Journal of Housing Economics 50 (2020) 101716

Contents lists available at ScienceDirect

Journal of Housing Economics

journal homepage: www.elsevier.com/locate/jhec

Stratified zoning in central cities

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ARTICLE INFO

JEL Classification:
R14
R21
R52

Keywords:
Residential
Zoning
Voting

ABSTRACT

I model a central city where citizens differ by income, and housing confers benefits on neighbors. Zoning separates citizens into neighborhoods by income. This maximizes total surplus and facilitates redistributing gains to a majoritarian governing coalition of citizens, which changes from rich to poor as the city grows. Non-members of the coalition may form suburbs. These theoretical results are supported by empirical facts compiled by Schnore and Varley (1955). It is difficult to justify stratified zoning on Paretian grounds, even when a municipal government can redistribute income. If stratified zoning is not a Pareto improvement before gains are redistributed, it will not be afterward under majority rule. Gains in total surplus increase as the distribution of income becomes less equal, which helps explain why neighborhood stratification has outpaced stratification of income in U.S cities in recent decades, as documented by Booza et al (2006) and Sampson (2016).

1. Introduction

Central cities impose many restrictions on the use of residential land. To explain, describe, and evaluate these restrictions, I construct an intra-municipal urban sorting model in which citizens differ by income and housing is semi-rival and semi-excludable in consumption, similar to a clubs good.[1] Previous literature on neighborhood externalities focuses on single neighborhoods, and most previous literature on urban sorting is based on multiple municipalities. I model multiple neighborhoods in a single municipality, since much sorting occurs within central cities.

A common restriction is a minimum on the size of lots, historically often a quarter of an acre in the U.S., or a maximum on the number of units on a given plot, but restrictions may also apply to architectural standards, such as façade requirements. Restrictions that decrease density necessarily increase commuting time. Locally, under some allocations, at least, people seem to prefer low density to high density even after netting out the extra commuting costs. This view was validated by Bucovetsky (1984), who concludes that minimum lot size zoning increases the price of new housing and decreases the price of residential land.

Henderson (1985b) is critical of zoning *only* land: "Of course, minimum lot size zoning is not an efficient way to raise h, in the sense that h could be raised more (for any alternative utility level) if, for example, all inputs into the production process could be zoned and raised by the same percentage." White (1975) compares zoning that restricts only consumption of land to zoning that restricts consumption of land and housing capital. In practice, using land alone to correct for the housing externality would favor citizens with strong preferences for land, both their own and that of their neighbors, relative to other inputs to housing. It would lead to high consumption of gasoline, pollution, and long travel times, and would make public transportation cost-ineffectual by making neighborhoods "spread out", so people would have far to drive to non-residential activities and far to walk to a bus or train.

Perhaps these concerns, but mostly the kind of increase in the cost of housing identified by Bucovetsky, have motivated a few U.S. cities, and one large state, recently to go the other way and ease maxima on density.[2] At other allocations, when the price of housing gets high enough, people may not prefer a combination of low density and high commuting costs. The present theoretical model is valid so long as the minima imposed on consumption of the various aspects of housing are targeted to those aspects for which the people intended are willing to pay the equilibrium price, which might reflect diminishing marginal utility of neighborhood as quality increased, but density declined. The suburban commuting cost specified in this model should be thought of as *additional* to the cost of an intra-city commute.

Work on neighborhood externalities includes that of Schall (1976) and Stahl (1976). They identify cooperative and non-cooperative equilibrium levels of investment in housing in dynamic contexts. The literature on urban sorting, beginning with Tiebout (1956), is extensive.

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[1] See Buchanan (1965; pp. 4-5).

[2] See Biron (2019) and Hansburg and Rutherford (2020).

https://doi.org/10.1016/j.jhe.2020.101716
Received 22 December 2016; Received in revised form 14 April 2020; Accepted 18 July 2020
Available online 31 July 2020

Glossary of variable names

N	number of citizens in the city
M	number of neighborhoods in the city
l	index of neighborhoods$l = 1, ...,M$; also, the set of citizens residing in Neighborhood$l$
$\#l$	number of citizens residing in Neighborhood l
U^i	utility of Citizen i
z_i	consumption of composite good by Citizen i
h_i	consumption of housing by Citizen i
l_i	neighborhood where Citizen i resides; also, the set of citizens residing in that neighborhood
$\bar{h}_l$	average consumption of housing in Neighborhood l
$\bar{h}_{l_i}$	average consumption of housing in the neighborhood where Citizen i resides
α	every citizen's relative preference for the composite good
β	every citizen's relative preference for housing
δ	every citizen's relative preference for average consumption of housing in her neighborhood
y_i	Citizen i's income of the composite good
p	price of housing when the external benefits of housing are not priced
p_h	price of housing when the external benefits of housing are priced
p_l	price of the external benefits of housing
m	index of income groups;$m = 1, ...,M$
y_m	income of a citizen in Income Group m
m_i	neighborhood corresponding to the income group to which Citizen i belongs
z_i^*	Citizen i's private utility maximizing consumption of the composite good
h_i^*	Citizen i's private utility maximizing consumption of housing
$\bar{h}^*_{l_i}$	the externality experienced by Citizen i in laissez-faire
$\bar{y}_l$	average income in Neighborhood l
V^i	indirect utility of Citizen i
K	a constant whose definition depends on context
H	total consumption of housing citywide
$\underline{h}$	uniform minimum consumption of housing
N_B	number of citizensbound by uniform minimum
σ_i	share of Citizen i's budget devoted to housing
$(z_{ia}, h_{ia}, \bar{h}_{ia})$	consumption of the composite good and the externality experienced by Citizen i in Allocation a
z_i^c	consumption of the composite good by Citizen i that leaves her indifferent to an alternative allocation
CV_i^X	Citizen i's willingness to pay to change to Zoning Policy X from an alternative policy;$z_i^X - z_i^c$, where z_i^X is i's consumption of the composite good under Policy X.
GTS	sum of CV_i over all i; gain in total surplus
z_i^{**}	Citizen i's consumption of the composite good consistent with maximum GTS
h_i^{**}	Citizen i's consumption of housing consistent with maximum GTS
$\bar{h}^{**}_{l_i}$	externality experienced by Citizen i at maximum GTS
$h_l^{\min}$	minimum consumption of housing in Neighborhood l
t	commuting cost from a Tiebout-style, income-homogeneous suburb as a fraction of income
V^m	indirect utility of a citizen in Income Group m

Much of it focuses on trade-offs between existence of equilibrium and "efficiency" in the provision of local public goods when citizens self-select to income-homogeneous municipalities. There is also substantial literature on zoning itself, which overlaps with that on neighborhood effects and sorting. Historical reviews of theoretical and empirical literature on zoning are Pogodzinski and Sass (1990) and Pogodzinski and Sass (1991), respectively.

Much of this literature, including Epple and Platt (1998), also allowed for locational differences in the price of housing. Like Epple and Platt, I assume that land is homogeneous. Though I generally assume that the marginal product of land is equalized across uses, so that all land is priced alike, I show that the allocations I derive are robust to this assumption. I show in Appendix A that there exist prices that do vary by location that also support the allocations I derive without locational variation in the price of land.

Henderson (1985a; p. 249) argues that models without an "adjustment mechanism for land uses across the fixed number of communities to equalize the marginal products of land" do not derive "long-run solutions, but might deal with what could be termed temporary equilibria in an unspecified dynamic context." He cites empirical evidence that numbers and boundaries of communities are flexible in the United States. In the long-run, a city, including its business districts and neighborhoods, need not exist. Their costs are not sunk, and the business districts, neighborhoods, and suburbs may reform or relocate to supplant any part of surrounding land, whose marginal value in agricultural use may be assumed not to vary by location or quantity supplied. Therefore, the long-run opportunity cost of urban land use need not vary by location or quantity consumed, either. It is analytically convenient to assume that the price of urban land equals this long-run opportunity cost, so the price of housing equals its long-run marginal cost, but, again, the allocations derived are also equilibria at prices that may vary by location.

Zoning is about as common as stratification, which is quite common, suggesting that zoning is not superfluous. According to Epple (2003; p. 189), before Ellickson (1971), "there was a widespread view among urban economists that zoning or other governmental restrictions were needed to maintain the stratification of households by income across municipalities that is commonplace in U.S. metropolitan areas." With Ellickson, the urban sorting literature began to derive equilibria with separation that existed without restrictions on consumption of housing but, rather, that depended on a "single-crossing property" derived from some modest restrictions on preferences. "Epple and Platt rely on a Cobb-Douglas specification for utility and use a numerical example to demonstrate that a sorting equilibrium [with incomplete income stratification across communities and income heterogeneity within communities] exists."[3] I also use Cobb-Douglas, which satisfies Ellickson's single-crossing property in $(\bar{h}, p)$ space, where $\bar{h}$ is a neighborhood housing externality, and $p$is the price of housing.

Therefore, when I, like Epple and Platt, allow the price of land to differ by location, I find that stratification is an equilibrium without zoning, but that, in the present model, this equilibrium is neither unique nor stable. Zoning, therefore, will be necessary and will be used to effect sorting in a long-run economic and political outcome. I return to the pre-Ellickson view, but in an intra-municipal model. The pre-Ellickson view may be theoretically retrograde, but the idea that zoning regulations both bind and stratify has some up-to-date empirical support. According to Chapple et al (2017),

> [Accessory dwelling unit] (ADU) owners in Portland, Seattle, and Vancouver value their ADUs as small and flexible spaces that can be built relatively quickly. Though built for a variety of purposes, *the majority are actually affordable housing.*
>
> Three factors are key to the success of ADU implementation. First, in cities that have reformed their *zoning regulations* (particularly minimum lot size and floor area) *production has jumped.* Also important are minimizing design review and easing owner occupancy

[3] See Kuminoff, Smith, and Timmons (2013), p. 1018.

requirements. (p. 23, emphasis added)

While implementation of ADU's reduces stratification, the observation that MLR's and other regulations are binding and stratifying provides some support for a hypothesis that a long trend toward greater stratification in the U.S. was implemented through zoning.[4]

The contribution of this article, however, is not mainly to revive the pre-Ellickson view. An interesting implication of the model is that governing coalitions in small (large) central cities will be formed from the upper (lower) portion of the distribution of income, and poorer (richer) people may form suburbs. In mid-size central cities, governing coalitions will be formed from the extremes of the distribution of income, and not from the middle class, and the middle class may form suburbs. This theoretical result explains empirical facts documented by Schnore and Varley (1955). Further, I prove that if stratified zoning is not a Pareto improvement before redistribution of its social benefits, it will not be a Pareto improvement afterward, either, under a majoritarian government. Losers *after* redistribution will be non-members of the governing coalitions: the poor in small cities; the middle class in mid-size cities; and, interestingly, the rich in large cities. Thus, though stratified zoning maximizes total surplus, which is the objective function of a Kaldor-Hicks welfare criterion[5], it is difficult to justify on the grounds that maximization of that criterion makes a Pareto improvement possible, under majoritarian democracy. I conclude that stratified zoning would be easier to justify if it required the approval of representatives from all parts of a city.

The U.S. Supreme Court upheld the right of municipalities to impose restrictions on consumption of housing that differed by location in Euclid v. Amber (1926). Justice Sutherland described negative externalities associated with low consumption of housing, which is economically equivalent to positive externalities of high consumption:

> …With particular reference to apartment houses, it is pointed out that the development of detached house sections is greatly retarded by the coming of apartment houses, which has sometimes resulted in destroying the entire section for private house purposes; that in such sections very often the apartment house is a mere parasite, constructed in order to take advantage of the open spaces and attractive surroundings created by the residential character of the district.Moreover, the coming of one apartment house is followed by others, interfering by their height and bulk with the free circulation of air and monopolizing the rays of the sun which otherwise would fall upon the smaller homes, and bringing, as their necessary accompaniments, the disturbing noises incident to increased traffic and business, and the occupation, by means of moving and parked automobiles, of larger portions of the streets, thus detracting from their safety and depriving children of the privilege of quiet and open spaces for play, enjoyed by those in more favored localities-until, finally, the residential character of the neighborhood and its desirability as a place of detached residences are utterly destroyed.Under these circumstances, apartment houses, which in a different environment would be not only entirely unobjectionable but highly desirable, come very near to being nuisances. U.S. Supreme Court 1926

Sutherland does not regard an apartment house as "residential". Moreover, he does not explain why traffic, darkness, noise, air pollution, danger, and the deprivation of children of quiet and open spaces for play would be "highly desirable" if concentrated in an environment other than "more favored localities". I will argue here that central cities will use zoning to internalize the positive externalities of housing, which benefits all citizens, and to separate rich neighborhoods from poor ones, which improves the neighborhoods of the rich and makes the neighborhoods of the poor worse, and that the net of these two effects on the poor is empirically negative. I will also argue that, under majority rule, in small cities the poor will not be able to compel others to make them whole for the losses they incur. In large cities, they will, but they will not stop there. They will tax the rich until the gains of the rich become losses.

