# Secular perturbations in the four-body system with anisotropically varying masses

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We consider the classical problem of four bodies attracting each other according to Newton's law of universal gravitation. The masses of the bodies are assumed to vary anisotropically with different rates, which leads to the appearance of reactive forces. Since the differential equations of motion of the system are not integrable, the problem is studied in the framework of the perturbation theory methods, where quite cumbersome symbolic calculations are involved (see [1], [2] and [3]). An exact solution to the two-body problem with variable masses describing the aperiodic motion of the bodies along quasi-conical section is used as the first approximation. The equation of the perturbed motion are obtained in terms of the osculating orbital elements. Averaging these equations over the mean longitudes of the bodies in the absence of mean motion resonances, we derive the differential equations describing the long-term evolution of the orbital parameters. All relevant symbolic calculations are performed with the aid of the computer algebra system Wolfram Mathematica.

**Keywords**: Four-body problem, Variable mass, Evolutionary equations, Secular perturbations, Wolfram Mathematica

## References

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