Who Becomes an Inventor in Italy? The Role of Firms in Talent Discovery

Sabrina Di Addario
Bank of Italy

Alice H. Wu
Harvard University

April 18, 2024

Abstract

This paper investigates the role of firms in discovering new inventors, that is, the employees who apply for a patent for the first time. Using employer-employee data from the Italian Social Security Institute matched with patent applications from 1987 to 2009, we identify more than one hundred thousand *potential* inventors, who either apply for a patent on the job or are predicted to ever invent based on observable characteristics. We find substantial heterogeneity in the discovery of new inventors across firms. Younger potential inventors are much less likely to apply for their first patents at a lower-wage firm. The gap between low-wage and high-wage firms in patenting disappears, however, among "established" inventors who have previous patent applications. Further, there is on average a 3-8 log-point increase in the annual wage when a worker files her first patent application. We interpret the empirical findings through a model where heterogeneous firms invest in talent discovery and use wage incentives to elicit effort from workers. When a firm's investment and a worker's effort are substitutable in innovation production, the model can explain why lower-wage firms set a higher return to patenting despite limited job mobility among inventors.

1 Introduction

Younger workers in Italy have been experiencing a decline in economic opportunities relative to older cohorts in the past decades. Labor market reforms aiming at increasing flexibility have been shown to reduce higher-paid and stable jobs available to younger workers (Daruich, Di Addario, and Saggio 2023), and older workers who delay retirement have negative spillovers on younger counterparts (Bianchi, Bovini, Li, Paradisi, and Powell 2023; Goldin, Koutroumpis, Lafond, and Winkler 2024). Despite a growing literature that studies the widening inequality between younger and older workers in Italy, relatively little is known about the role of firms in training and discovering higher-ability individuals. When employers under-invest in talent discovery, uncertainty about the true ability of a worker increases, hurting the career mobility of younger workers (Pallais 2014) and reducing aggregate productivity growth (Terviö 2009; Cetrulo, Cirillo, and Guarascio 2019).

Analyzing the role of firms in talent discovery is often difficult when researchers have little information about the ability or productivity of an individual. This paper overcomes this challenge by focusing on Italy's labor market for potential inventors, for whom we observe patent applications produced on the job, wages, and other job characteristics from administrative data. We provide an empirical assessment of the differences between lower- and higher-wage firms in the discovery of new inventors (i.e. applying for a patent for the first time), and of the differential wage returns to a patent application. We interpret the findings with an employer learning model that takes into account the dynamic incentives of firms and workers.

The Italian labor market for inventors is highly concentrated. Less than 5% of the firms with at least one inventor employ about 90% of inventors younger than 35 who apply for a patent on the job (Figure 1). The distribution of younger workers who will apply for a patent elsewhere in the future is relatively more balanced across firms, suggesting that a large fraction of firms underinvest in discovering new inventors. Although inventors are a highly selected group of workers in the labor force, faster discovery of a person's ability to invent can enhance the allocation of labor and boost innovation productivity (Wu 2023).

Using the employer-employee data matched between Italian Social Security Institute (INPS) and European patent applications (Depalo and Di Addario 2014), we identify roughly 112,000 *potential* inventors, who either contribute to a patent application assigned to their employers during the sample period (1987-2009) or are estimated to have a high probability of ever inventing based on observable characteristics. Relative to a typical worker in the INPS data, potential inventors on average earn 20 log-point higher wages, are more likely to work in white-collar jobs and less likely to have a temporary contract.

We classify firms based on the quartile of coworker mean wages and estimate the probability that a potential inventor files her first patent application in each quartile. Potential inventors who are younger than 35 and have not applied for any patent are 42% significantly less likely to become an inventor when they are employed by a firm in quartile 1 rather than quartile 4 that pays higher wages. The probability of a person ever inventing flattens and slowly declines above age 35. Older potential inventors who have not patented yet are also less likely to apply for patents at lower-wage firms, but the gap in filing one's very first patent application between quartile 1 and quartile 4 shrinks to 7%-11%.

Once workers have applied for a patent, however, there is no longer a gap between lower-wage and higher-wage firms in subsequent patent applications even among younger workers. Inventors employed in the middle quartiles 2 and 3 are in fact more likely to continue patenting than those in quartile 4. From the perspective of employer learning, established inventors with prior patent applications have publicly revealed their ability, and firms would find it less risky to assign them to innovation tasks or increase their R&D investment. Importantly, lower-wage firms do not necessarily have fewer patenting opportunities, but they discover new inventors at slower rates relative to higher-wage firms, a gap that affects younger workers disproportionately.

If we consider patent applications as positive signals on a worker's research ability, employer learning models predict a wage increase when a worker becomes an inventor (e.g., Altonji and Pierret 2001; Farber and Gibbons 1996; Lange 2007; ; Kahn 2013; Schönberg 2007). We can test this prediction in the Italian labor market, given data on annual wages from INPS. We estimate an OLS regression of log wage on whether a worker applies for a patent this year and its interaction with the quartile of her employer, conditional on heterogeneity across industries and geographic regions and over time. There is on average a 32 log-point gap in wages between quartile 1 and quartile 4 among younger potential inventors. Half of the gap can be closed in the year a worker files her first-ever patent application. If we take into account unobserved individual heterogeneity by controlling for person-fixed effects, we find a 3.6 log-point significant increase in wages when a younger potential inventor files her first patent application in quartile 4, and a 5.3 log-point further increase if she does so in quartile 1. Younger potential inventors in quartiles 2 and 3 on average experience 1.3-2.0 log-point higher wage increase than those in quartile 4.

Older workers who file their first patent application in quartile 4 experience a 3.0 log-point wage increase in the same year. The excess wage return in quartile 1 is approximately half that for younger workers but remains significant. In contrast with workers who apply for a patent for the first time, established inventors (i.e. those who have already contributed to patent applications before) do not enjoy a significant wage increase for a new application, except for older inventors in lower quartiles.

These findings suggest that firms reward workers when they apply for a patent for the first time, which is a greater information shock than subsequent patent applications filed by established inventors. A patent with monopoly rights over an invention is typically not granted until a few years later. The wage increase experienced by a new inventor upon application is more aligned with employer learning than rent sharing (e.g., Kline et al. 2019).