In Section 2, I describe a model of a city with multiple neighborhoods and citizens who differ by income. In Section 3, I derive outcomes first without zoning, second when zoning merely bans "substandard" housing throughout the city, and third when the government can impose restrictions that differ by location. I discuss the properties of these outcomes in terms of efficiency and implementability. In Section 4, I describe the outcome of citywide voting on zoning policy. In Section 5, I discuss the distributional consequences of this outcome. In Section 6, I use the model to interpret the facts in a report from the Brookings Institution.[6] I conclude in Section 7.

2. Model

N citizens reside in a city of M neighborhoods, across which the citizens are mobile. Each neighborhood may expand in area, absorbing non-residential land, without contacting other neighborhoods.

Citizens consume housing and a composite good and differ only by income of the composite good. Each citizen resides at a single locational node, which I have called a "neighborhood". The utility function for Citizen i, $i = 1. ..N$, is

$$U^i = \alpha \ln z_i + \beta \ln h_i + \delta \ln \bar{h}_{l_i} \tag{2.1}$$

where z_i is i's consumption of the composite good, h_i is i's consumption of housing, $\bar{h}_{l_i}$ is average consumption of housing in i's neighborhood, Neighborhood l_i, α, β, and$\delta > 0$, and $\alpha + \beta + \delta = 1$. I will also sometimes refer to "Neighborhood l" and to $\bar{h}_l$ as average consumption of housing in that Neighborhood.[7]$(l_1, ..., l_N)$ will refer to the locations of all citizens in an allocation $A = (z_1, ...,z_N, h_1, ...,h_N, l_1, ...,l_N)$. In an abuse of notation, l_i sometimes also refers to the set of citizens residing in Neighborhood l_i. $\#l_i$ citizens reside in Neighborhood l_i.[8]

Citizen i's utility increases in average consumption of housing in her neighborhood. She prefers low density, dislikes "crowding". She also enjoys looking at clean, well-built, well-maintained, artful dwellings when she comes and goes from hers. When she visits her neighbors, she may enjoy the interiors of their dwellings. She may benefit if her neighbor's house is better protected from fire or pests. In the case of attached housing, she may benefit from a neighbor whose house is better insulated for temperature and sound, and, if one includes it in the definition of "housing", from her neighbor's consumption of artificial heat. Consumption of housing can also proxy for other desirable characteristics in one's neighbors.

Citizen i has income y_i. She faces a budget constraint

$$z_i + ph_i = y_i \tag{2.2}$$

where p is the price of housing, equal to the long-run marginal cost of housing. p is the same throughout the city. (In Appendix A, I explore the implications of allowing the price of housing to vary by location depending on the external benefits of housing generated in each location, $\bar{h}_{l_i}$.) I write (2.2) as an equality because preferences are locally non-satiated. I define units such that

[4] See Booza et al (2006) and Sampson (2016).

[5] See Griffin (1995), p. 2.,¶ 1.

[6] Booza et al. (2006)

[7] The specification of utility as log-linear follows that of Fujita (1989; p. 182). Epple and Platt also use Cobb-Douglas. Specification of the externality as average consumption of housing is similar to Schall's formulation for quality of land use. The difference between myh_i and Schall'sQ_i is that h_i increases in the amount of land attached to i's dwelling unit, while Q_i measures improvements to a fixed plot of land. Therefore, unlike Schall's$\bar{Q}_i$, $\bar{h}_{l_i}$, like Justice Sutherland, ascribes a positive externality to lower density of neighborhood population.

[8] A glossary of variable names appears at the end of the paper.

$$\sum_{i=1}^{N} y_i = 1 \tag{2.3}$$

I assume a simple distribution of income consisting of exactly M groups indexed by $m = 1. ..M$. There is the same number of neighborhoods as income groups. Every citizen in Income Group m has income y_m. y_m increases in m; that is, $y_m \leq y_{m+1}$ for all $m = 1, ...,M - 1$.

A city planner governs the city. I assume that the planner and all citizens know the utility function (2.1) and the distribution of income, but not the income of any individual citizen before the fact. Municipal governments control assessed values and tax rates on residential property. This gives them two tools with which to redistribute income whenever citizens of different incomes consume different amounts of housing.

Municipal governments also control the provision of local public goods (and bads) to each neighborhood. If citizens of different incomes reside in different neighborhoods, then that also enables the planner to redistribute income. Examples include quality of primary and secondary schools, police, and fire protection, parks and other urban greenspace, sidewalks, pothole repair, on-street parking privileges for neighborhood residents, mixed recycling, range of items (e.g. mattresses) included in, and the frequency of, trash collection, community centers, freeways, transit centers and public transportation, airports, halfway houses for convicted criminals, landfills, and zoning for adjacent or inter-mingled non-residential uses, including public, religious, commercial, and industrial uses, which may involve traffic, emissions, effluents, or noise. Commercial uses may include amenities such as restaurants, cafés, and grocery stores; see Couture and Handbury (2017). There are myriad ways in which a municipal government can confer benefits or costs on a neighborhood in practice, and to distribute them across different neighborhoods. I will assume that the planner can redistribute the composite good to or from any subset of citizens who consume a different quantity of housing from all others, in an equal amount per citizen, in an income-homogeneous neighborhood.

I have not specified the process by which the planner comes to power, but, whatever that process is, the successful candidate for planner will be the one who can deliver the greatest benefits to those on whose support her power depends. Each citizen of our model city is willing to pay an amount of the composite good to effect a change from one allocation to another. This amount may be negative. Call the sum over the citizens of these amounts the "gain in total surplus" of the change.[9] Since the planner can redistribute income between citizens who consume different amounts of housing or reside in different neighborhoods, I assume that she will maximize total surplus over the set of allocations that she can implement, and in which citizens with different incomes either consume different amounts of housing or reside in different neighborhoods. She will do this whether she comes to power by majority vote, consensus, plutocracy (support of only the top income group), or dictatorship of the proletariat (the bottom income group). If, under the allocation that maximizes total surplus overall, she can distinguish among citizens of different incomes, and she can implement this allocation, then the planner will maximize total surplus over the set of all allocations.

I assume that the price of non-residential land is geographically uniform and equal to its long-run market value in non-residential use, and that all land is of the same quality, except insofar as location-specific amenities are included in the composite good. If there is no restriction on the supply of residential land, then the price of residential land will equal the price of non-residential land. Residences are worth more when situated adjacent to other residences, commercial structures when situated near other commercial activity, and so, too, with industrial structures. Thus, agglomerative economies in residential, commercial, and industrial uses, respectively, will render neighborhoods contiguous in theory, and to tend that way in practice.

Alternatively, suppose the planner sets a boundary between each neighborhood and surrounding, non-residential areas. A municipal government interested in maximizing total surplus will want all land to be priced at its long-run marginal cost. (See the quote above from Henderson [1985a].) The planner, then, will set the land area of each neighborhood so as to equate the price of land in that neighborhood to the long-run opportunity cost of land; ultimately, its value in agricultural use, which I take to be constant. I assume that every input to housing other than land is also priced at its long-run marginal cost. The long-run marginal cost of housing is the price of land plus the prices of the other inputs to housing. The long-run marginal cost of housing, then, is constant over the range of quantities demanded under the different policies I consider. The price of housing, p, equals the long-run marginal cost of housing, and is the same throughout the central city and the suburbs, and under all policies I consider.[10] I show in Appendix A, however, that the allocations I derive assuming that all land goes for the same price are also supported as equilibria when the price of land varies with the neighborhood externality. That is, though I assume a uniform price for land, I also show that my results are robust to this assumption.

3. Policies

3.1. Laissez-faire

Suppose that there are no restrictions on the use of land and that the planner does not redistribute income. I will refer to this policy as "laissez-faire". In practice, owners of residential property could do what they pleased with it. Instead of having to put up a four-bedroom house on a half-acre lot, a developer could construct row houses. A homeowner would be free to rent out a granny flat over the garage, or to turn her upstairs into a separate unit.

3.1.1. Consumption in laissez-faire

Suppose Citizen i chooses h_i independently, taking as given, without attempting to cooperate with her neighbors in an effort to take advantage of the external benefits of housing. She maximizes (2.1) over z_i and h_i subject to (2.2), so h_i=$(y_i - z_i)/p$. For the moment, suppose she holds l_i fixed.[11] She then maximizes $\alpha \ln z_i + \beta \ln(y_i - z_i)/p$ by choosing z_i. Differentiating with respect to z_i and setting the result equal to zero gives $\alpha/z_i - \beta p/(y_i - z_i) = 0$. The left hand side is decreasing in z_i, so solving this for z_i and substituting the result into (2.2) give her private utility-maximizing consumption of the composite good and housing:

$$z_i^* = \frac{\alpha y_i}{\alpha + \beta} \text{and} h_i^* = \frac{\beta y_i}{(\alpha + \beta)p} \tag{3.1}$$

(3.1) will hold for Citizen i for any choice of location l_i, so it will hold for all $i = 1. ..N$ in equilibrium in laissez-faire.

[9] Here, total surplus of housing and neighborhood equals consumer surplus, because the supply curve is horizontal at the long-run marginal cost of housing.

[10] An alternate assumption would be that the planner allocates enough land to residential use to equate the marginal social benefit of housing, rather than the price of housing, to its marginal cost. That is, the planner would internalize the external benefits of housing not by imposing minima on the consumption of housing, but by allocating sufficient land to residential use. (The marginal social benefit of housing declines in the quantity of land allocated to residential use.) Assuming the planner would use land alone to correct for the externality would be an abuse of the assumption of constant marginal costs of housing, since it would allow for a large expansion of citizens' budget sets as land went into residential use and the price of housing fell, and the price of residential land fell below the price of non-residential land.

[11] I assume that Citizen i's consumption of housing, h_i, is a small fraction of the total consumption of housing in her neighborhood so that I may neglect the effect of h_i on the externality, $\bar{h}_{l_i}$, in identifying her private utility-maximizing consumption of housing.

Proposition 1. *In laissez-faire, every citizen consumes her private utility maximizing bundle of the composite good and housing as described in*(3.1).

3.1.2. Location in laissez-faire

In choosing l_i in laissez-faire, all citizens, regardless of income, have the same preference ordering over neighborhoods. All citizens want to live in neighborhoods with a high proportion of rich citizens, who generate high external benefits. No two citizens will choose to reside in separate neighborhoods unless average consumption of housing is the same in both neighborhoods:

$$\bar{h}_a = \bar{h}_b \text{ for all } a, b \tag{3.2}$$

(3.2) is a necessary condition for equilibrium in laissez-faire. Citizens will only be indifferent among neighborhoods, and, therefore, stay put, if the neighborhoods are of the same quality. All neighborhoods are, therefore, of average quality.

Proposition 2. *In laissez-faire, average consumption of housing is the same in every neighborhood throughout the city.*

3.1.3. Equilibrium

In laissez-faire, then,

$$\left.\begin{aligned} z_i^* &= \tfrac{\alpha y_i}{\alpha+\beta} \\ h_i^* &= \tfrac{\beta y_i}{(\alpha+\beta)p} \\ \bar{h}_{l_i}^* &= \tfrac{\beta}{(\alpha+\beta)p}\tfrac{1}{N} \end{aligned}\right\} \text{for all } i \tag{3.3}$$

where $1/N$ is average income citywide. $1/N$ appears in the expression for $\bar{h}_{l_i}^*$ in the same place that y_i appears in the expression for h_i^*, reflecting that each citizen experiences a neighborhood externality equal to average consumption of housing citywide.

