Geometric Foundations for Transformer in Gröbner Basis Computation

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We provide a theoretical foundation for Transformer-based computation of Gröbner bases by proving the geometric generality of existing dataset generation algorithms and introducing an extended sampling method. Under a mild heuristic and assuming a Hilbertian base field, we show that the training examples constructed via random elementary matrix transformations are Zariski dense in the space of generating sets of a fixed ideal. This guarantees that Transformers trained on such data can, in principle, learn any generic Gröbner basis. Our extended algorithm controls sparsity, interaction, and coefficient distributions, further improving dataset richness and empirical performance.