pi, i \models \varphi_1$ or $\pi, i \models \varphi_2$
- $\pi, i \models X\varphi$ iff $\pi, i+1 \models \varphi$
- $\pi, i \models \varphi_1 \mathcal{U} \varphi_2$ iff $\exists j \geq 0$ such that $\pi, j \models \varphi_2$ and $\pi, k \models \varphi_1$ for all $0 \leq k < j$
- $\pi, i \models E\varphi$ iff there is an infinite path $\pi' = s'_0, s'_1, s'_2, ...$ s.t. $s'_0 = s_i$ and $\pi', 0 \models \varphi$
- $\pi, i \models A\varphi$ iff for every infinite path $\pi' = s'_0, s'_1, s'_2, ...$ s.t. $s'_0 = s_i$, we have $\pi', 0 \models \varphi$

Solving Model Checking Problem

Let define:

$$pre_{\exists}(Y) = \{s \in S \mid \exists s' \in Y \ s.t. \ (s, s') \in \mathcal{R}\}$$

$$pre_{\forall}(Y) = \{s \in S \mid \mathcal{R}(s) \subseteq Y\}$$

Compute $[\![\varphi]\!] = \{s \in S \mid s \models \varphi\}$

$$[\![p]\!] = \{s \in S \mid p \in L(s)\}$$

$$[\![\neg\varphi]\!] = S \setminus [\![\varphi]\!]$$

$$[\![\varphi_1 \vee \varphi_2]\!] = [\![\varphi_1]\!] \cup [\![\varphi_2]\!]$$

$$[\![EX\varphi]\!] = pre_{\exists}([\![\varphi]\!])$$

$$[\![AF\varphi]\!] = MC^{AF}_{CTL}(\varphi)$$

$$[\![E(\varphi_1 \mathcal{U} \varphi_2]\!] = MC^{EU}_{CTL}(\varphi_1, \varphi 2)$$

Test if the input state $s \in [\![\varphi]\!]$.

$MC^{AF}_{CTL}(\varphi)$ is computed as:

- $Y := S;\ Z := [\![\varphi]\!]$;
- **while** $Y \neq Z$ do:
  - $Y = Z$;
  - $Z = Z \cup pre_{\forall}(Z)$;
- return Y;

$MC^{EU}_{CTL}(\varphi_1, \varphi_2)$ is computed as:

- $Y := \emptyset;\ Z := [\![\varphi_2]\!]$;
- **while** $Z \not\subseteq Y$ **do**:
  - $Y = Y \cup Z$;
  - $Z = pre_{\exists}(Y) \cap [\![\varphi_1]\!]$
- return Y;

Having these definitions at hand will help us solve the exercise:
Let S be the set of states of the transition system: $S = \{1, 2, 3, 4, 5, 6\}$, where $L(1) = \{p, q\}$; $L(2) = \emptyset$; $L(3) = \{q\}$; $L(4) = \{p\}$; $L(5) = \{q\}$; $L(6) = \emptyset$.

1. $[\![EFp]\!] \equiv E(\top \mathcal{U} p) = MC^{EU}_{CTL}(\top, p)$

   $[\![\varphi_1]\!] = [\![\top]\!] = S$

   $$\begin{array}{ccl} Y = \emptyset & Z = [\![\varphi_2]\!] = [\![p]\!] = \{1,4\} & \\ Y = \{4\} & Z = pre_{\exists}(Y) \cap [\![\varphi_1]\!] = \{3\} & \\ Y = \{4,3\} & Z = pre_{\exists}(\{4,3\}) \cap S = \{4,3,2,1\} & \\ Y = \{4,3,2,1\} & Z = pre_{\exists}(\{4,3,2,1\} \cap S = \{4,3,2,1\} & Z \subseteq Y \Rightarrow \text{stop here.} \end{array}$$

   $[\![EFp]\!] = \{4,3,2,1\}$.

2. $[\![AFq]\!] = MC^{AF}_{CTL}(q)$

   $$\begin{array}{ccl} Y = S = \{1,2,3,4,5,6\} & Z = [\![\varphi]\!] = [\![q]\!] = \{1,3,5\} & \\ Y = \{1,3,5\} & Z = \{1,3,5\} \cup pre_{\forall}(\{1,3,5\}) = \{1,3,5\} & Y = Z \Rightarrow \text{stop here.} \end{array}$$

   $[\![AFq]\!] = \{1,3,5\}$