3.1.4. Weakness and inefficiency in the laissez-faire equilibrium

I derived (3.1) assuming that all Citizens i take $\bar{h}_{l_i}$ as given. That is, they do not attempt to cooperate with their neighbors in deciding how much housing to consume. However, Pareto improvements on (3.3) are attainable if citizens can cooperate, so (3.3) is not a strong equilibrium. For example, a Pareto improvement would result from raising consumption of housing to $h_i = (\beta + \delta)y_i/p$ for all Citizens i to take advantage of the external benefits of housing. Suppose citizens reside in the same locations that they do in laissez-faire; that is, where there is complete mixing by income. Suppose also that they all devote the same share of their budgets to housing, denoted σ. The utility for any Citizen i, then, would be

$$U^i(\sigma) = \alpha \ln(1-\sigma)y_i + \beta \ln \frac{\sigma}{p} y_i + \delta \ln \frac{\sigma}{p}\frac{1}{N}$$

where $1/N$ is average income citywide. To maximize i's utility, I differentiate with respect to σ and set the result equal to zero, which gives

$$\frac{\partial}{\partial \sigma} U^i(\sigma) = \frac{-\alpha}{1-\sigma} + \frac{\beta}{\sigma} + \frac{\delta}{\sigma} = 0$$

Since the derivative decreases in σ, solving this equation maximizes i's utility. The solution is

$$\sigma = \beta + \delta \tag{3.4}$$

This exceeds $\beta/(\alpha + \beta)$, the budgetary share every citizen devotes to housing when laissez-faire is a non-cooperative game. Thus, a Pareto improvement over the non-cooperative equilibrium occurs when all Citizens i increase their consumption of housing to $(\beta + \delta)y_i/p$.

Proposition 3. *There exist allocations that are Pareto improvements over the non-cooperative laissez-faire equilibrium.*

How reasonable is the assumption of non-cooperation in laissez-faire? In practice, having a single firm own a large block of rental property can partially address the problem of maintaining neighborhood quality without reliance on zoning. This approach is complicated by the agency problems of rental agreements, the difficulties faced by a single firm in acquiring an entire neighborhood, and the market power such a firm might exercise if it did. Owner-occupied housing accounts for over half of expenditure on housing in the United States, according to data from the 2015 Consumer Expenditure Survey (CES)[12], despite any potential for large blocks of rental property to form neighborhoods of relatively high quality.

In the case of owner-occupied housing, social pressure and neighborhood associations can improve the quality of neighborhoods. Gated communities and condominiums can enforce standards under contract using the threat of civil penalties. However, political restrictions on consumption of housing are far more common than contractual ones, suggesting that barriers to private cooperation are substantial, or that the costs of public enforcement are relatively low.

The city of Houston, Texas is known for its libertarian zoning policies. According to McDonald (1995; p. 137), Houston is "the only major city in the U.S. that remains unzoned". However, only 27% of households surveyed there in 1998 reported paying a cooperative, condominium, or neighborhood association fee. The average over 28 MSA's surveyed in 1998 and 2002 was 13%, and the standard deviation was 8.3% overall and 8.8% above the mean. Houston was above average, but not an outlier.[13] Moreover, the level of private cooperation in lieu of zoning was modest. Non-cooperative behavior still appears to predominate. Houston residents sometimes employ deed restrictions in lieu of zoning, but McDonald (p. 138) writes "In Coasian terms, the 'transactions costs' of setting up a system of deed restrictions in developed areas are high." It is reasonable, then, to model laissez-faire as a non-cooperative game.

3.2. A uniform minimum

Suppose the planner has the power to impose a uniform minimum on consumption of housing throughout the city. The uniform minimum could represent restrictions on use of residential land that include only prohibitions on "substandard" housing, such as shanties. Since the minimum is the same in every neighborhood, (3.2) is a necessary condition for equilibrium with a uniform minimum; average consumption of housing is the same in every neighborhood.

If the minimum binds more than one income group, then all such groups will consume the same amount of housing, and the planner will not be able to distinguish among them. Due to this restriction, I assume that the planner sets the uniform minimum at either the level that maximizes total surplus, or the private utility maximizing consumption of housing for the second lowest income group, whichever is lower. This policy preserves the planner's ability to redistribute income. A minimum slightly lower than what is optimal for members of the second lowest income group results in citizens with different incomes consuming different amounts of housing, enabling the planner to redistribute income.

3.2.1. Gains from a uniform minimum relative to laissez-faire

The gain in total surplus associated with a change from an Allocation 0 to another Allocation 1 are the sum over $i = 1. ..N$ of variations in consumption of the composite good that leave each citizen indifferent to the change. Define $(z_{ia}, h_{ia}, \bar{h}_{ia})$ as consumption of the composite good and housing and the externality experienced by Citizen i under Allocation $a = 0, 1$. Define z_i^c as consumption of the composite good, when housing and the externality are given by Allocation 1, that leaves Citizen i indifferent to Allocation 0:

$$\alpha \ln z_i^c + \beta \ln h_{i1} + \delta \ln \bar{h}_{i1} = U_0$$

[12] See .

[13] See U.S. Bureau of the Census (1998) and (2002).

or

$$(z_i^c)^{\alpha} h_{i1}^{\beta} \bar{h}_{i1}^{\delta} = \exp U_0$$

where U_0 is utility under Allocation 0. The amount of the composite good that Citizen i will pay to change from Allocation 0 to Allocation 1, then, is

$$CV_i = z_{i1} - z_i^c = z_{i1} - \left(\frac{\exp U_0}{h_{i1}^{\beta} \bar{h}_{i1}^{\delta}} \right)^{\frac{1}{\alpha}} \tag{3.5}$$

where "*CV*" stands for "compensating variation". The gain in total surplus associated with a change to Allocation 1 are the sum over all i of CV_i.

$$GTS \equiv \sum_{i=1}^{N} CV_i = \sum_{i=1}^{N} (z_{i1} - z_i^c) \tag{3.6}$$

Define CV_i^{UM} as the gain to Citizen i from a uniform minimum relative to laissez-faire. There are two cases to consider. First is the case when Citizen i is bound by the uniform minimum, and second if she is not bound. If Citizen i is bound by the uniform minimum,

$$\alpha \ln z_i^c + \beta \ln \underline{h} + \delta \ln \bar{h} = \alpha \ln \frac{\alpha y_i}{\alpha + \beta} + \beta \ln \frac{\beta y_i}{(\alpha + \beta)p} + \delta \ln \frac{\beta}{(\alpha + \beta)p} \frac{1}{N}$$

where $\underline{h}$ is the uniform minimum and $\bar{h}$ is average consumption of housing citywide. From (3.5),

$$CV_i^{UM} = y_i - p\underline{h} - K_B y_i^{\frac{\alpha+\beta}{\alpha}} \tag{3.7}$$

where K_B is positive and constant with respect to y_i. I hold $\underline{h}$ constant with respect to y_i. $\bar{h}$, then, is constant with respect to y_i because, with $\underline{h}$ binding on Citizen i, she will devote any marginal increase in her income entirely to consumption of the composite good. Her consumption of housing does not change, so neither does average consumption of housing. Differentiating twice with respect to y_i gives

$$\begin{aligned} \frac{\partial CV_i^{UM}}{\partial y_i} &= 1 - \frac{\alpha+\beta}{\alpha} K_B y_i^{\frac{\beta}{\alpha}} \\ \frac{\partial^2 CV_i^{UM}}{\partial y_i^2} &= -\frac{\alpha+\beta}{\alpha} \frac{\beta}{\alpha} K_B y_i^{\frac{\beta-\alpha}{\alpha}} < 0 \end{aligned} \tag{3.8}$$

Thus, for citizens bound by a uniform minimum, welfare gains relative to laissez-faire are strictly concave in income. "Gains" may be negative: For the very poor, the minimum moves them well above their individually optimal consumption of housing, and their welfare losses, therefore, are large. Banning "substandard" housing may render them homeless.

If Citizen i is not bound by the uniform minimum,

$$\alpha \ln z_i^c + \beta \ln \frac{\beta y_i}{(\alpha + \beta)p} + \delta \ln \bar{h} = \alpha \ln \frac{\alpha y_i}{\alpha + \beta} + \beta \ln \frac{\beta y_i}{(\alpha + \beta)p} + \delta \ln \frac{\beta}{(\alpha + \beta)p} \frac{1}{N}$$

Solving for z_i^c gives

$$z_i^c = K_{NB} y_i^{\alpha}$$

where K_{NB} is also positive and constant with respect to y_i. Holding $\bar{h}$ constant with respect to y_i, the gain to Citizen i is

$$CV_i^{UM} = \frac{\alpha y_i}{\alpha + \beta} - K_{NB} y_i^{\alpha} \tag{3.9}$$

Differentiating twice gives

$$\begin{aligned} \frac{\partial CV_i^{UM}}{\partial y_i} &= \frac{\alpha}{\alpha + \beta} - \alpha K_{NB} y_i^{\alpha-1} \\ \frac{\partial^2 CV_i^{UM}}{\partial y_i^2} &= -\alpha(\alpha - 1) K_{NB} y_i^{\alpha-2} > 0 \end{aligned} \tag{3.10}$$

The derivative of $\bar{h}$ with respect to $y_i$is $\beta/((\alpha + \beta)Np)$. Assuming N is large, this effect is negligible. Gains from a uniform minimum to citizens not bound by the minimum, unlike those of citizens bound by the minimum, are convex in income. The higher the minimum, the more citizens are bound by it, and the less convex are societal gains relative to laissez-faire. They become concave as the minimum rises.

3.3. Minima by neighborhood

Now suppose the planner has the authority to set a minimum on consumption of housing by neighborhood. She will use this authority to maximize total surplus if, when she does so, citizens with different incomes consume different amounts of housing or reside in different neighborhoods, under a variety of processes by which she may come to power. In this section of the paper, I solve for and describe the allocation that maximizes total surplus and show that a zoning policy exists to implement that allocation as a unique, coalition-proof equilibrium.

3.3.1. The allocation that maximizes total surplus

Competitive markets maximize total surplus; they leave no "deadweight loss". In order to find the allocation that maximizes total surplus, let us assume that competitive markets exist for z_i, h_i, and $\bar{h}_{l_i}$. Let z_i continue to be the numeraire, and let p_h and p_l be the prices of $h_i$and $\bar{h}_{l_i}$, respectively. A necessary condition for equilibrium in a competitive market is that each citizen maximizes her utility subject to her budget constraint

$$z_i + p_h h_i + p_l \bar{h}_{l_i} = y_i \tag{3.11}$$

which I write as an equality due to local non-satiation of preferences. Substituting (3.11) into the utility function gives

$$U_{CM}^i = \alpha \ln(y_i - p_h h_i - p_l \bar{h}_{l_i}) + \beta \ln h_i + \delta \ln \bar{h}_{l_i} \tag{3.12}$$

where "*CM*" stands for "complete markets". Differentiating with respect to h_i and $\bar{h}_{l_i}$ gives the first order conditions

$$\begin{aligned} \frac{-\alpha p_h}{y_i - p_h h_i - p_l \bar{h}_{l_i}} + \frac{\beta}{h_i} &= 0 \\ \frac{-\alpha p_l}{y_i - p_h h_i - p_l \bar{h}_{l_i}} + \frac{\delta}{\bar{h}_{l_i}} &= 0 \end{aligned} \tag{3.13}$$

These imply that

$$\bar{h}_{l_i} = \frac{\delta}{\beta} \frac{p_h}{p_l} h_i \tag{3.14}$$

In *any* allocation,

$$\sum_{i=1}^{N} \bar{h}_{l_i} = \sum_{l=1}^{M} \#l \bullet \bar{h}_l = \sum_{l=1}^{M} \#l \frac{1}{\#l} \sum_{k \in l}^{\#l} h_k = \sum_{l=1}^{M} \sum_{k \in l}^{\#l} h_k = \sum_{i=1}^{N} h_i \tag{3.15}$$

The sum over all citizens of the neighborhood externalities experienced by each equals the sum over all citizens of consumption of housing by each. By (3.14) and (3.15), then,

$$p_l = \frac{\delta}{\beta} p_h \tag{3.16}$$

Substituting (3.16) into (3.14) gives

$$\bar{h}_{l_i} = h_i \tag{3.17}$$

To maximize total surplus, the externality experienced by each citizen should equal her own consumption of housing.

Substituting (3.16) into the first equation in (3.13) gives

$$h_i = \frac{\beta y_i}{p_h} \tag{3.18}$$

By (3.17),

$$\bar{h}_{l_i} = \frac{\beta y_i}{p_h} \tag{3.19}$$

Substituting (3.16), (3.18), and (3.19) into (3.11) gives

$$z_i = \alpha y_i \tag{3.20}$$

To convert back to a situation in which there is no market for $\bar{h}_{l_i}$, I substitute (3.18) and (3.20) into (2.2). This implies that

$$p_h = \frac{\beta}{\beta + \delta} p \tag{3.21}$$

Substituting (3.21) into (3.18) gives

$$h_i = \frac{(\beta + \delta) y_i}{p} \tag{3.22}$$

Proposition 4. All citizens in the city must devote share $\beta + \delta$ of their budgets to housing to maximize total surplus.

