

## 作业3

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### 6

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 1 & -4 \\ -1 & -3 & 1 & 0 \\ 2 & 6 & 2 & -8 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & 0 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{rank}(\mathbf{A}) = 2$$

$$\mathbf{A}^T \mathbf{A} = \begin{pmatrix} 1 & -1 & -2 \\ 3 & -3 & 6 \\ 1 & 1 & 2 \\ -4 & 0 & 8 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 & -4 \\ -1 & -3 & 1 & 0 \\ 2 & 6 & 2 & -8 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 18 & 4 & -20 \\ 18 & 54 & 12 & -60 \\ 4 & 12 & 6 & -20 \\ -20 & -60 & -20 & 80 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & 0 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{rank}(\mathbf{A}^T \mathbf{A}) = 2$$

$$\mathbf{A} \mathbf{A}^T = \begin{pmatrix} 1 & 3 & 1 & -4 \\ -1 & -3 & 1 & 0 \\ 2 & 6 & 2 & -8 \end{pmatrix} \begin{pmatrix} 1 & -1 & -2 \\ 3 & -3 & 6 \\ 1 & 1 & 2 \\ -4 & 0 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 27 & -9 & 54 \\ -9 & 11 & -18 \\ 54 & -18 & 108 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{rank}(\mathbf{A} \mathbf{A}^T) = 2$$

所以

$$\text{rank}(\mathbf{A}) = \text{rank}(\mathbf{A}^T \mathbf{A}) = \text{rank}(\mathbf{A} \mathbf{A}^T) = 2$$

### 9

最小二乘法拟合有公式

$$\mathbf{A}^T \mathbf{A} \mathbf{x} = \mathbf{A}^T \mathbf{b}$$

其中

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & \dots \\ t_1 & t_2 & t_3 & \dots \\ t_1^2 & t_2^2 & t_3^2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}^T$$

误差

$$err = (\mathbf{Ax} - \mathbf{b})^T (\mathbf{Ax} - \mathbf{b})$$

对于用一次多项式拟合有

$$\begin{aligned}\mathbf{A} &= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \end{pmatrix}^T \\ \mathbf{b} &= (2 \ 7 \ 9 \ 12 \ 13 \ 14 \ 14 \ 13 \ 10 \ 8 \ 4)^T \\ \mathbf{x} &= (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b} \\ &= (9.6364 \ 0.1818)^T\end{aligned}$$

对应的拟合多项式为

$$\hat{y} = 0.1818 + 9.6364x$$

误差

$$\begin{aligned}err_1 &= (\mathbf{Ax} - \mathbf{b})^T (\mathbf{Ax} - \mathbf{b}) \\ &= 162.9091\end{aligned}$$

对于用二次多项式拟合有

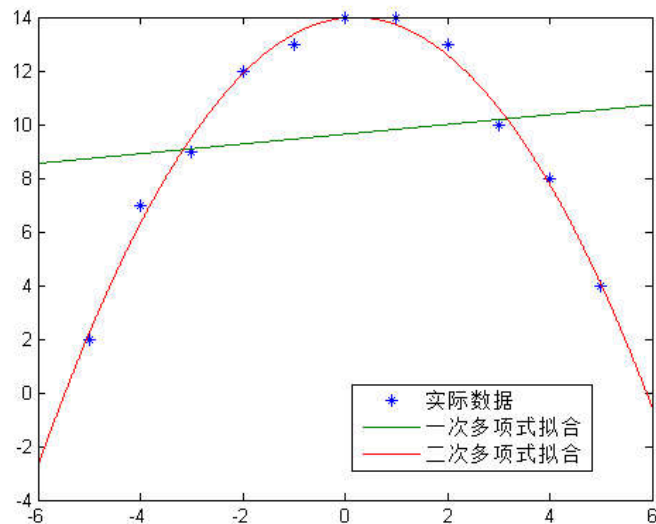
$$\begin{aligned}\mathbf{A} &= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 25 & 16 & 9 & 4 & 1 & 0 & 1 & 4 & 9 & 16 & 25 \end{pmatrix}^T \\ \mathbf{b} &= (2 \ 7 \ 9 \ 12 \ 13 \ 14 \ 14 \ 13 \ 10 \ 8 \ 4)^T \\ \mathbf{x} &= (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b} \\ &= (13.9720 \ 0.1818 \ -0.4336)^T\end{aligned}$$

对应的拟合多项式为

$$\hat{y} = -0.4336 + 0.1818x + 13.9720x^2$$

误差

$$\begin{aligned}err_2 &= (\mathbf{Ax} - \mathbf{b})^T (\mathbf{Ax} - \mathbf{b}) \\ &= 1.6224\end{aligned}$$



$err_1 > err_2$ 并且从示意图也可以清晰看出二次拟合效果更好