Why is the wage return to a person's first patent application higher at lower-wage firms, especially in quartile 1? We point out that employer learning alone cannot provide a satisfactory answer in a labor market with limited job mobility. Wu (2023) shows that non-top firms in the U.S. tech industry face higher turnover of workers who publish new computer science papers.[1] When lower-wage firms anticipate an increase in labor market competition for new inventors, setting a higher bonus may improve retention. However, on average workers who become inventors in Italy, unlike computer scientists in the U.S., are less likely to move to a new firm than their coworkers even a few years later. Furthermore, the average mobility responses to a patent application are similar across quartiles. If a new inventor in quartile 1 has a more credible threat to move away than a new inventor in quartile 4, she could have used it to bargain for a higher wage, but we do not find evidence that supports this hypothesis.

To reconcile the empirical findings, we formulate a dynamic model of talent discovery where firms post wages and invest in employees' research, workers choose efforts accordingly, and jointly they can produce an innovation. Information on worker ability is imperfect but symmetric between all players, and an innovation output will fully reveal someone as a high-ability inventor. In equilibrium, the more productive firms would set a higher base wage and invest more in a worker's research than their less productive counterparts. This equilibrium result is consistent with the empirical finding that workers are less likely to become an inventors at lower-wage firms.

A worker's effort on innovation is not contractible. Each person chooses an effort that maximizes her expected earnings today plus her option value on the labor market next period, which is higher if she is recognized as an inventor. When there are fewer opportunities for new inventors to move between firms, the increase in option value is lower and firms would have to set a larger bonus for innovation to elicit effort from workers. It provides an explanation for the significantly positive wage increase around a worker's first patent application, despite limited mobility between firms. Depalo and Di Addario (2014) shows that the average increase in wages around an application is in fact higher than the increase around a patent grant, which is likely to benefit managers more than the inventors themselves (e.g., Kline et al. 2019).

We also show the conditions under which lower-wage firms set higher wage returns to patent applications despite limited job mobility among inventors. An important necessary condition is that a worker's effort on innovation and her employer's R&D investment are substitutes rather than complements in the innovation production function. By setting a higher wage incentive conditional

[1]Non-top firms are defined as firms that are not Google/Meta/Amazon/Microsoft/Apple/IBM.

on innovation output, less productive firms can increase effort from workers and compensate for the lower investment they make. Admittedly, with neither measures of a firm's R&D investment nor information on the effort of a worker, we are unable to estimate if they are indeed substitutes. This mechanism does not rule out other explanations, but it helps make sense of the relatively higher wage returns to a worker's first patent application at lower-wage firms, even when they are less likely to move away in the short term than workers who are not inventing.

1.1 Related Literature

This paper contributes to several strands of literature. First, our empirical assessment of the discovery of new inventors is related to the growing literature on the career challenges faced by younger workers in Italy. The wage gap between younger and older cohorts has been widening in Italy since the 1990s (Rosolia and Torrini 2007; Rosolia and Torrini 2016). A series of reforms aimed at increasing the flexibility of the labor market led to a higher share of temporary contracts and wage depression that hurt younger workers disproportionately (Daruich et al. 2023). Even the highly educated younger cohorts earned much less on average than earlier cohorts (Naticchioni et al. 2016). The aging population and delayed retirement further reduce opportunities available to younger workers (Bianchi and Paradisi 2023; Bianchi et al. 2023). We contribute to this literature by showing that there is a substantial gap in the discovery of new inventors between lower-wage and higher-wage firms, and the gap for workers younger than 35 is almost four times as large as the gap for older workers.

Second, the matched data between social security records and patent applications in Italy allow us to contribute to the literature on the career paths of inventors and researchers, an important group of workers for knowledge creation and innovation growth. The wage return to patent application is comparable with estimates from other countries (e.g., Aghion et al. 2018). The Italian labor market for inventors is highly concentrated as in the US, and we see similar patterns that established inventors employed by higher-wage firms tend to experience a decline in subsequent patent activity (Akcigit and Goldschlag 2023). We do not find, however, an increase in between-firm or upward job mobility among workers who become an inventor, which is a key finding from the U.S. labor market for computer scientists (Wu 2023). The limited mobility of Italian inventors makes it difficult to explain the heterogeneous wage returns to patenting across firms via employer learning alone.

Third, our model of talent discovery contributes to the employer learning literature as a new framework that considers the dynamic incentives of firms and workers simultaneously. The tradeoff between employer learning and retention, as in Acemoglu and Pischke (1998), continues to matter when firms make R&D investments. The introduction of worker effort also lets us consider the optimal incentive contract as in Holmstrom and Milgrom (1991), and makes it possible to make sense of the higher wage return to patenting at lower-wage firms.

The remainder of this paper is structured as follows. Section 2 describes the matched INPS-PATSTAT data and the selection of potential inventors. Section 3 presents our empirical findings on the heterogeneity in talent discovery and wage returns to patent applications between firms. Section 4 presents a two-period model of talent discovery that reconciles our main empirical findings. Section 5 discusses a policy implication and concludes.

2 Data

We build a panel data on the employment and innovation history of more than 100,000 potential inventors in Italy, using the database in Depalo et al. (2023) that matched the employer-employee data from the Italian Social Security Institute (INPS) with the Worldwide Patent Statistical Database (PATSTAT). The original database in Depalo et al. (2023) contains the employment and patent records for about sixteen thousand Italian inventors and the employment history of their coworkers across employers from 1987 to 2009.

We summarize the matching between PATSTAT and INPS originally done by the team Depalo et al. (2023). Then we discuss the identification of potential inventors in the database who either file a patent application during the sample period or are predicted to do so based on observable characteristics. Throughout the empirical analysis, we focus on the panel of potential inventors to study the role of firms in discovering new inventors.

2.1 INPS-PATSTAT Matched Database

PATSTAT contains the universe of patent applications ever submitted to the European Patent Office (EPO) and has detailed information on the inventors of an application, and the assignees or owners of the patent, which are typically the inventors' employers. Depalo et al. (2023) selected patent applications submitted by firms located in Italy, as recorded in PATSTAT 2009.[2] Between 1987 and 2009, EPO received more than 50,000 patent applications from the private sector, submitted, overall, by 36,000 inventors from $16,000$ firms.