Define the following:

$$\left.\begin{aligned} z_i^{**} &\equiv \alpha y_i \\ h_i^{**} &\equiv \tfrac{(\beta+\delta) y_i}{p} \end{aligned}\right\} \text{for all } i \tag{3.23}$$

Given the endowments, the total surplus (Kaldor-Hicks) criterion necessitates (3.23). Since citizens with different incomes consume different amounts of housing, the planner can redistribute income, so it is reasonable to assume that she allocates land to neighborhoods such that p equals the long-run marginal cost of housing in order to maximize total surplus. The marginal social benefit of housing under (3.23), then, equals the long-run marginal cost of housing. This leads to a corollary to Proposition 4:

Corollary. to Proposition 4: When total surplus is maximized, the external benefits of housing are "internalized".

Combining (3.17) and (3.22) implies that all of Citizen i's neighbors have the same income as she. Therefore, $l_i = m_i$, for all i, in the allocation that maximizes total surplus. There is complete separation by income into neighborhoods. I put this in the form of a proposition.

Proposition 5. *Complete separation by income into neighborhoods is necessary to maximize total surplus.*

The allocation that maximizes total surplus follows:

$$\left.\begin{aligned} z_i^{**} &\equiv \alpha y_i \\ h_i^{**} &\equiv \tfrac{(\beta+\delta) y_i}{p} \\ \bar{h}_i^{**} &\equiv \tfrac{(\beta+\delta) y_i}{p} \\ l_i &= m_i \end{aligned}\right\} \text{for all } i \tag{3.24}$$

When citizens with different incomes reside in different neighborhoods, as in (3.24), municipal governments can direct benefits to particular groups through the provision of local public goods to each neighborhood. The ability of the planner to redistribute income and, therefore, her incentive to maximize total surplus, is especially strong because that maximization leads to such an outcome.

3.3.2. Pareto efficiency of maximizing total surplus

Every allocation that maximizes total surplus is Pareto efficient because citizens as a group are always willing to pay for a Pareto improvement. If a Pareto improvement is possible, those who gain utility will pay to realize the gain. Those left indifferent need not be compensated.

If winners always fully compensate losers when allocations change, every Pareto efficient allocation maximizes total surplus. That is, an allocation that does not maximize total surplus is not Pareto efficient. If an increase in the sum of variations in consumption of the composite good is possible, and it is distributed so as to raise the utility of all citizens, then a Pareto improvement occurs. Since the derivation of (3.24) holds for an arbitrary distribution of income, by varying the distribution of income, I can trace out the set of Pareto efficient allocations when compensation is guaranteed.[14]

3.3.3. Tiebout equilibria and maximum total surplus

When total surplus is maximized, a Tiebout-like outcome obtains *within* a municipality. The Tiebout literature predicts Pareto efficient provision of local public goods, which implies internalizing the external benefits of housing and maximum total surplus. The Tiebout literature also predicts separation by income, which is necessary to maximize total surplus. Such results are derived by Hamilton (1975), Epple et al (1988), and Nechyba (1997).[15] That the allocation that maximizes total surplus differs from equilibrium in laissez-faire echoes Epple et al's result in a model of inter-municipal sorting that "households will pay more to enter a [municipality] where zoning constrains their housing consumption…than they will pay to enter a community where housing choices are unconstrained".[16] Epple et al's result adds support to the argument I make next, which is that the planner can maximize total surplus with a policy that I will refer to as "stratified zoning".

3.3.5. Implementation of the allocation that maximizes total surplus

The city planner would like a mechanism with which to implement (3.24). I define "stratified zoning" in terms of the present model as follows:

In stratified zoning,

the city planner imposes a minimum consumption of housing of $h_l^{\min} = (\beta + \delta) y_m / p$ on each dwelling in Neighborhood l, $l = m = 1. ..M.$ (3.25)

She sets consumption of housing necessary to maximize total surplus for a member of each income group as the minimum throughout one neighborhood.

I will show that (3.25) implements (3.24), even when citizens can cooperate in their choices of consumption and location.

Proposition 6. The allocation that maximizes total surplus (3.24) is a unique, coalition-proof equilibrium under stratified zoning (3.25).[17]

Proof of Proposition 6:(3.24) is a coalition-proof equilibrium: The allocation that maximizes total surplus is Pareto efficient. With the minima in place under (3.25), no set of citizens can lower the utility of any other set of citizens by permissibly moving or changing consumption. Therefore, no set of citizens can do anything to raise the utility of all its members, since that would amount to a Pareto improvement.

I relegate the proof of uniqueness to Appendix B.

Under (3.24), the planner has enhanced ability to redistribute income because citizens with different incomes both consume different amounts of housing and reside in different neighborhoods, so she can direct the welfare gains from stratified zoning to whoever puts her in power. Since (3.24) also maximizes those gains, the planner who implements it can deliver more benefits to her supporters than a planner who implements any other allocation. Regardless of the process by which the planner comes to power, if she has the authority to impose

[14] Varying incomes here would be similar to obtaining the set of efficient allocations in the Herbert-Stevens (1960) model of optimal land use by varying target utility levels.

[15] That said, unique Tieboutequilibria do not always exist under general conditions. See Conley and Wooders (1998).

[16] See page 144.

[17] According to Davis (1963; p. 384), "it can be shown that aggregate gains [from zoning] are almost always greater than aggregate losses". Conley and Konishi (2002) discuss tradeoffs between existence and uniqueness of equilibria in Tiebout-based sorting models.

minima on consumption of housing by neighborhood, (3.24), possibly with some redistribution of income, is the political equilibrium.[18] Zoning has long been ubiquitous in U.S. cities[19], and separation by income has increased rapidly in recent decades as U.S. incomes have stratified[20].

3.3.6. Linearity in income of gains from stratified zoning relative to the suburbs

Suppose that when the planner imposes minima on consumption of housing by neighborhood, members of Group m may also form a Tiebout-style, income-homogeneous suburb, and the suburban government restricts consumption of housing so as to maximize the utility of its citizens. The existence of these suburbs is also supported by "Westoff (1977) [who] proved that a sorting equilibrium exists in a model where households in each community vote to determine public goods provision and community-specific tax rates."[21] The cost of commuting from a suburb is ty_m, where t is the fraction of income that the additional suburban commuting cost represents. (I do not specify a commuting cost for citizens of the central city, though the cost of intra-municipal transportation, in both the city and the suburbs, can be thought of as part of the diminishing marginal utility of neighborhood when additional consumption of housing implicitly lowers density.) The opportunity cost of time spent commuting from the suburbs is proportional to income. Utility of a member of Group m in such a suburb is

$$U_s^m(\sigma) = \alpha \ln((1-\sigma)(1-t)y_m) + (\beta+\delta)\ln\left(\frac{\sigma}{p}(1-t)y_m\right)$$

where σ is the budgetary share spent on housing in the suburb. I differentiate with respect to σ and set the result equal to zero, which gives

$$\frac{\partial}{\partial\sigma}U_s^m(\sigma) = \frac{-\alpha}{1-\sigma} + \frac{\beta+\delta}{\sigma} = 0$$

Since the derivative decreases in σ, solving this equation maximizes utility. The solution is

$$\sigma = \beta + \delta \tag{3.26}$$

In the suburbs, then, utility for a member of Income Group m is

$$V_s^m = \alpha \ln(\alpha y_m(1-t)) + (\beta+\delta)\ln\frac{(\beta+\delta)(y_m(1-t))}{p} \tag{3.27}$$

Define CV_i^{SZ} as the gain to Citizen i from stratified zoning, before any redistribution of income, relative to an allocation in which she resides in the suburbs. z_i^c is consumption of the composite good under stratified zoning that leaves i indifferent between stratified zoning and the suburbs if

$$\alpha \ln z_i^c + (\beta+\delta)\ln\frac{(\beta+\delta)y_i}{p} = \alpha\ln(\alpha y_i(1-t)) + (\beta+\delta)\ln\frac{(\beta+\delta)(y_i(1-t))}{p}$$

Solving this for z_i^c gives

$$z_i^c = \alpha(1-t)^{\frac{1}{\alpha}}y_i$$

So

$$CV_i^{SZ} = \alpha\left(1-(1-t)^{\frac{1}{\alpha}}\right)y_i \tag{3.28}$$

Welfare gains from stratified zoning relative to life in the suburbs are linear and increasing in income, so they do not depend on the distribution of income. Nor do they depend on the relative values of β and δ, but only on the value of α. They are also increasing in t.

4. Citywide voting on zoning policy

Suppose the citizens choose the planner by majority vote. Suppose one candidate is committed to zoning in some form and the other to no zoning in the city, though the Tiebout-style suburbs exist regardless of the electoral outcome in the city. Assume that, with zoning, members of different income groups consume different amounts of housing or live in different neighborhoods, so that the candidate committed to zoning may redistribute income. Define the income of a member of income-group m after redistribution as $y_m + s_m$, where s_m is the subsidy received by a member of that group, and s_m may be positive or negative. Define the transferrable gain, before redistribution, to a member of Income Group m under zoning relative to living in a suburb as CV_m^Z.

A majority coalition will consist of $(M+1)/2$ income groups, where M is taken to be an odd number. If Group nm is a non-member of the majority coalition, then the subsidy received by a member of Group nm under zoning must be $s_{nm} = -CV_{nm}^Z$: The coalition will tax non-members to the point of indifference between remaining in the city and commuting from a suburb. Conversely, if a suburb exists for Income Group m in long-run equilibrium, then $s_m = -CV_m^Z$. Otherwise, citizens in that group would move. We would expect suburbs, then, to be formed by the groups excluded from the central city's governing coalition. If Group mem is a member of the governing coalition, then the subsidy s_{mem} received by a member of Group mem must follow

$$-(CV_{mem}^Z - CV_{mem}^{NZC}) < s_{mem} < \min_{nm}[-(CV_{nm}^Z - CV_{nm}^{NZC}) + CV_{nm}^Z - CV_{mem}^Z] \tag{4.1}$$

Where CV_m^{NZC} is the gain to a member of Income Group m from a move to an unzoned central city from an income-homogeneous suburb. The left hand side of (4.1) is the cost to a member of Group mem of zoning. She will vote to zone only if she is made more than whole for her loss, or all of her gains are not taken away. The right hand side of (4.1) is the cost per vote to the coalition of replacing Income-Group mem with a different income-group. (This cost may be negative.) The coalition will not pay its members more than it costs to bring members of a different group into the coalition. Cancelling terms in (4.1), it must be the case that

$$CV_{mem}^{NZC} < \min_{nm}[CV_{nm}^{NZC}] \tag{4.2}$$

for every member mem in the coalition. (4.2) will hold for all mem in the coalition if

$$\max_{mem}[CV_{mem}^{NZC}] < \min_{nm}[CV_{nm}^{NCZ}] \tag{4.3}$$

This leads to a proposition.

Proposition 7. *When citizens choose the planner by majority vote, the governing coalition will consist of the* $(M+1)/2$ *income groups m for whom* CV_m^{NZC} *are the lowest.*

The city planner will zone and distribute the benefits to those who have the most to gain, or the least to lose, when moving from a city with no zoning to an income-homogeneous suburb. The coalition's political strength in the city depends on how good an alternative it has in the form of suburbs. This is analogous to having a stronger negotiating position when one has a higher reservation wage. Those who can most easily "walk away" to the suburbs get the best deal from zoning in the city.

The majority coalition will consist of the half of the citizens for whom the function $CV^{NZC}(y, t)$ assumes the lowest values. The compensating variation is based on the following equation:

[18] Davis (1963; p. 385) argues that "those who favor zoning will almost always outnumber those who oppose it in any metropolitan area", where zoning is considered to eliminate external diseconomies.

[19] See Manvel (1967).

[20] See Booza et al.

[21] See Kuminoff, Smith, and Timmins (2013), p. 1016.

