On some properties of the Gray map

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We discuss the properties of the Gray map and its generalizations. Within our new framework, one can recover many well-known properties of the Gray map, but also identify some original behaviors. First, we show that the Gray map can be factored as a mapping to a multivariate polynomial ring followed by an evaluation over a projective point set. This provides an interpretation of the Gray map in terms of evaluation of functions. Under this association, it follows that the linearity defect of the image of the code, e.g., its rank and its kernel (in the nonlinear sense), can be characterized by the structure of this set of functions. In particular, we derive some local principles that allow to reduce the study of the image code to a shortened code over a logarithmically smaller support. In parallel, we pay a specific attention to the practical costs for manipulating these invariants: for example, we provide a very efficient algorithm to invert the Gray map.