After cleaning the names of inventors and firms applying for patents, Depalo et al. (2023) asked INPS to match the firms with employers (by name and location) in their administrative data, and to match the inventors with individual employees (by name and municipality of residence). A person is defined as a matched inventor if she is acknowledged so on a patent application submitted by her employer at the time of initial filing. INPS returned a de-identified database of about $16,000$

[2] This restriction excludes patent applications by individuals, universities, or public entities, which could not be matched with INPS data, which only covers the private sector in Italy.

matched inventors, as well as 4.5 million coworkers who have ever been employed by the same firm as a matched inventor. We refer to Depalo et al. (2023) for further details on the matching process.

The matched INPS-PATSTAT database contains information on a worker's annual wage at her employer in the private sector each year, type of contract (permanent or temporary), occupation group (blue or white collar), and basic demographic information such as gender or year of birth. INPS also provided information at the establishment level, such as size, sector, location, and dates of business opening and closure. The matching between patent applications and employment records also allows us to build a measure of a worker's on-the-job innovation outputs, and identify the employer at which a worker files her first patent application ever, if it happens during the sample period.

2.2 Potential Inventors

Among the 4.5 million coworkers of matched inventors, the vast majority are unlikely to ever become an inventor. For example, an inventor with patents at the largest automobile company in Italy would have thousands of coworkers who work in factories rather than in R&D departments. Unfortunately, we do not have access to a person's educational background or detailed occupation codes in the INPS sample. Instead, we rely on observable information such as broad occupation groups (white collar/blue collar), type of contract (permanent/temporary/seasonal), and demographic characteristics to predict how likely is a worker to ever file a patent application.

To do so, we first restrict to workers who entered the INPS sample at 14-55 years of age, who were employed in the private sector for at least five years during $[1987, 2009]$, and who spent more years in white-collar than in blue-collar occupations. This initial selection leaves us with 1.5 million workers. We then fit a Poisson model of ever-inventing on observable demographic and employment characteristics, specified as follows:

$$E[Inv_i|x_i, z_{it}, j(i,t), t] = exp\left(x_i'\lambda + z_{it}'\gamma + \psi_{j(i,t)} + \theta_t\right) \tag{2.1}$$

where $Inv_i = 1$ if worker i has any patent application matched with her employment record between 1987 and 2009. The person-level controls, x_i, are whether the person is female, the age at which she first shows up in the INPS data, and its interaction with whether her employment records are left-truncated in 1987 (due to data constraint). The time-varying controls at person-year level, z_{it}, include a cubic polynomial of age, a cubic polynomial of tenure at the worker's current employer $j(i,t)$, characteristics of her current job (white vs. blue collar, permanent vs. temporary contracts) and their interactions with her age. Finally, to take into account heterogeneity in patenting across firms and over time, we control for firm fixed effects $\{\psi_{j(i,t)}\}$ and calendar year fixed effects $\{\theta_t\}$.

Female workers on average are 30% as likely to ever apply for a patent as male co-workers (column 2 of Table B1). Younger workers also have a higher chance of ever inventing, consistent with the findings of Figure 2. Regarding job characteristics, comparing workers within a firm, we find that workers with longer tenure are more likely to apply for a patent, and blue-collar workers are 20% less likely to ever invent relative to the base group (missing occupation). Workers on permanent contracts are significantly more likely to invent (column 1), but conditional on firm fixed effects, the relationship is no longer significant at 95% confidence level.

We rank the 1.5 million workers in the estimation sample by the predicted probability of ever-inventing from the model above. As shown in Figure 3, the distribution of the estimated probabilities among the matched inventors is skewed more to the right than the distribution among all workers. We label a worker as a potential inventor if any one of of the following conditions holds:[3]

1. $max\{\hat{p}_{it} : age_{it} <= 35\} \geq 0.05$: the maximum estimate of one's propensity to invent at age≤35 exceeds the median estimate among matched inventors younger than 35;
2. $max\{\hat{p}_{it} : age_{it} > 35\} \geq 0.06$: the maximum estimate of one's propensity to invent at age>35 exceeds the median estimate among matched inventors older than 35;
3. any matched inventor with at least one patent application matched with her employer in the INPS data 1987-2009.

This selection rule results in about 112,000 potential inventors in total, including about 97,000 workers who did not have a matched patent during [1987, 2009], but are estimated to ever invent with a relatively high probability. In comparison with the full estimation sample of 1.5 million workers, the vast majority of potential inventors are male (Table 1). They also are two years younger on average, tend to be more mobile across firms, more likely to be white-collar than blue-collar, and more likely to have an indefinite/permanent contract. On average there is a 22 log point wage between potential inventors and the estimation sample. The average wage gap at age 30 is about 14 log points and is increased to over 30 log points at age 45 and above. Table 1 also compares potential inventors with matched inventors who have at least one patent application assigned to their INPS employers. Matched inventors earn about similar wages as potential inventors at age 30, but 15 log points more at age 45.

Figure 2 shows the age profile for becoming an inventor among potential inventors. We fit a logistic regression of becoming an inventor on age dummies, gender and calendar year fixed effects. The rate at which a worker files her first patent is increasing the fastest in age in her mid-20s, and at slower rates in her late-20s. It is estimated to peak at age 32 and starts to decline afterward. After age 35, the probability that a worker becomes an inventor remains flat around 4 log points higher than the rate at age 25. Throughout this paper we estimate separate models for workers younger than 35 of age versus older, given the age profile shown above.

[3]The age 35 threshold is selected based on Figure 2, which shows the probability of a worker becoming an inventor increases faster in the early career, peaks around age 32, and begins to flatten after age 35.

In our empirical analysis, we investigate the heterogeneity across firms in identifying talent from potential inventors selected above, excluding workers who are considerably different from inventors and may specialize in tasks unrelated to R&D.

3 Empirical Findings

We analyze the heterogeneity between firms in talent discovery. First, we study the differential rates at which potential inventors apply for patents at lower-wage versus higher-wage firms. We find a significant gap between firms in the filing of a worker's first ever patent application. The gaps in patenting close between lower-wage and higher-wage firms among established inventors who have already applied for patents. Second, we estimate the wage returns to a new patent application at different tiers of firms.

We interpret the empirical findings from the employer learning perspective, where patent applications are positive signals of one's innovation ability. However, given a lack of between-firm mobility among inventors, employer learning alone cannot make sense of the higher wage return to patenting at lower-wage firms.