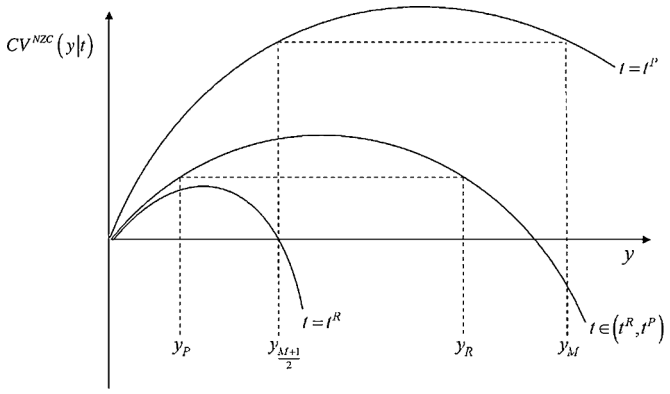

Fig. 1. No zoning in the city relative to suburbs and implied majority coalitions.

$$\alpha \ln z^c + \beta \ln \frac{\beta y}{(\alpha+\beta)p} + \delta \ln \bar{h} = \alpha \ln(\alpha y(1-t)) + (\beta+\delta)\ln \frac{(\beta+\delta)(y(1-t))}{p}$$

Solving for z^c gives

$$z^c = K_{NZC}(1-t)^{1/\alpha} y^{(\alpha+\delta)/\alpha}$$

where K_{NZC} is positive and constant with respect to y. The gain to the citizen from residing in the unzoned city is

$$CV^{NZC}(y,t) = \frac{\alpha y}{\alpha+\beta} - K_{NZC}(1-t)^{1/\alpha} y^{(\alpha+\delta)/\alpha} \tag{4.4}$$

Differentiating twice with respect to y gives

$$\frac{\partial CV_i^{NZC}(y,t)}{\partial y_i} = \frac{\alpha}{\alpha+\beta} - \frac{\alpha+\delta}{\alpha} K_{NZC}(1-t)^{1/\alpha} y_i^{\frac{\delta}{\alpha}}$$
$$\frac{\partial^2 CV_i^{NZC}(y,t)}{\partial y_i^2} = -\frac{\delta}{\alpha}\frac{\alpha+\beta}{\alpha} K_{NZC}(1-t)^{1/\alpha} y_i^{\frac{\delta-\alpha}{\alpha}} < 0 \tag{4.5}$$

Denote the value of this function for a given value of $t$as $CV^{NZC}(y|t)$. By (4.4) and (4.5), $CV^{NZC}(y|t)$ goes through the origin, is increasing when y is low, and is concave. If $t \in [0, 1)$, it is strictly concave and, at some level of income, crosses the horizontal axis because richer citizens are better off in the suburbs, where the externalities of housing are internalized, than in the city without zoning, where they are not, and where poorer citizens generate lower external benefits. If t is 1, then $CV^{NZC}(y|t)$ is increasing and linear in y. Note also that, for a given value of y,

$$\frac{\partial CV^{NZC}(t|y)}{\partial t} = \frac{1}{\alpha} K_{NZC}(1-t)^{\frac{1-\alpha}{\alpha}} y^{\frac{\alpha+\delta}{\alpha}} > 0 \tag{4.6}$$

Fig. 1 shows $CV^{NZC}(y|t)$ for three values of t.

If $t \leq t^R$, then the majority coalition will consist of those citizens whose incomes equal or exceed median income, which is $y_{(M+1)/2}$. If $t \geq t^P$, then the majority coalition will consist of those citizens whose incomes equal or are less than median income. If $t \in (t^R, t^P)$, then the majority coalition will consist of those Citizens i such that $y_i \notin (y_P, y_R)$, where these citizens make up just over half the electorate. The majority coalition will not be formed only from the interior of the distribution of income.

Commuting costs increase as cities grow.[22] Small cities will redistribute gains from zoning and other income upwards until the lower middle class and the poor are indifferent between living in town and on the outskirts, and the outskirts will be poorer than the city. Medium-size cities will redistribute gains from zoning and other income from the middle to the extremes until those in the middle are indifferent between the central city and the suburbs. Suburbs will have a corresponding middle-income character, and may be poorer or richer than the central city. Large cities will redistribute gains from zoning and other income downward until the upper middle class and the rich are indifferent between the central city and the suburbs, and the suburbs will be relatively well-to-do.[23]Schnore and Varley (1955) include Fig. 2 on page 413, where "centers" refers to cities proper, "rings" refers to suburbia, "large" central cities had at least 500,000 inhabitants, "middle-size" central cities had between 100,000 and 500,000 inhabitants, and "small" central cities had 50,000 to 100,000 inhabitants. In practice, poor people may self-select to larger cities, where they have a good chance of being part of a governing coalition, and richer people may self-select to smaller cities. In the 30 most populous cities proper in the U.S. in 2005, 19 percent of people lived below the poverty line. In the next 462 most populous, 15 percent lived in poverty.[24]

TABLE 12. CITY SIZE AND MEDIAN FAMILY INCOME IN CENTERS AND RINGS, 1950, IN PERCENTAGES

Median Income	Large Cities	Middle-Size Cities	Small Cities
Ring higher	95	37	19
City higher	5	63	81
Total	100	100	100

Fig. 2. Table from Schnore and Varley.

5. Distributional consequences

Table 1 gives compensating variations in dollars under stratified zoning, a uniform minimum, and laissez-faire, calculated in accordance with (3.5), based on 2015 CES data[25], for a city in which the additional commute associated with living in the suburbs is 40 minutes per day, round-trip.

Table 1 also shows income and compensating variations relative to the suburbs under stratified zoning and a uniform minimum *after* redistribution. I assume that members of the governing coalition divide the income that is redistributed to them equally. If $\beta = \delta = 1/10$, then the governing coalition consists of Quintiles 3, 4, and 5. If $\beta = 2/15$ and $\delta = 1/15$, the coalition consists of Quintiles 1, 2, and 5. If $\beta = 3/20$ and $\delta = 1/20$, it consists of Quintiles 1, 2, and 3.

I use annual expenditure as a measure of permanent income. The income share spent on housing under stratified zoning is $\beta + \delta$. U.S. citizens spend about a fifth of their incomes on housing, so I assume that $\beta + \delta = 1/5$ in each case.[26] I set the price of housing such that a citizen with mean income, $\bar{y}$, derives a third of her utility under stratified zoning from housing and neighborhood:

$$\alpha \ln \alpha \bar{y} = 2(\beta+\delta)\ln \frac{(\beta+\delta)\bar{y}}{p}$$

With $\beta + \delta = 1/5$and $\bar{y} = 55,925$, $p = 559 \times 10^{-8}$.

5.1. Laissez-faire versus stratified zoning

In Table 1, stratified zoning (before any redistribution) is an improvement on laissez-faire for an upper income majority in the first

[22] SeeU.S. Bureau of the Census (2004)http://www.census.gov/Press-Release/www/releases/archives/american_community_survey_acs/001695.html

[23] Fisch (1975) uses a clubs model to show that a majority, upper-income coalition can form to exclude a lower-income minority from the *outer ring* (emphasis added) of a monocentric city.

[24] See U.S. Bureau of the Census (2008; Table 686).

[25] See U.S. Bureau of Labor Statistics, Consumer Expenditure Survey.

[26] In the case where $\beta = \delta$, the citizen is indifferent between living in a trailer in a neighborhood of mansions and in a mansion in a trailer park.

Table 1
Compensating Variations (CV) Relative to Suburb Under Three Land-Use Policies, 2015 Distribution of Income

Annual Income by Quintile U.S. 2015	Policy Stratified Zoning*		Uniform Minimum				Laissez-Faire		
				After Redistribution					
	CV Before Redistribution	CV After Redistribution	CV Before Redistribution	CV	Spending on Housing	Housing's Share	CV	Spending on Housing	Housing's Share
$\beta = \frac{1}{10}, \delta = \frac{1}{10}$					Minimum = $2,893 6492.722				
$ 110,508	$9,111	$10,747	($1,710)	$652	$12,541	11.1%	($1,780)	$12,279	11.1%
$ 63,671	$5,249	$6,886	$2,850	$5,212	$7,337	11.1%	$2,812	$7,075	11.1%
$ 45,912	$3,785	$5,421	$3,607	$5,969	$5,364	11.1%	$3,581	$5,101	11.1%
$ 35,063	$2,891	$0	$3,696	$0	$3,485	11.1%	$3,677	$3,896	11.1%
$ 24,470	$2,017	$0	$3,390	$0	$2,893	13.7%	$3,410	$2,719	11.1%
$\beta = \frac{2}{15}, \delta = \frac{1}{15}$					Minimum = $5,009 5457.4939				
$ 110,508	$9,111	$12,123	$3,712	$6,614	$16,201	14.3%	$3,430	$15,787	14.3%
$ 63,671	$5,249	$0	$4,494	$0	$8,454	14.3%	$4,338	$9,096	14.3%
$ 45,912	$3,785	$0	$4,211	$0	$5,957	14.3%	$4,102	$6,559	14.3%
$ 35,063	$2,891	$5,902	$3,812	$6,714	$5,424	14.3%	$3,731	$5,009	14.3%
$ 24,470	$2,017	$5,029	$2,722	$5,624	$5,009	18.3%	$3,146	$3,496	14.3%
$\beta = \frac{3}{20}, \delta = \frac{1}{20}$					Minimum = $4,574 3365				
$ 110,508	$9,111	$0	$5,366	$0	$16,601	15.8%	$5,278	$17,449	15.8%
$ 63,671	$5,249	$0	$4,803	$0	$9,295	15.8%	$4,754	$10,053	15.8%
$ 45,912	$3,785	$8,572	$4,176	$7,565	$7,784	15.8%	$4,141	$7,249	15.8%
$ 35,063	$2,891	$7,678	$3,629	$7,019	$6,071	15.8%	$3,603	$5,536	15.8%
$ 24,470	$2,017	$6,804	$2,774	$6,164	$4,574	16.4%	$2,917	$3,864	15.8%

Price of housing = $.559E-05. Additional suburban commute costs = 8.33% of income.
* Housing's share of income under stratified zoning is 20% for all income groups.

case, where δ is high. In the second and third cases, where δ is lower, a lower income majority prefers laissez-faire. (Subtract "CV" under laissez-faire from "CV Before Redistribution" under stratified zoning.)

From (3.3), (3.5), and (3.24), the compensating variation for Citizen i in a change from laissez-faire to stratified zoning is

$$CV_{i\ LF\to SZ} = \alpha y_i - \frac{\alpha}{\alpha+\beta}\left(\frac{\beta}{(\alpha+\beta)(\beta+\delta)}\right)^{\frac{\beta+\delta}{\alpha}}\left(\frac{1}{N}\right)^{\frac{\delta}{\alpha}} y_i^{\frac{\alpha-\delta}{\alpha}} \tag{5.1}$$

In Table 1, willingness to pay for a change from laissez-faire to stratified zoning is monotonically increasing in income. In going from laissez-faire to stratified zoning, every citizen gains from the increased consumption of housing by the citizens in her income group.[27] Poorer citizens lose utility because they are separated from the rich, who consume more housing than they. Richer citizens gain utility because they experience a higher externality generated by only rich citizens. While both increased consumption of housing and separation raise utility for richer citizens, they work in opposite directions for poorer citizens. Either effect may predominate for poorer citizens, depending on parameter values and on the distribution of income.

Define V_m^{LF} as indirect utility of a citizen in Income Group m in laissez-faire and V_m^{SZ} as that under stratified zoning, before any redistribution.

$$V_m^{LF} = \alpha \ln \frac{\alpha y_m}{\alpha+\beta} + \beta \ln \frac{\beta y_m}{(\alpha+\beta)p} + \delta \ln \frac{\beta}{(\alpha+\beta)Np}, \text{and}$$

$$V_m^{SZ} = \alpha \ln \alpha y_m + (\beta+\delta)\ln \frac{(\beta+\delta)y_m}{p} \tag{5.2}$$

Stratified zoning, before any redistribution, is an improvement on laissez-faire for members of income group m if and only if $V_m^{LF} \le V_m^{SZ}$. That is, after some algebra, if and only if

$$Ny_m \ge \left(\frac{1}{1-\delta}\left(\frac{\beta}{\beta+\delta}\right)^{\beta+\delta}\right)^{\frac{1}{\delta}} \tag{5.3}$$

The left hand side of (5.3) is the income of a citizen in Income Group m as a fraction of average income in the city, which is $1/N$. I prove in Appendix B that the right-hand side is less than 1. If a citizen's income is above the mean, then she will benefit from a change from laissez-faire to stratified zoning. When the mean is low relative to the median, income is less concentrated at the top. I, therefore, state the following proposition:

Proposition 8. *The more equally income is distributed, the greater the number of citizens who gain in a move from laissez-faire to stratified zoning.*

If $\beta = \delta = 1/10$, then the right hand side of (5.3) is 0.72. If $\beta = 2/15$ and $\delta = 1/15$, the right hand side of (5.3) is 0.83. If $\beta = 3/20$ and $\delta = 1/20$, the right hand side of (5.3) is 0.88. From the 2015 CES data, one would not expect stratified zoning (before any redistribution) to be an improvement on laissez-faire for the poor in the United States, where total annual expenditure in the bottom quintile is less than half of average expenditure over all households.

The right hand side of (5.3) depends only on citizen preferences, and it is decreasing in δ, which also follows from the proof in Appendix B. Therefore, so is the number of citizens who are better off with stratified zoning than with laissez-faire.

