Iterating Generalized Cyclotomic Mappings of Finite Fields

Daniel Panario
Carleton University, Canada

30th Applications of Computer Algebra - ACA 2025

When we iterate functions over finite structures, there is an underlying natural functional graph. For a function f over a finite field $\mathbb{F}_q$, this graph has q nodes and a directed edge from vertex a to vertex b if and only if $f(a) = b$. It is well known, combinatorially, that functional graphs are sets of connected components, components are directed cycles of nodes, and each of these nodes is the root of a directed tree.

Some functions over finite fields when iterated present strong symmetry properties. These symmetries allow mathematical proofs of some dynamical properties such as the period and preperiod of a generic element, (average) "rho length" (number of iterations until a cycle is formed), number of connected components, cycle lengths, and permutational properties (including the cycle decomposition).

We briefly survey the main problems and results in this area. Then, we concentrate on the functional graph of generalized cyclotomic mappings of finite fields. These are a natural and manageable generalization of monomial functions. We study periodic points, cycle structure, and rooted trees attached to periodic points. We provide both theoretical results on the structure of their functional graphs as well as algorithms for solving basic problems, such as parametrizing the connected components of the graph, or describing the structure of a connected component given by a representative vertex.

Based on the following papers:

1. "A survey on iterations of mappings over finite fields", R. Martins, D. Panario and C. Qureshi; Radon Series on Computational and Applied Mathematics, de Gruyter, 23, 135–172, 2019.

2. "Functional graphs of generalized cyclotomic mappings of finite fields", A. Bors, D Panario and Q. Wang; `https://arxiv.org/abs/2304.00181`, 219 pages.