3.1 Heterogeneity in Patenting between Firms

We begin by estimating the probability that a worker files her first patent application at an employer. For workers without a prior application, the first one at her current employer is also the first patent application ever, and she will be referred to as an established inventor afterwards.

We analyze the differences in becoming an inventor at lower-wage versus higher-wage firms. Specifically, given the person×year panel for potential inventors, we rank firms by mean coworker wages each year, leaving out a person's own wage. Denote by $j(i,t)$ the primary employer of worker i in year t, and by $Q(j(i,t))$ the quartile of coworker wages to which an employer belongs.

Coworker wages are used as a proxy for firm productivity, which we cannot directly measure in the data. This choice of ranking is consistent with the microfoundation for the AKM models where more productive firms set a higher wage premium in an imperfectly competitive labor market (Abowd et al. 1999; Card et al. 2018). In this setting, workers in higher quartiles on average are more likely to apply for patents. 7% of younger workers employed in the top quartile become an inventor by filing their first patent application, whereas 4% of younger workers become an inventor in the bottom quartile (Figure 4a). There is also a 0.7 pp (or 50%) gap in the rate at which a worker applies for any new patent per year between the bottom and the top quartiles. The raw gaps in patenting between older workers at different employers are smaller (Figure 4b or d).

We estimate a Poisson regression of a person becoming an inventor as follows:

$$E[y_{it}|j(i,t),x_{it}] = exp\left(\beta_0 + \sum_{q<4} \underbrace{\beta_q \times 1[Q(j(i,t)) = q]}_{\text{if employer in quartile } q} + \underbrace{x_{it}\Gamma}_{\text{controls}} + \underbrace{\phi_{G(j(i,t))} + \theta_t}_{\text{fixed effects}}\right) \tag{3.1}$$

where y_{it} is an indicator for worker i filing her first patent application at employer $j(i,t)$. The coefficient of interest β_q represents a proportional increase in the mean outcome when a worker is employed by a firm in the q-th quartile, from the mean in the top quartile ($q = 4$) that pays the highest wages. The covariates x_{it} include sex, a cubic polynomial in age (relative to age 35), indicators for white/blue collar and permanent/temporary contract, and their interactions with age. We also control for calendar year to absorb any common trend, 2-digit industry fixed effects to account for time-invariant heterogeneity in patenting across industries, and geographic region fixed effects to absorb unobserved heterogeneity across regions. For each person i, the estimation sample includes all years she is employed according to the INPS data until y_{it} changes from 0 to 1 at employer $j(i,t)$.

3.1.1 Becoming an Inventor

We fit separate models for workers younger versus older than thirty-five, a cutoff selected based on the age profile of becoming an inventor in Figure 2. Younger potential inventors who have not applied for any patent are 42% significantly less likely to become an inventor when they are employed by a firm in the bottom quartile, $Q1$, rather than the top quartile, $Q4$, as shown in the first column of Table 2. The gap shrinks to 12% but remains significant between $Q2$ and $Q4$. Potential inventors in $Q3$, in contrast, are 9% more likely to become an inventor than those in $Q4$.

We find similar patterns among younger matched inventors who have not patented but will have at least one patent application by 2009, the end of INPS sample. Column 2 shows that workers in $Q1$ are 22% less likely to become an inventor relative to the base $Q4$, approximately half the gap among all potential inventors in column 1, but remaining significant with a $t \approx 6$. The estimated relationship between firm ranking (quartile) and the rate of becoming an inventor is also monotonic. Relative to observably similar workers at the top $Q4$, younger matched inventors in $Q2$ are 17% less likely to become an inventor, and those in $Q3$ are 13% less likely to do so.

These findings suggest that lower-wage firms provide less opportunities for younger potential inventors to work on innovation tasks that could lead to patents. It is consistent with the task allocation or investment decisions made by firms that are heterogeneous in productivity. Wu (2023) presents an employer learning framework in which less productive firms assign fewer innovation tasks than more productive ones. Firms do not take into account the positive externality of publicly revealing a high-ability employee. As a result, the gap in innovation tasks between firms is expanded when less productive firms face higher turnover of talent.

Do lower-wage firms assign fewer innovation tasks for everyone, not just potential inventors whose innovation ability is not yet revealed? The answer is no according to the estimates among established inventors who have already applied for patents elsewhere. Column 3 of Table 2 shows that the gap in filing one's first patent application at her current employer in $Q1$ vs. $Q4$ closes. Established inventors at $Q2$ or $Q3$ are about 25% more likely to start applying for patents at their current employers than those in $Q4$. These estimates show a different picture from columns 1-2 among workers who have not applied for patents elsewhere. Lower-wage firms do not necessarily have less patenting opportunities, but they discover new inventors at lower rates relative to higher-wage firms.

Workers who are aged above 35 and have not patented, as shown in Figure 2, are less likely to ever become an inventor than younger workers in their late 20s or early 30s. Lower-wage firms also provide fewer opportunities to patent for older potential inventors, but the gap between workers in $Q1$ and $Q4$ in becoming an inventor shrinks to 7%-11%, as shown by columns 4-5 of Table 2. Potential inventors in $Q2$-$Q3$ are also about 6-10% less likely to become an inventor than in $Q4$, but the gaps are reversed among older established inventors (column 6).

Taken together, workers are less likely to apply for their first patent ever and become an inventor at firms that pay lower wages. Younger workers are particularly affected when they are employed in the bottom quartile of employers, ranked by average wages as a proxy for productivity. Less productive firms may be particularly concerned about publicly revealing an inventor and losing her to competitors that pay higher wages, and therefore assign fewer innovation tasks to begin with (Wu 2023). This learning concern is often argued to matter more for workers earlier in their career whose ability is more uncertain (e.g., Altonji and Pierret 2001; Farber and Gibbons 1996; Schönberg 2007), providing an explanation for why the gaps between firms are more salient among younger than older workers.

3.1.2 Additional Results: Intensive Margin Beyond the First Patent Application

We also consider any new patent application rather than just the first one at the current employer. Define $y_{it} = 1$ if a worker applies for any patent application in year t while being employed by firm $j(i,t)$. We estimate (3.1) on the person $\times$ year panel that includes all years a potential inventor has an employment record in INPS.

