Scattered trinomials of $\mathbb{F}_{q^6}[X]$ in even characteristic

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30th Applications of Computer Algebra - ACA 2025

(joint work with D. Bartoli, G. Marino and M. Timpanella)

In the last decades, Algebraic Geometry over finite fields has emerged as a powerful tool for investigating various objects closely associated with Galois Geometry, Coding Theory and Cryptography. In this talk, we will show an example of this approach through the study of a family of scattered polynomials defined over a finite field of even characteristic. Although several families of scattered polynomials have been investigated in recent years, most of them only exist in odd characteristics. In particular, in [1] and [2] the authors proved that the trinomial

$$f_c(X) = X^q + X^{q^3} + cX^{q^5}$$

of $\mathbb{F}_{q^6}[X]$ is scattered under the assumptions that q is odd and $c^2 + c = 1$. However, they explicitly noted that this is not the case when q is even.

Using tools of Algebraic Geometry in positive characteristic, we explore a different set of conditions on c under which this trinomial is scattered in the case of even q and we show that when q is sufficiently large, there are roughly q^3 elements $c \in \mathbb{F}_{q^6}$ such that $f_c(X)$ is scattered.

References

[1] B. Csajbók, G. Marino, F. Zullo. New maximum scattered linear sets of the projective line, *Finite Fields Appl.*, **54**:133-150, 2018.

[2] G. Marino, M. Montanucci, F. Zullo. MRD-codes arising from the trinomial $x^q + x^{q^3} + cx^{q^5} \in \mathbb{F}_{q^6}[x]$, *Linear Algebra Appl.* **591**:99-114, 2020.