Notes on How to Prove It: A Structured Approach

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Chapter 1

Introduction

This is a book about deductive reasoning in mathematics and mathematical proofs. Fundamental theorems regarding prime numbers are introduced in this chapter to provide an idea of what to expect later in the book.

As a first proof, let's test the conjecture that there are infinitely many prime numbers. If we assume that there is a finite list of prime numbers denoted as $p_1, p_2, \ldots, p_n$, we can create a number m as follows:

$$m = p_1 p_2 \ldots p_n + 1$$

Note that m is not divisible by any of the primes p_n.[1]

If we acknowledge the fact that every integer larger than 1 is either a prime or can be written as a product of two or more primes, then if m is a prime, it can't be on the list because that would contradict the first assumption that all primes are on the list. Now, assume m is a product of primes. Take q as one of the primes in that product and an element on the list, so m is divisible by q. But then again, we reach a contradiction because m is not divisible by any p_n.

Since the assumption that there is a finite amount of prime numbers reached a contradiction, there must be an infinite amount of prime numbers.

1.1 Exercises

*1. (a) Factor $2^n - 1 = 32,767$ into a product of two smaller positive integers.

$n = log_2(32,767+1) = 15$, we know that if n is not prime then $2^n - 1$ is not prime, so 32,767 is a composite number. One possible solution is $4,681 * 7 = 32,767$.

(b) Find an integer x such that $1 < x < 2^{32,767} - 1$ and $2^{32,767} - 1$ is divisible by x.

[1] This is because the remainder when dividing m by any of the primes p_n is 1, which means m is not divisible by them.

$2^{32,767} - 1$ is composite so there exist at least one x that can solve the problem. From the proof of *Conjecture 2*, if n is not prime then $n = a * b$, also $2^n - 1 = (2^b - 1) * (1 + 2^b + 2^{2b} + \ldots + 2^{(a-1)b})$. So one possibility is $x = 2^7 - 1$.

2. Make some conjectures about the values of n for which $3^n - 1$ is prime or the values of n for which $3^n - 2^n$ is prime. (You might start by making a table similar to *Figure I.1*)

Chapter 2

Chapter 1: Sequential Logic

2.1 Deductive Logic and Logical Connectives