Younger potential inventors are 27.2% less likely to apply for a new patent while being employed by $Q1$ relative to similar workers in $Q4$, as shown in column 1 of Appendix Table B2.[4] The gap between $Q2$ and $Q4$ is 15.1%, smaller but significant. Workers in $Q3$ appear to apply for patents

[4]Previously, Table 2 presents the results from a "survival" regression that is estimated on years before a person's first patent application at an employer. In contrast, column 1 of Appendix Table B2 includes all years a potential inventor shows up in INPS rather than years $\leq$ the first year of a patent application.

more frequently than those in $Q4$, even among younger workers who have not become an inventor. The gaps in applying for new patents between the bottom two quartiles and $Q4$ are smaller among younger matched inventors who will eventually patent.

Established inventors at $Q1$ are 15% significantly more likely to apply for a new patent than similar workers at $Q4$ (columns 3 and 6 of Appendix Table B2). Workers who have invented in $Q2$ and $Q3$ are also more likely to continue patenting, despite a smaller difference. This result, in combination with the estimates for established inventors in Table 2, further highlights that the differences in patenting between firms are most salient at the *extensive* margin of discovering new inventors. Younger workers are particularly unlikely to cross the threshold of filing her first ever patent application at low-wage firms. But once they become an inventor, they will continue patenting at similar rates as those who start at higher-wage firms.

3.2 Wage Returns to Patent Applications

Are workers rewarded for filing new patent applications? We can answer this question given the annual wages of potential inventors in the INPS data. To estimate the wage return to a new patent in each quartile of firms, we specify an OLS regression as follows:

$$\begin{aligned} ln(w_{it}) = \mu_0 + &\underbrace{\sum_{q<4} \mu_q \times 1[Q(j(i,t)) = q]}_{\text{avg wage difference rel. to Q4}} + \gamma_0 \times y_{it} + \underbrace{\sum_{q<4} \gamma_q \times y_{it} \times 1[Q(j(i,t)) = q]}_{\text{excess returns to new patent rel. to Q4}} \\ &+ \underbrace{x_{it}\,\Lambda}_{\text{controls}} + \underbrace{\alpha_i + \phi_{G(j(i,t))} + \theta_t}_{\text{fixed effects}} + e_{it} \end{aligned} \tag{3.2}$$

in which $y_{it} = 1$ if worker i applies for a new patent in year t while being employed by firm $j(i,t)$. The coefficient $\underline{\gamma}$ represents the average wage return to a new patent among workers in the base group, $Q4$. $\{\gamma_q : q < 4\}$ represent the excess returns in other quartiles of employers, relative to the return in $Q4$. The regression includes the same set of time-varying controls, x_{it}, as in (3.1). In addition to year, industry, and region fixed effects, we include person fixed effects to absorb unobserved individual heterogeneity that matters for wages. Therefore, the excess returns $\{\gamma_q\}$ can be interpreted as within-person wage increases when a person produces a patent application.

Among potential inventors younger than 35 who have not patented, there is a 20 log-point wage gap between those employed by a firm in $Q1$ vs. $Q4$ (column 1 of Table 3). There is a 3.6 log-point significant increase in wages when a young potential inventor produces a patent application at the top quartile ($Q4$). The excess return to a new application in $Q1$, denoted by γ_1 in (3.2), is estimated to be 5.3 log points with a $t > 6$. Workers in $Q2$ and $Q3$ on average experience 1.3-2 log-point higher wage increase than those in $Q4$.[5]

The excess returns in $Q2$ or $Q3$ disappear if we focus on matched inventors who will eventually patent by 2009, the end of INPS sample (column 2 of Table 3). But matched inventors in $Q1$ receive a 2.8 log-point significant higher wage increase from those in $Q4$ in the same year as a new patent application. It shows that firms reward workers for their first patent applications, although it will take a few more years to know if a patent will be granted successfully. The wage returns among younger potential inventors are significantly higher at lower rungs of the job ladder, especially those in the bottom quartile of firms ($Q1$) that pay lower wages on average.

The wage returns to a new patent application among established inventors who have already applied before are no longer significant (column 3 of Table 3). The mean wage gap between established inventors in $Q1$ versus $Q4$ are 16 log points (column 3), relatively lower than the gap among all potential inventors (column 1). Established inventors who file a new application in $Q1$ appear to receive a 2.5 log-point wage increase in the same year, but the estimate is no longer statistically significant.

Wage returns to new patent applications are smaller for potential inventors who are older than 35, as shown in column 4 of Table 3. When an older potential inventor files her first patent application at a firm in $Q4$, there is a 3.0 log-point significant increase in wage, conditional on individual, industry and geographic heterogeneity as before. There is a 2.6 significant excess return to the first patent application in $Q1$, roughly half of the excess return $\hat{\gamma}_1$ among younger potential inventors (column 1). Among older established inventors, there is no wage return to another patent application in $Q4$, but 1-2 log points significantly higher in lower quartiles (column 6).

3.2.1 Interpreting the Wage Returns to Patenting via Employer Learning

The contrast in wage returns between potential inventors who have not applied for patents and established inventors, and the contrast between younger and older workers, are consistent with an employer learning mechanism. A new patent application is a positive signal about a worker's research or innovation ability. As employers revise upward the belief about a potential inventor's ability, employer learning models suggest the wage bids would go up. In a perfectly competitive labor market, wages would be increased to fully match the marginal revenue product of labor expected from a worker (Altonji and Pierret 2001; Farber and Gibbons 1996; Lange 2007; ; Kahn 2013; Schönberg 2007). Wages would be marked down from the marginal revenue product in a monopsonistic market, but when there is public information about talent, a wage increase is also expected (Wu 2023).

[5]The differences in wage returns to a worker's first patent application between firm quartiles are about 3-5 times larger if we ignore individual heterogeneity (dropping person fixed effects from 3.2), as shown in Appendix Table B3.

In contrast with potential inventors, there is less room for upward belief updating about the ability of established inventors who have already showcased their ability via earlier patent applications. A similar argument can be said for older workers who have been observed in the labor market longer. The lack of belief updating provides one explanation for why the wage returns to new patent applications are lower for established inventors and older workers.

3.2.2 Why is there a higher wage return to patenting at low-wage firms?

Can the employer learning mechanism explain the higher wage return to a patent application at the bottom of the job ladder? The answer is mixed.

