

作业

3,4

3(a)

方程组对应矩阵系数 \mathbf{A} 为

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 1 & 3 \\ 3 & 6 & 1 & 4 \end{pmatrix}$$
$$\sim \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -2 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

对应的齐次方程组为

$$\begin{cases} x_1 = -2x_2 - x_4 \\ x_2 = x_2 \\ x_3 = -x_4 \\ x_4 = x_4 \end{cases}$$

齐次方程组的通解为

$$\mathbf{x} = \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} m + \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} n$$

3(b)

方程组对应系数矩阵 \mathbf{A} 为

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 4 & 2 & 1 \\ 6 & 3 & 1 \\ 8 & 4 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 2 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & -2 \\ 0 & 0 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

对应的齐次方程组为

$$\begin{cases} x_1 = -\frac{1}{2}x_2 \\ x_2 = x_2 \\ x_3 = 0 \end{cases}$$

齐次方程组的通解为

$$\mathbf{x} = \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \\ 0 \end{pmatrix} m$$

4(a)

方程组对应增广矩阵 $(\mathbf{A} \mid \mathbf{b})$ 为

$$\begin{pmatrix} 1 & 2 & 1 & 2 & 3 \\ 2 & 4 & 1 & 3 & 4 \\ 3 & 6 & 1 & 4 & 5 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & -2 & -2 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

对应的非齐次方程组为

$$\begin{cases} x_1 = -2x_2 - x_4 + 1 \\ x_2 = x_2 \\ x_3 = -x_4 + 2 \\ x_4 = x_4 \end{cases}$$

非齐次方程组的通解为

$$\mathbf{x} = \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} m + \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} n + \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$$

4(b)

方程组对应增广矩阵 $(\mathbf{A} \mid \mathbf{b})$ 为

$$\begin{pmatrix} 2 & 1 & 1 & 4 \\ 4 & 2 & 1 & 6 \\ 6 & 3 & 1 & 8 \\ 8 & 4 & 1 & 10 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 1 & 4 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & -2 & -4 \\ 0 & 0 & -3 & -6 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{1}{2} & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

对应的齐次方程组为

$$\begin{cases} x_1 = -\frac{1}{2}x_2 + 1 \\ x_2 = x_2 \\ x_3 = +2 \end{cases}$$

齐次方程组的通解为

$$\mathbf{x} = \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \\ 0 \end{pmatrix} m + \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$$