Proposition 9. *The greater the external benefits of housing, the greater the number of citizens who benefit from stratified zoning, relative to laissez-faire.*

Average expenditure in the third quintile was $45,912 in 2015, and average expenditure over the five quintiles was $55,978, so the left hand side of (5.3) would be 0.82. With $\alpha = 4/5$, and setting the right

[27] This is similar to Schall's (1976) result that greater improvements to some or all plots of land characterize a cooperative, as opposed to competitive, equilibrium when improvements generate positive externalities. See his Appendix 2, Proposition B.

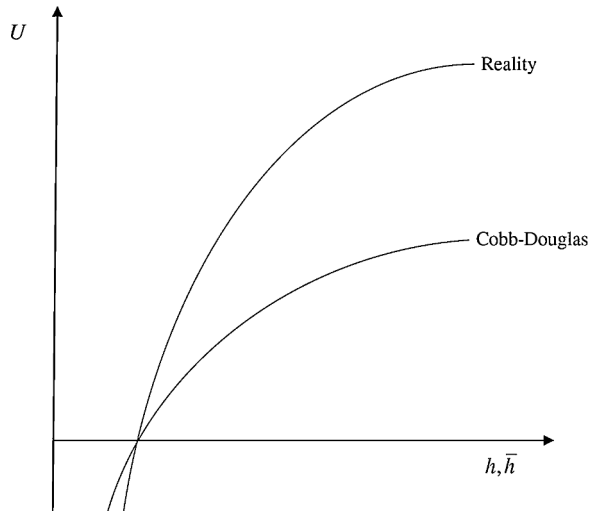

Fig. 3. Utility of housing and neighborhood.

hand side equal to 0.82, I find that most citizens would benefit if $\delta > 0.07$.

In both laissez-faire and stratified zoning, demand for housing is unit elastic, with respect to both price and income, given the Cobb-Douglas utility function. Empirically, this is excessive. Hanushek and Quigley (1980) estimate price elasticity to be around -0.4. Eq. (5.3) does not involve p, the price of housing, which always equals its long-run marginal cost. If this cost changes, and with it, the price of housing, it would not alter the conclusions about ordinal welfare in Propositions 8 and 9 and the surrounding text. Hansen, Formby, and Smith (1996) estimate housing demand to be income-inelastic, and increasingly so as income falls. This is confirmed by falling shares of income spent on housing in the lower half of the income distribution in the CES data.

If stratified zoning is the norm, then observed expenditures on housing are really expenditures on the bundle of housing and neighborhood. Assume, then, that $\bar{h} = h$. Assume, also, that housing's share in reality declines with income because the true utility function is more concave than Cobb-Douglas, as shown in Fig. 3.

Let us measure the concavity of a utility function using the coefficient of relative risk-aversion. For our Cobb-Douglas utility function, with the budget constraint, (2.2), substituted in, and, again, $\bar{h} = h$, this is

$$RRA = \frac{\frac{\beta+\delta}{h} + \frac{\alpha h p^2}{(y-ph)^2}}{\frac{\beta+\delta}{h} - \frac{\alpha p}{y-ph}} \tag{5.4}$$

(5.4) is not defined under (3.24) because of the first order condition for maximizing total surplus. Let us assume, then, that the citizen resides in an income-homogeneous neighborhood where all residents consume $h = \bar{h} = \beta y/(\alpha+\beta)p$, the unconstrained individual optimum. It can be shown that

$$RRA(h = \bar{h} = \beta y/(\alpha+\beta)p) = \frac{\beta + \delta + \frac{\beta^2}{\alpha}}{\delta} \tag{5.5}$$

Hall (1988) estimates relative aversion to risk to total consumption to be between two and four (when it is constant with respect to consumption). When $\beta = \delta = 5/58$, $RRA = 2.10$ in (5.5), the right-hand-side of (5.3) equals 0.71, implying that those with incomes below 71% of the mean lose from stratified zoning, and housing's share under (3.24), stratified zoning, is 0.17. 0.17 is mean spending on shelter *minus* one standard error over mean total expenditure *plus* one standard error for the *top* quintile in the 2015 CES data. I use this as a soft lower bound on housing's share.

When $\beta = \delta = 5/39$, $RRA = 2.17$ in (5.5), the right-hand-side of (5.3) equals 0.73, implying that those with incomes below 73% of the mean lose from stratified zoning, and housing's share under (3.24), stratified zoning, is 0.26. 0.26 is mean spending on shelter *plus* one standard error over mean total expenditure *minus* one standard error for the *bottom* quintile in the 2015 CES data. I use this as a soft upper bound on housing's share. The implication of this is that using the Cobb-Douglas utility function may lead to counting those with incomes between 71% and 73% of the mean as winners from stratified zoning, when they are actually losers, before any side-payments are made.

Differentiating (5.1) twice,

$$\frac{\partial \underset{LF\to SZ}{CV_i}}{\partial y_i} = \alpha - \frac{\alpha-\delta}{\alpha+\beta}\left(\frac{\beta}{(\alpha+\beta)(\beta+\delta)}\right)^{\frac{\beta+\delta}{\alpha}}\left(\frac{1}{N}\right)^{\frac{\delta}{\alpha}} y_i^{\frac{-\delta}{\alpha}}$$

$$\frac{\partial^2 \underset{LF\to SZ}{CV_i}}{\partial y_i^2} = \frac{\delta}{\alpha}\frac{\alpha-\delta}{\alpha+\beta}\left(\frac{\beta}{(\alpha+\beta)(\beta+\delta)}\right)^{\frac{\beta+\delta}{\alpha}}\left(\frac{1}{N}\right)^{\frac{\delta}{\alpha}} y_i^{\frac{-\delta}{\alpha}-1} > 0$$

$\underset{LF\to SZ}{CV_i}$ is convex in income. I put this in the form of a proposition.

Proposition 10. *Citizen i's willingness to pay for a change from laissez-faire to stratified zoning is strictly convex in i's income.*

Strict convexity in income of willingness to pay for stratified zoning results from the external benefits of housing. From (5.1), $\underset{LF\to SZ}{CV_i}$ is linear in income, y_i, only when $\delta = 0$, and strictly convex otherwise. My willingness to pay to live in a rich neighborhood reflects both my willingness to pay for my own housing and my willingness to pay to live in a neighborhood where others consume a lot of housing.

Because Citizen i's willingness to pay for stratified zoning is convex in income, the gains in total surplus, $\sum_i \underset{LF\to SZ}{CV_i}$, afforded by stratified zoning relative to laissez-faire increase as the distribution of income becomes less equally distributed.

Fig. 4 illustrates the phenomenon. Citizens i and j have the same income in Case A, $y_{i,j}^A$, but different incomes in Case B, y_i^B and y_j^B. The sum of incomes over both citizens is the same in both cases; that is, $y_i^B + y_j^B = 2y_{i,j}^A$. Both are indifferent to a change to stratified zoning in Case A: $\underset{LF\to SZ}{CV_{i,j}^A} = 0$. Because their willingness to pay for stratified zoning is convex in y, the sum of their willingnesses to pay for stratified zoning is positive in Case B: $\underset{LF\to SZ}{CV_i^B} + \underset{LF\to SZ}{CV_j^B} > 0$.

Table 2 is similar to Table 1, but the distribution of income is shaped like the U.S. distribution of income in 1972-73, although it is scaled up so that average income is the same as in 2015.[28] The income distribution in 2015 is noticeably less equal than in 1972-73. When $\delta = 1/15$, the middle quintile gains from stratified zoning relative to laissez-faire in 1972-73, but loses in 2015, reflecting how a less equal distribution of income shrinks the set of winners (before redistribution) from stratified zoning, by Proposition 8.[29] However, the sums of the compensating variations relative to both laissez-faire and a uniform minimum are higher in 2015, reflecting the convexity in income of gains together with the less equal distribution of income in 2015. (Subtract "CV" under laissez-faire or a uniform minimum from "CV Before Redistribution" under stratified zoning.)

5.2. Laissez-faire versus a uniform minimum

In going from laissez-faire to a uniform minimum, upper income citizens not bound by the minimum experience an increase in utility. They continue to consume their privately optimal bundle of the composite good and housing according to (3.1), as in laissez-faire, and experience higher external benefits due to increased consumption of

[28] U.S. Bureau of Labor Statistics (1972-73)

[29] The case where $\delta = 1/10$ is not consistent with Proposition VIII because the uppermost quintile prefers the suburbs to the central city in laissez-faire in 2000. When the city zones, the rich move in, bringing external benefits.

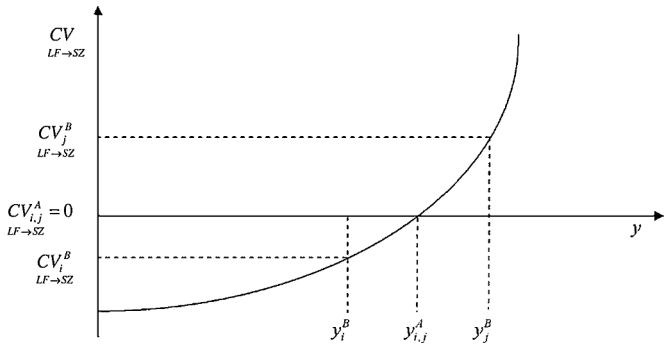

Fig. 4. Gains from stratified zoning relative to laissez-faire by income.

housing by poorer citizens. Among those citizens bound by the minimum, citizens with relatively high incomes gain because the constraint does not move them far from their individual optima, and they experience greater external benefits. However, a move from laissez-faire to a uniform minimum can hurt the poor. If the minimum is low, it binds only the poor, moving them away from their individual optima, but doing little to increase the external benefits they experience from their neighbors' consumption of housing, since the vast majority of their neighbors, who are of all income levels, are not bound by the minimum. If the minimum is high, it raises external benefits, but it requires the poor to consume far more housing than is optimal for them individually. In practice, if they are liquidity-constrained and consume less than their permanent income, or are simply very poor, it may render them homeless. Shanties are common in some countries, but not generally allowed in the U.S.A.

In Table 1, I set the uniform minimum either at the level that maximizes total surplus or at the individually optimal consumption of housing for a member of Income Group 2 after redistribution, whichever is less. Before redistribution, the poor are worse off under a positive uniform minimum than in laissez-faire, and a majority of relatively rich citizens are better off.

5.3. A uniform minimum versus stratified zoning

Willingness to pay for a change from a uniform minimum to stratified zoning is $CV_i^{LF\to SZ} - CV_i^{UM}$.

By Proposition 10, $CV_i^{LF\to SZ}$ is convex in income. By (3.8) and (3.10), CV_i^{UM} is concave if the minimum binds, and convex if the minimum is not binding. For those citizens bound by a uniform minimum, willingness to pay for a change to stratified zoning from the uniform minimum is convex in income. If the uniform minimum is less than the minimum under stratified zoning, the additional restriction is much less painful for those with relatively high incomes. If the uniform minimum exceeds the minimum under stratified zoning, then those with relatively high incomes become less constrained. In Table 1, willingness to pay for a change from a uniform minimum to stratified zoning is convex in income for all values of parameters shown.

The welfare gains from a positive uniform minimum are quite modest compared to those from stratified zoning, but only because of the unequal distribution of income. If all citizens had the same income, then stratified zoning and a uniform minimum that maximized total surplus would be the same policy. As incomes diverge, a uniform minimum set to maximize total surplus becomes an increasingly less accurate approximation to stratified zoning. The shares of expenditures devoted to shelter in the Consumer Expenditure Survey data were higher in 2015 than in 1972-73 for all quintiles.