First, it depends on whether labor turnover varies across firms when workers apply for patents. Wu (2023) shows that non-top firms in the U.S. tech industry face higher turnover of workers who publish new computer science papers.[6] A higher bonus for patent application, as shown in Table 3, may raise the probability that a firm in the bottom quartile retains inventors. However, it appears that on average workers who become inventors are less likely to move to a new firm (Figure 5) or move upward to a higher-wage quartile (Figure 6). By contrast, established computer scientists with patent applications or publications in the U.S. are as mobile as "potential inventors" who are yet to do so (Appendix Figures B1, B2). There is also an increase in upward mobility from non-top to top firms after computer scientists publish or apply for patents (B3, B4).

To show the lack of between-firm mobility among Italian inventors more precisely, we estimate a Poisson regression of job mobility between firms in 1-3 years on whether a worker files a patent application this year, interacted with quartiles of firms:

$$ln\left(E[\text{Move}_{it}|j(i,t),x_{it}]\right) = m_0 + \underbrace{\sum_{q<4} m_q \times 1[Q(j(i,t))=q]}_{\text{avg mobility rel. to Q4}} + \underbrace{\eta_0 \times y_{it}}_{\triangle\text{mobility in Q4}} + \underbrace{\sum_{q<4} \eta_q \times y_{it} \times 1[Q(j(i,t))=q]}_{\text{excess mobility response rel. to Q4}} + \underbrace{x_{it}\Psi}_{\text{controls}} + \underbrace{\phi_{G(j(i,t))} + \theta_t}_{\text{fixed effects}} \tag{3.3}$$

Given $\text{Move}_{it} := 1[j(i,t+1) \neq j(i,t)]$, Table 4 shows that on average workers who file a new patent application are significantly less likely to move in 1-3 years than other similar workers. There is no evidence that workers who become an inventor in $Q1$ are more mobile between firms than inventors in higher wage quartiles. Similar findings hold for upward mobility, defined as the quartile of one's employer next year $Q(j(i,t+1))$ is higher than that of her employer this year $Q(j(i,t))$, or $Q(j(i,t+1)) = Q(j(i,t)) = 4$, as shown in Table 5. Looking out three years, workers with a new patent in year t are significantly less mobile than others without a patent (Appendix Tables B4,

[6]Non-top firms are defined as firms that are not Google/Meta/Amazon/Microsoft/Apple/IBM. See Apppendix Figures B* for mean job mobility among Ph.D. computer scientists in the United States.

B5). Furthermore, there are no significant differences in mobility responses to a patent application between Q1 and Q4 that could explain the 5 pp higher wage return we see in Table 3, among younger potential inventors for example.

Admittedly, the baseline job mobility among younger potential/matched inventors is higher in the bottom quartile (see the coeffient on $Q1$ in columns 1-2 of Table 4). The higher baseline turnover may pose a threat to employers in $Q1$ and pressure them to raise wages more than others when workers produce a patent application. But this threat does not hold for workers in $Q2$, for example, who also see a 2 log-point higher wage increase upon patent filing (colum 1 of Table 3). Neither does it apply to older potential inventors at lower quartiles who are less mobile than those in $Q4$ but still will see a higher wage return to patent application.

Employer learning may still be able to explain the differential wage returns to a patent if workers employed by the bottom quartile are believed to have lower innovation ability in the beginning. A negative selection of workers at the bottom may explain the 17 log-point excess return to a patent application in $Q1$ rel. to $Q4$ in Appendix Table B2 without person fixed effects. The gap in wage returns among younger potential inventors shrinks to 5 log points in our preferred specification (3.2), which relies on within-person variation and hence cannot be fully explained by initial selection of workers across quartiles. The 5 log-point difference, conditional on person fixed effects, suggests on average firms in $Q1$ set a higher reward for patent applications.

In summary, we find that younger potential inventors are 42% less likely to file their first patent application at an employer in the bottom quartile of wages than similar workers in the top quartile. The gap between low-wage and high-wage firms in patent applications disappear, however, among established inventors who have already applied for patents before. Further, there is a significant 3-4 log point increase in a person's annual wage when she files her first patent application, and the wage returns are significantly higher at firms that pay lower wages. From the perspective of employer learning, at least part of the wage increase upon one's first patent application reflects a upward change in employers' belief about her ability. Nevertheless, employer learning alone cannot explain why less productive firms set a higher reward for patenting even when there is no substantial increase in turnover among new inventors.

4 A Model of Talent Discovery

We formulate a dynamic framework where employers invest in learning the innovation ability of workers, and use wage incentives to influence their efforts on innovation. Taking into account the dynamic decisions of firms and workers simultaneously, this model helps us reconcile the empirical findings above that could not be explained by employer learning alone. In particular, it shows the conditions under which lower-wage firms set higher wage returns to patent applications despite limited job mobility among inventors.

4.1 Model Environment

We introduce the model environment that features workers who vary in innovation ability and firms that vary in productivity. We describe the labor market matching process across two periods. To keep it simple, the information about workers is symmetric between all players in each period.

4.1.1 Workers

Workers (indexed by i) are endowed with a binary one-dimensional ability α_i, which can be high H or low L. In the first period, there is public information I_{i1} on the innovation ability of a worker. Workers and all potential employers share a prior belief $\pi_{i1} = Pr(\alpha_i = H|I_{i1})$.

L-ability workers cannot produce a patent, while H-ability can with probability $h(\tau, e)$ when their employer decides to invest τ on innovation and workers themselves choose effort e. A worker's innovation effort e is not contractible, whereas the investment τ is set by employers and specified in the contracts.

If a worker applies for a patent at $t = 1$, denoted by $y_{i1} = 1$, she will be publicly known as H-ability the next period. Otherwise, the common belief is updated to $\pi_{i2} = Pr(\alpha_i = H|I_{i2}) < 1$, conditional on new information $I_{i2} = I_{i1} \cup \{j(i, 1)\} \cup \{y_{i1} = 0\}$, which includes her $t = 1$ employer $j(i, 1)$ and the fact that there is no innovation output.

4.1.2 Employers

Employers (indexed by j) are endowed with publicly known productivity f_j. Employers simultaneously post contracts based on public information at the beginning of each period. A contract contains a base wage $\underline{w}_{itj} \in \mathbb{R}^+$, a proportional increase in wage, γ_{itj}, if i produces a patent application ($y_{i1} = 1$), and an investment in innovation $\tau_{itj} \geq 0$.

The total production at a firm each period is the sum of the outputs across individual employees.[7] The marginal revenue product expected from a worker at j, given belief π of being H-ability, investment τ and worker effort e, can be written as:

$$MP_j(\pi, \tau, e) := \underbrace{f_j \times}_{\text{Productivity}} \left(\underbrace{1}_{\text{Routine}} + \underbrace{\theta \times \pi \times h(\tau, e)}_{\text{Expected Innovation}} \right) - \underbrace{\zeta/2 \times \tau^2}_{\text{Innovation Cost}} \tag{4.1}$$

[7]This model assumes away the joint production by workers. Identifying talent from team output can be difficult and requires more careful analysis in future work.

where $\theta > 0$ represents the return to innovation in proportion to a firm's productivity f_j, and e is the amount of effort chosen by the worker. There is a convex cost of allocating workers to innovation tasks, determined by parameter $\zeta > 0$.[8]

4.1.3 Labor Market Dynamics

Figure 7 illustrates the model timeline.[9] Every worker is on the labor market at $t = 1$. Each worker observes contracts $\{\underline{w}_{i1j}, \gamma_{i1j}, \tau_{i1j}\}$ posted by potential employers and draws idiosyncratic preferences for employers i.i.d. from a type-I extreme value distribution:

$$F(\{\epsilon_{itj}\}) = exp\left(\sum_j exp(-\epsilon_{itj})\right) \tag{4.2}$$

Worker i chooses her initial employer $j(i,1)$ and chooses her effort e_{i1j} that maximizes her expected utility conditional on entering firm $j = j(i,1)$. Following a dynamic extension of Card et al. (2018), a worker can get back on the market at $t = 2$ with probability $\lambda \in [0,1]$. Conditional on re-entering the market, a worker redraws her preferences across potential employers from (4.2), independent of her preferences at $t = 1$. Other workers who are not on the market at $t = 2$ stay put.

4.2 Optimization by Workers and Firms

We state the problems of workers and firms in each period. The model is solved backward in Appendix A1, and we discuss the model results below.

4.2.1 Workers' Problem

At the beginning of $t = 2$, given contracts $\{(\underline{w}_{i2j}, \gamma_{i2j}, \tau_{i2j})\}$ from employers, a worker who re-enters the job market chooses her employer $j(i,2)$ as follows:

$$j(i,2) = argmax_j\ \boldsymbol{u_{i2j}} + \epsilon_{i2j} = E_y[b \times ln(w_{i2j}) \,|\, \pi_{i2}, \tau_{i2j}, e_{i2j}] - c(e_{i2j}) + \epsilon_{i2j} \tag{4.3}$$

$$\text{where } w_{i2j} = \underline{w}_{i2j} \times \left(1 + y_{i2} \times \gamma_{i2j}\right)$$

$$e_{i2j} = argmax_e\ \underbrace{\pi_{i2}\, h(\tau_{i2j}, e) \times b\, ln(1 + \gamma_{i2j})}_{\text{expected bonus}} - \underbrace{\frac{c}{2}e^2}_{\text{effort cost}}$$

$$= \pi_{i2} \frac{\partial h(\tau, e)}{\partial e} \times \frac{b\, ln(1 + \gamma_{i2j})}{c}$$

[8]This cost may include investment in computing power that often grows in a convex way as employees spend more time on innovation. It may also absorb the management costs of moving workers away from routine activities at a firm. For example, a firm may have to establish an in-house research lab, hire new managers, and establish a new performance evaluation system for workers who are increasingly involved in innovation tasks.

[9]Appendix A0 describes the model timeline and information structure each period in detail.

The probability of her choosing firm j is:

$$p_{i2j} = \frac{exp(E[b \times ln(w_{i2j})] - c(e_{i2j}))}{\sum_{j'} exp(\boldsymbol{u_{i2j'}})} \tag{4.4}$$

The labor supply of an incumbent worker is thus $p^{(1)}_{i2j} = 1 - \lambda(z_{i1}) \times (1 - p_{i2j})$, and the labor supply of a worker from a different firm is $p^{(0)}_{i2j} = \lambda(z_{i1}) \times p_{i2j}$.

At $t = 1$, every worker is on the market, observes initial contracts $\{(\underline{w}_{ij}, \gamma_{ij}, \tau_{i1j})\}$, and draws her preferences over employers from (4.2). Let $\beta_W \in [0, 1]$ denote the exponential discount factor shared by all workers. When $\beta_W > 0$, workers take into account their option value at $t = 2$ when choosing an employer at $t = 1$. The discrete choice facing a worker with an initial belief π_{i1} is summarized as follows:

$$j(i,1) = argmax_j\ \boldsymbol{u_{i1j}} + \epsilon_{i1j} = \underbrace{E_y[b \times ln(w_{i1j})|\ \pi_{i1},\ \tau_{i1j},\ e_{i1j}]}_{\text{expected utility from wage at t=1}} + \beta_W \times \underbrace{E_y[\Omega_j\ (\pi_{i2})\ |\ \pi_{i1},\ \tau_{i1j},\ e_{i1j}]}_{\text{option value at t=2}} - c(e_{i1j}) + \epsilon_{i\,1\,j}$$

$$\text{where } e_{i1j} = argmax_e\ \underbrace{\pi_{i1}\ h(\tau_{i1j}, e)}_{Pr(y_{i1}=1|...)} \times \underbrace{\left(b\ ln(1 + \gamma_{i1j}) +\ \beta_W\ \triangle_y \Omega_j\right)}_{\text{increase in expected utility}} - \frac{c}{2} e^2 \tag{4.5}$$

in which the expectation is over y_{i1} - whether a worker produces a patent during $t = 1$. The option value of choosing firm j at $t = 1$, Ω_j, is a function of worker's effort z on job search during $t = 1$ and the next-period public belief π_{i2}:

$$\Omega_j(\pi_{i2}) = \underbrace{(1 - \lambda) \times u_{i2j}(\pi_{i2})}_{\text{not on market, stay at j}} + \underbrace{\lambda \times E[max\{u_{i2j'}(\pi_{i2}) + \epsilon_{i2j'}\}]}_{\text{on market}} \tag{4.6}$$