5.4. Distributing the gain in total surplus from stratified zoning

Proposition 7 and Eqs. (4.5) and (4.6) imply that central cities who decide zoning policy and distribute the associated welfare gains by

Table 2
Compensating Variations (CV) Relative to Suburb Under Three Land-Use Policies, 1972-73 Distribution of Income

Annual Income by Quintile U.S. 2015	Policy Stratified Zoning*		Uniform Minimum				Laissez-Faire		
				After Redistribution					
	CV Before Redistribution	CV After Redistribution	CV Before Redistribution	CV	Spending on Housing	Housing's Share	CV	Spending on Housing	Housing's Share
$\beta = \frac{1}{10}, \delta = \frac{1}{10}$					Minimum = $4.310 6492.722				
$ 95,480	$7,872	$9,598	$571	$2,831	$10,860	11.1%	$27	$10,609	11.1%
$ 67,927	$5,600	$7,326	$2,905	$5,165	$7,798	11.1%	$2,534	$7,547	11.1%
$ 53,412	$4,404	$6,130	$3,622	$5,882	$6,186	11.1%	$3,339	$5,935	11.1%
$ 38,788	$3,198	$0	$3,879	$0	$4,310	12.3%	$3,681	$4,310	11.1%
$ 24,017	$1,980	$0	$2,900	$0	$4,310	20.4%	$3,388	$2,669	11.1%
$\beta = \frac{2}{15}, \delta = \frac{1}{15}$					Minimum = $4.886 5457.4939				
$ 95,480	$7,872	$11,207	$4,150	$7,109	$14,063	14.3%	$3,918	$13,640	14.3%
$ 67,927	$5,600	$0	$4,499	$0	$9,061	14.3%	$4,338	$9,704	14.3%
$ 53,412	$4,404	$0	$4,375	$0	$7,005	14.3%	$4,252	$7,630	14.3%
$ 38,788	$3,198	$6,533	$3,969	$6,927	$5,964	14.3%	$3,881	$5,541	14.3%
$ 24,017	$1,980	$5,315	$2,709	$5,667	$4,886	18.1%	$3,115	$3,431	14.3%
$\beta = \frac{3}{20}, \delta = \frac{1}{20}$					Minimum = $4,482 3365				
$ 95,480	$7,872	$0	$5,323	$0	$14,235	15.8%	$5,250	$15,076	15.8%
$ 67,927	$5,600	$0	$4,912	$0	$9,950	15.8%	$4,861	$10,725	15.8%
$ 53,412	$4,404	$8,894	$4,475	$7,887	$8,972	15.8%	$4,436	$8,433	15.8%
$ 38,788	$3,198	$7,689	$3,832	$7,244	$6,663	15.8%	$3,804	$6,124	15.8%
$ 24,017	$1,980	$6,471	$2,744	$6,156	$4,482	16.3%	$2,883	$3,792	15.8%

Price of housing = $.559E-05. Additional suburban commute costs = 8.33% of income.
* Housing's share of income under stratified zoning is 20% for all income groups.

Table 3
Governing Coalitions by Commuting Time from Suburbs

	Additional Suburban Commuting Time minutes/day round-trip	Governing Coalition (Quintiles of Income)				
		1,2,3	1,2,5	2,3,4	1,4,5	3,4,5
$\beta = \frac{1}{10}, \delta = \frac{1}{10}$	0					X
	10					X
	20					X
	30					X
	40				X	
	50				X	
	60		X			
	70		X			
	80	X				
	90	X				
$\beta = \frac{2}{15}, \delta = \frac{1}{15}$	0					X
	10					X
	20					X
	30				X	
	40		X			
	50	X				
	60	X				
	70	X				
	80	X				
	90	X				
$\beta = \frac{3}{20}, \delta = \frac{1}{20}$	0					X
	10					X
	20				X	
	30		X			
	40	X				
	50	X				
	60	X				
	70	X				
	80	X				
	90	X				

majority vote will not be governed by a coalition of citizens in the interior of the distribution of income. This is illustrated in Table 3. Table 3 indicates which quintiles of income in the 2015 CES data form a governing coalition in accordance with (4.3), under different suburban commutes. The mean daily trip to work in the U.S.A. in 2006 was 50 minutes round-trip.[30] Table 3 shows the commuting time associated with moving from the central city to the suburbs, ranging from zero to 90 minutes round-trip. I assume that citizens work 2000 hours per year. The governing coalition shifts from upper- to lower-income as the city grows and commuting time increases. Because the rich place a higher monetary value on their time than the poor, the monetized attractiveness of the suburbs deteriorates faster for them than for the poor as commuting time increases. Since a coalition's political strength in the city depends on the attractiveness of its suburban alternative, the rich become politically weaker as commuting time increases. In the middle of this process, the governing coalition is formed by the extremes of the distribution of income, and not by the middle. The governing coalition, be it rich, extremes, or poor, taxes non-members to the point of indifference between living in the city and in the suburbs. The larger the city, the greater the commuting cost, the poorer the governing coalition, and the greater the taxes on upper-income citizens. The smaller the city, the lower the commuting cost, the richer the governing coalition, and the greater the taxes on lower-income citizens.

In 1962, the citizens of Houston, Texas rejected a proposal to introduce zoning. Upper-income Anglos voted 65 percent in favor of the proposal. In 1993, voters again rejected a zoning proposal. This time, the vote was close, but upper-income Anglos voted 66 percent *against* the proposal, and turnout was highest among these voters.[31] The population of the city proper increased 74% between 1960 and 1990.[32] The Houston MSA grew 179% at the same time.[33] McDonald (p. 138) writes "Under Texas law Houston has extraterritorial rights over areas within five miles of its city limits. In this area new municipalities cannot be created and existing municipalities cannot annex territory without the approval of Houston. Houston has taken advantage of this system to prevent the creation of separate suburban municipalities. Only a few small municipalities in the metropolitan area have zoning ordinances." Thus, not only had Houston become much larger by 1993, but the cost of commuting from an income-homogeneous suburb must have been high, even for a large city. A coalition of poor and middle class citizens would have been closer to viable in 1993, prompting the lower middle class to support the zoning proposal and upper-income Anglos to vote against it.

When zoning laws are decided by majority vote, non-members nm of the governing coalition, (those citizens for whom CV_m^{NZC}, by

[30] See U.S. Bureau of the Census (2006;GCT0801)

[31] See McDonald, p. 137.The support for the proposal in 1993 came mainly from the middle class, with the lower-income people voting against. This is different from anything seen in Table 3, but not necessarily inconsistent. Low-income voters turned out in very low numbers, so, although they did not support the measure, they did not oppose it strongly. Also, the distribution of income in Houston is much more concentrated in the middle than it is nationally. The coefficient of variation of income in Houston is 0.35, and 0.55 nationally. See Freitag and 2000 CES Table 45, respectively.

[32] See http://physics.bu.edu/~redner/projects/population/cities/houston.html (Boston University Department of Physics).

[33] See U.S. Bureau of the Census (1961; Table 10) and U.S. Bureau of the Census (2006; Table 20).

Proposition 7, is the highest) will net

$$(CV_{nm}^{Z} - CV_{nm}^{NZC}) - CV_{nm}^{Z} = -CV_{nm}^{LF}$$

after redistribution in a change from laissez-faire to stratified zoning, because $s_{nm} = -CV_{nm}^{Z}$. Stratified zoning, then, after redistribution, is a Pareto improvement on laissez-faire if and only if $\max_{m}[CV_{m}^{NZC}] < 0$. This implies a proposition:

Proposition 11. *When zoning policy and distribution of the associated gains are decided by majority vote, stratified zoning, after redistribution, is a Pareto improvement on a city without zoning if and only if all citizens are better off in the suburbs than they are in a city without zoning. That is, if and only if* $CV_{m}^{NZC} < 0$ *for all m.*

All citizens are at least as well off in the city under stratified zoning before redistribution as they are in the suburbs because of transportation costs: $CV_{m}^{SZ} \geq 0$ for all m. Therefore, if $CV_{m}^{SZ} - CV_{m}^{NZC} < 0$ for some m, then $CV_{m}^{NZC} > 0$ as well, and, by Proposition 11, stratified zoning after redistribution is not a Pareto improvement on laissez-faire. This is the final proposition.

Proposition 12. *When zoning policy is decided by majority vote, stratified zoning after redistribution is a Pareto improvement on a city without zoning only if it is a Pareto improvement before redistribution.*

Even if it is a Pareto improvement before redistribution, it may not be after.

Define CV_{m}^{LF} as the gain to a member of Income Group m in a change from an allocation in which she resides in an income-homogeneous suburb to laissez-faire, which differs from no zoning in the city in that suburbs do not exist. The arguments above supporting Propositions 11 and 12 also hold if CV_{m}^{LF} is substituted for CV_{m}^{NZC}. In no case in Table 1 is stratified zoning a Pareto improvement on laissez-faire before redistribution, so it is not a Pareto improvement on laissez-faire after redistribution, either.

Therefore, it is extremely difficult to justify the maximization of total surplus under stratified zoning on the grounds that they make possible a Pareto improvement on no zoning, be it either no zoning in the central city or in the metropolitan area. Table 3 indicates, rather, that, in small cities, the poor and the lower middle class will be the losers, in mid-size cities, the middle class, and, in large cities, the upper middle class and the rich. In small cities, the losses of the poor after redistribution will be greater than their losses before redistribution. In large cities, the gains to the rich from stratified zoning will be more than negated.[34] This may be why the upper income citizens of Houston voted for zoning in 1962 and against it in 1993, when the city was much larger.

In practice, subjecting all new and existing restrictions on the supply and use of residential land to the ongoing and unanimous approval of lawmakers from all different parts of a city could improve things. Since stratified zoning maximizes total surplus in a unique, coalition-proof equilibrium, facilitates redistribution, and, therefore, is a potential Pareto improvement over any other policy, requiring unanimous approval should not present an unrealistic hurdle. A handful of city councilors should be able to find a Pareto improvement when one exists to be found. Since everyone *can* gain from stratified zoning, it is not obstructionist to require that everyone *does* gain. Stratified zoning would always generate enough benefits, to be distributed to districts in the form of lower taxes or local public goods, to attract the support of every member of a city council. Failing to agree to a zoning proposal would not be an equilibrium because any councilor holding up an agreement could do better by her constituents to change her strategy. Since cities tend to grow rather than shrink, it might behoove the well-to-do who govern small cities to subject zoning laws to ongoing and unanimous approval while they still have political control, before the poor take control and tax them into the suburbs.

Instituting such a no-losers policy would also slow down or reverse suburbanization and urban sprawl. There would be no minority of income-groups in the city who were taxed to the point of indifference between residing in the city and in the suburbs. When the gains from the Tiebout-like outcome in the central city are shared by all citizens, no one needs to escape to the suburbs in order to experience such gains. Sprawl may reflect efforts by well-off city dwellers to maintain their political strength as cities grow. When suburbanites cluster around arterials, they are minimizing the cost of commuting to the city which, as illustrated in Table 3, helps maintain the political control of well-off city dwellers. Subjecting zoning laws to unanimous approval would remove the incentive for well-off city dwellers to fund ever-expanding routes from the suburbs to downtown. Slowing or reversing suburbanization and sprawl would reduce the amount of fuel that metropolitan areas consume for transportation.

6. Explaining accelerated sorting in U.S. central cities

The Brookings report (Booza et al., 2006) describes how stratification by income among U.S. neighborhoods has been proceeding faster than stratification of income itself, especially in central cities. While the share of families categorized as “middle income” in central cities shrank from 26 percent in 1970 to 18 percent in 2000, the share of neighborhoods categorized as middle income shrank from 45 percent in 1970 to 23 percent in 2000. The report states that “families increasingly sorted into neighborhoods that reflected their own income profiles” and that “the magnitude of the neighborhood income shift exceeded that of the family income shift.” The gains in total surplus from stratified zoning increase as the distribution of income becomes less equal, by Proposition 10 (See Fig. 4), so the incentive for governments to foster separation by income was greater in 2000 than in 1970. The contemporaneous increase in average income must also have raised the gains from stratified zoning in absolute terms.

The report goes on to say that MSA's with high growth at the upper end of the income distribution had greater than proportional increases in neighborhood-level separation by income. This can be explained by the fact that it is the rich who gain the most from separation by income. Areas with more growth at the high end of the income distribution would see greater than proportional increases in willingness to pay for a policy that led to separation by income.

The process of increased neighborhood stratification by income continued between 2000 and 2012. Sampson (2016) reports census data showing that about 80 percent of neighborhoods with median income in the bottom and top quintiles remained in those quintiles between 2000 and 2012, but only half of neighborhoods with median income in the third quintile remained there in 2012, with 23% entering the second quintile and 24% entering the fourth. See Sampson's Table 1, Panel B, p. 269.

7. Conclusion

A zoning policy that internalizes the external benefits of housing and separates citizens into neighborhoods by income maximizes total surplus, the objective function of a Kaldor-Hicks welfare criterion. The total surplus maximizing allocation is a unique, coalition-proof equilibrium under this policy, which I refer to as “stratified zoning”. Because local governments control assessments, property tax rates, and provision of local public goods to each neighborhood, and because citizens with different incomes consume different amounts of housing and reside in different neighborhoods under stratified zoning, local governments can distribute the benefits of stratified zoning to the income groups who vote them into office. Thus, even if only a minority of citizens gain from stratified zoning before redistribution, the feasibility of redistribution along with maximization of total surplus means that stratified zoning will beat any other zoning policy in a pair-wise vote.

[34] Glaeser and Gyourko (2002) perform an empirical investigation of the effect of zoning on the affordability of housing in the United States.

If zoning policy and distribution of the associated gains are decided by majority vote, the governing coalition in the central city will consist of the half of the distribution of income with the least to lose, in terms of compensating variations, from moving to the suburbs, in the absence of zoning. Commuting costs increase in proportion to income and commuting time, which increases with city size. Thus, the balance of power in a city shifts from rich to poor as it gets larger. In small cities, an upper- and middle-income majority of citizens will determine how the gains from stratified zoning are distributed. Lower-income citizens will either subsidize them or escape to the outskirts of town. In medium-size cities, a coalition drawing from the upper and lower ends of the income distribution will determine how gains are shared, and middle-income citizens will either subsidize others or form suburbs. In large cities, where commuting times are greatest, lower- and middle-income citizens will decide how gains are shared. Upper-income citizens will subsidize others or live in suburbs. These results are validated by facts documented by Schnore and Varley (1955); see Figure 2.

Stratified zoning will not result in a Pareto improvement after redistribution of income if it does not do so before redistribution, or if any citizen is better off in a city without zoning than in the suburbs, under majoritarian democracy. Given U.S. data, stratified zoning is not a Pareto improvement on laissez-faire before redistribution. Thus, it is very hard to justify the maximization of total surplus afforded by stratified zoning on Paretian grounds when zoning policy and distribution of gains are decided by majority vote. Subjecting zoning laws to ongoing unanimous approval of a city council could improve this situation, and would be realistic since stratified zoning both maximizes total surplus and facilitates redistribution. It would also slow or reverse suburbanization and sprawl by protecting city-dwellers from being taxed to the point of out-migration to the suburbs.

That the gains to a citizen from stratified zoning are convex in income helps to explain the fact that neighborhood stratification by income in U.S. cities far outpaced stratification of income itself between 1970 and 2000, and stratification continued to increase between 2000 and 2012.

Author's statement

I have no conflict of interest that would affect my objectivity in performing this research.

Acknowledgements

I thank Talbot Page, Vernon Henderson, Qinghua Zhang, Harl Ryder, Allan Feldman, Kristin Aarland, Delia Furtado, Yukako Ono, Heinrich Hock, and Brian Bishop for helpful comments. I thank Nicholas Zwileneff for providing data. I thank the taxpayer for student loans. I thank EdWhitelaw, Edwin Mills, and Bruce Hamilton for inspiration. I thank Elizabeth Ferreira for her intervention when I might have given up. I thank God for blessing me with the chance to do this work. Any errors are my responsibility. The views expressed in this paper are mine and do not necessarily reflect the position of Elevation Direct Corporation or Brown University.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jhe.2020.101716.

Appendix A. Allowing the price of land and, therefore, housing, to vary by location depending on the external benefits of housing

I show in this appendix that the allocations I derive under laissez-faire and stratified zoning are also supported as equilibria when the price of land is allowed to vary by location, and that perfect separation by income is neither a unique nor a stable equilibrium without zoning.

The derivation of the allocation that maximizes total surplus in Eq. (3.11) through (3.23) can be used to relax the assumption that the price of land is the same in all neighborhoods. Consider Citizen i's problem when the external benefits of housing in each neighborhood are reflected in the price of land: Maximize

$$U^i_{CM} = \alpha \ln(y_i - p_h h_i - p_l \bar{h}_{l_i}) + \beta \ln h_i + \delta \ln \bar{h}_{l_i} \tag{A.1}$$

$p_l$is not the price of land, per se, but applying it to $\bar{h}_{l_i}$ reflects variation in the price of land that depends on the external benefits from housing in each neighborhood. It will cost more for i to reside in a neighborhood where others consume a lot of housing than in a neighborhood where others consume little housing, holding i's consumption of housing constant. If $p_l > 0$ and $\bar{h}_l \neq \bar{h}_k$ for some Neighborhoods l, k, then the price of land varies by neighborhood.

First, though, suppose that the status quo is perfect mixing by income, so that $\bar{h}_l$ is the same in every neighborhood, that $p_l = 0$, and that there are no zoning restrictions on consumption of housing. If $p_h = p$, the long-run marginal cost of housing, this reduces to the model of laissez-faire, whose equilibrium consumption of housing, by (3.3), is $\beta y_i/(\alpha + \beta)p$ for all i. If $p_h \neq p$, it is a short run equilibrium. Either way, the cost of locating a given dwelling in any neighborhood will be the same, and perfect mixing by income will remain the case in a non-cooperative solution. Though the price of residential land *could* vary by neighborhood, there exists an equilibrium in which it does not, and the external benefits of housing are not internalized.

($p_l = 0$ would also support an equilibrium allocation under a uniform minimum, as the external benefits experienced are the same in every location. As in laissez-faire, though the price of land *could* vary by location, it need not with a uniform minimum consumption of housing.)

Now suppose the status quo is perfect separation by income, there exists a neighborhood where $\bar{h}_{l_i} = \beta y_i/p_h$ for every level of y_i, but there is no zoning. According to Eqs. (3.17) and (3.18),

$$\bar{h}_{l_i} = h_i \tag{3.17}$$

$$h_i = \frac{\beta y_i}{p_h} \tag{3.18}$$

Citizen i will choose $h_i = \beta y_i/p_h$, and $\bar{h}_{l_i}$=$\beta y_i/p_h$, by residing in the correct neighborhood. By Eq. (3.16), $p_l = p_h\delta/\beta$ in equilibrium, so the cost of locating in Neighborhood i will be $p_l\bar{h}_{l_i} = \delta y_i$ (or $\delta(y_i + s_{m_i})$ if the planner redistributes income). Since $p_l > 0$ and $\bar{h}_l \neq \bar{h}_k$ for some Neighborhoods l, k, then the price of land varies by neighborhood. There exists a vector of prices for urban land that vary by location and support perfect separation by income as an equilibrium without zoning. If $p_h = \beta p/(\beta + \delta)$, where p is the long-run marginal cost of housing, including the price of agricultural land, the perfect separation also maximizes total surplus. In that case, the price of land varies according to $p_l = \delta p/(\beta + \delta)$, so $p_h + p_l = p$, the long-run marginal cost of housing, including the price of agricultural land, which does not vary by location, and also maximizes total surplus:

$$h_i = \frac{\beta y_i}{p_h} = \beta y_i \frac{\beta+\delta}{\beta p} = \frac{(\beta+\delta)y_i}{p}$$
$$\bar{h}_{l_i} = \frac{\beta y_i}{p_h} = \beta y_i \frac{\delta}{\beta p_l} = \beta y_i \frac{\delta}{\beta}\frac{\beta+\delta}{\delta p} = \frac{(\beta+\delta)y_i}{p}$$

which is the same as in (3.24).

Whether $p_h = \beta p/(\beta+\delta)$ or not, however, this perfect separation is not a unique equilibrium without zoning, even though the price of land varies by location: As I showed above, perfect mixing by income and private utility-maximizing consumption of housing is also an equilibrium when the price of land may vary by location.

Nor is perfect separation a stable equilibrium without zoning. Suppose p_l falls below $p_h\delta/\beta$. Because the "price of admission" to the neighborhood has fallen, Citizen i will then choose to reside in a Neighborhood j, where $y_j > y_i$, in order to experience greater external benefits from the housing of neighbors. Citizen j will then no longer be willing to pay δy_j for the privilege of living there because i's low consumption of housing causes $\bar{h}_{l_j} < \beta y_j/p_h$. Since this is a deviation in p_l, similar incursions of lower income Citizens will occur in neighborhoods throughout the city. These will lower willingness to pay for land in all neighborhoods, and p_l will remain below $p_h\delta/\beta$, incenting further incursions. As Justice Sutherland wrote, "the coming of one apartment house is followed by others".

Though an equilibrium, perfect separation is neither unique nor stable without zoning. If stratified zoning is applied, however, perfect separation becomes a unique, coalition-proof equilibrium, and it can be supported as such either with or without locational variation in the price of land.

Appendix B:. Proof of uniqueness of allocation that maximizes total surplus as a Nash equilibrium under stratified zoning

If (3.25) is in effect, no allocation other than (3.24) is Nash because some citizen will have incentive to move. Consider Neighborhood n, where $h_n^{\min} = (\beta+\delta)y_n/p$, and y_n is the endowment of a member of the corresponding Income Group n. There are two cases to consider.

Case 1: Suppose the private utility-maximizing consumption of housing $h_k^* < h_n^{\min}$ for all $k \in n$, so that the minimum is binding on all citizens in Neighborhood n, and $\bar{h}_n = h_n^{\min}$. Consider the function

$$V^i(y) \equiv \alpha \ln(y_i - (\beta+\delta)y) + (\beta+\delta)\ln((\beta+\delta)y/p)$$

Citizen i's utility is $V^i(y_n)$ if she resides in Neighborhood n and at least $V^i(y_i)$ if she resides in her "home" Neighborhood m_i, where $h_{m_i}^{\min} = (\beta+\delta)y_i/p$, and the externality $\bar{h}_{m_i}$ may be higher if the minimum does not bind all citizens in Neighborhood m_i. Differentiating $V^i(y)$ gives

$$\frac{\partial V^i(y)}{\partial y} = \frac{-\alpha(\beta+\delta)}{y_i - (\beta+\delta)y} + \frac{\beta+\delta}{y}$$

This is decreasing in y. Setting it equal to zero, then, maximizes $V^i(y)$at $y = y_i$, so $V(y_i) > V(y_n)$ if $y_i \neq y_n$. Therefore, if $y_i \neq y_n$ for some $i \in n$, then Citizen i can raise her utility by moving to her home Neighborhood m_i.

Case 2: Suppose $h_j^* > h_n^{\min}$ for some Citizen(s) $j \in n$; the minimum does not bind all citizens in Neighborhood n. Consider the Citizen $i \in n$ such that $h_i^* \geq h_k^*$ for all $k \in n$ (the citizen with the largest endowment in the neighborhood, whose private utility maximizing consumption of housing exceeds that of all her neighbors, and the neighborhood minimum). In Neighborhood n, i will consume $h_i^* = \beta y_i/(\alpha+\beta)p$ and experience an externality $\bar{h}_n < h_i^*$, since she consumes more housing than any of her neighbors. It follows that i's utility in Neighborhood n is no greater than if all her neighbors consumed the same amount of housing as she.

$$U^i|_{l_{i=n}} \leq \alpha \ln \frac{\alpha y_i}{\alpha+\beta} + (\beta+\delta)\ln \frac{\beta y_i}{(\alpha+\beta)p} \tag{B.1}$$

If she moves to Neighborhood m_i, her utility will be at least as great as when all her neighbors consume the minimum there.

$$U^i|_{l_{i=m_i}} \geq \alpha \ln \alpha y_i + (\beta+\delta)\ln \frac{(\beta+\delta)y_i}{p} \tag{B.2}$$

an inequality because I allow that some of her new neighbors may consume more than the minimum $h_{m_i}^{\min} = (\beta+\delta)y_i/p$. If the right hand side of the second inequality exceeds the right hand side of the first, then $U^i|_{l_{i=m_i}} > U^i|_{l_{i=n}}$, and Citizen i will have incentive to move from Neighborhood n to Neighborhood m_i. Consider the function

$$U^i(\sigma) = \alpha \ln(1-\sigma)y_i + (\beta+\delta)\ln \frac{\sigma y_i}{p} \tag{B.3}$$

In (B.1), $\sigma = \beta/(\alpha+\beta)$. In (B.2), $\sigma = \beta+\delta$. Differentiating (B.3) with respect to σ gives

$$\frac{\partial U^i(\sigma)}{\partial \sigma} = \frac{-\alpha}{1-\sigma} + \frac{\beta+\delta}{\sigma}$$

This derivative is decreasing in σ, so setting it equal to zero gives the unique global maximizer $\sigma^*=\beta+\delta$. It follows that

$$\alpha \ln \alpha y_i + (\beta+\delta)\ln \frac{(\beta+\delta)y_i}{p} > \alpha \ln \frac{\alpha y_i}{\alpha+\beta} + (\beta+\delta)\ln \frac{\beta y_i}{(\alpha+\beta)p} \tag{B.4}$$

So, $U^i|_{l_{i=m_i}} > U^i|_{l_{i=n}}$. Citizen i can gain by moving to her "home" Neighborhood m_i. Intuitively, if the citizen derives no utility from neighborhood effects ($\delta = 0$; $\alpha+\beta = 1$; $\sigma^*=\beta+\delta = \beta/(\alpha+\beta)$), she is indifferent among neighborhoods, but if she derives utility from neighborhood effects ($\delta > 0$), a high-income citizen will prefer to reside with other high-income citizens, who generate greater external benefits.

These two cases cover all allocations that may obtain when stratified zoning is in force, other than (3.24), so only (3.24) is Nash under (3.25), stratified zoning ∴

Note that (B.4) reduces to

$$\frac{1}{1-\delta}\left(\frac{\beta}{\beta+\delta}\right)^{\beta+\delta} < 1 \tag{B.5}$$

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