Define $\triangle_y f := E[f|y = 1] - E[f|y = 0]$, and hence $\triangle_y \Omega_j = \Omega_j(\pi_2(1)) - \Omega_j(\pi_2(0))$ represents the change in a worker's option value at $t = 2$ when she produces a patent application at $t = 1$.[10]

Forward-looking workers conjecture how beliefs about their ability evolve. The option value can also be interpreted as a preference for innovation tasks, where workers with a higher prior π_{i1} have a stronger preference for higher τ that would increase the chance of being revealed as H-ability the next period.[11] The labor supply to firm j can be written as:

$$p_{i1j} = \frac{exp\left(E[b \times ln(w_{i1j}) + \beta_W\ \Omega_j(\pi_{i2}]\right)}{\sum_{j'} exp\left(\boldsymbol{u_{i1j'}}\right)} \tag{4.7}$$

4.2.2 Employers' Problem

Firms set optimal contracts $\left(\underline{w}_{itj},\ \gamma_{itj},\ \tau_{itj}\right)$ given imperfect but symmetric information about a worker's research ability. At the beginning of $t = 2$, conditional on belief $\pi_{i2} = Pr(\alpha_i = H|I_{i2})$,

[10] According to the information structure in Section 4.1.1., $\pi_{i2}(1) = 1$ and $\pi_{i2}(\pi_2(0)) = Pr(\alpha_i = H|\pi_{i1},\ j(i,1)\ y_{i1} = 0) < 1$.

[11] See Appendix A4 of Wu (2023), which allows $\beta_W > 0$ and expresses the option value as a preference for innovation tasks.

each firm solves:

$$v_{2j}^{(\delta_{ij})}(\pi_{i2}) = max_{(\underline{w},\gamma,\tau)} \quad \underbrace{p_{2j}^{(\delta_{ij})}}_{\text{labor supply}} \times \underbrace{\left(MP_j(\pi_{i2},\,\tau,\,e) - E_y[w_{i2j}|\pi_{i2},\tau,\,e]\right)}_{\text{expected profits}} \tag{4.8}$$

$$\text{s.t. worker effort } e = \pi_{i2}\frac{\partial h(\tau,e)}{\partial e} \times \frac{b\;ln(1+\gamma)}{c} \text{ solves (4.3)}$$

in which $\delta_{ij} = 1[j(i,1) = j]$ indicate if i is employed by j in the first period, at the beginning of $t = 2$. The labor supply of incumbent workers is different from that of an outside worker:

$$p_{2j}^{(\delta_{ij})} = \begin{cases} 1-\lambda\times(1-p_{2j}) & \text{if } \delta_{ij} = 1\text{, incumbent} \\ \lambda\times p_{2j} & \text{if } \delta_{ij} = 0\text{, new employee} \end{cases}$$

At $t = 1$, given prior π_{i1}, firms set contracts that maximize their profits at $t = 1$ and expected returns from an incumbent employee at $t = 2$. Letting $\beta_J \in (0,1]$ denote the exponential discount factor shared by all employers, each firm solves:

$$v_{1j}(\pi_{i1}) = max_{\underline{w},\gamma,\tau} \quad \underbrace{p_{1j}}_{\text{labor supply}} \times \left(\underbrace{MP_j(\pi_{i1},\tau,e) - E_y[w_{i1j}|\,\pi_{i1},\tau,\,e]}_{\text{expected profit at } t=1} + \underbrace{\beta_J \times E_y[v_{2j}^{(1)}(\pi_{i2}(y))\,|\pi_{i1},\tau,\,e]}_{\text{continuation value}} \right) \tag{4.9}$$

$$\text{s.t. worker effort } e = \pi_{i1} \times \frac{\partial h(\tau,e)}{\partial e} \times \frac{\left(b\;ln(1+\gamma)\;+\beta_W \triangle_y \Omega_j\right)}{c} \text{ solves (4.5)}$$

Firms are in Bertrand competition with each other, where the strategic variables are base wage $\underline{w}$, return to patent γ, and investment τ on innovation. In equilibrium, workers on market take the contracts from firms as given and choose their employer and effort in innovation e optimally. Firms take into account the efforts of employees when deciding on the investment. The equilibrium concept is subgame perfect Nash as in Wu (2023). We complete the backward induction in Appendix A1.

4.3 Equilibrium Results and Links to Empirical Findings

We derive three equilibrium results from the model, and discuss the assumptions under which they can explain the empirical findings in Section 3. The proof of each proposition can be found at Appendix A2.

Proposition 1 (Positive Relationship between Firm Productivity and Base Wages) *Assume the labor supply is not perfectly elastic with respect to wages at both periods, $b < \infty$. We have the base wages set by firms at both periods to be strictly increasing in firm productivity f_j*

We do not have a direct measure of firm productivity from the INPS data. But Proposition

1 suggests we can preserve the ranking of firm productivity by using wages alone. It justifies our decision to rank firms by leave-self-out coworker wages throughout the empirical analysis.

Proposition 2 (Heterogeneity in Firm Investment on Innovation) *Assume at any level of worker effort, the patent production function is increasing in a firm's investment on innovation: $\forall e \in \mathbb{R}^+ : \frac{dh}{d\tau} = \frac{\partial h}{\partial \tau} + \frac{\partial h}{\partial e} \times \frac{\partial e}{\partial \tau} > 0$. Given any belief about a worker's innovation ability $\pi \in (0,1]$, we have each firm's investment, τ_{tj}, to be increasing in firm productivity f_j at both periods.*

In equilibrium, the more productive a firm is (higher f_j), the more investment it would make on innovation. At $t = 2$, firms no longer have a dynamic concern regarding worker turnover in the future. From the solution to the firm's problem (4.8, 7.5), we have τ_{2j} increasing in f_j. At $t = 1$, forward-looking firms consider how investment in innovation today affects the information revelation in the next period. Firms that anticipate higher turnover of publicly revealed H-ability workers would set fewer innovation tasks initially. The tradeoff between employer learning and retention is discussed and empirically tested in the U.S. labor market for computer scientists in Wu (2023) (see mobility patterns in Appendix Figure B*). In the Italian market for potential inventors, we do not find evidence that successful inventors at lower-ranked firms move elsewhere faster (see Figure 5 or Table 4). Therefore, the dynamic trade-off that would have widened the gap in τ_{1j} between low- and high- productivity firms may not be as important as in the U.S. labor market.

Despite the limited mobility, the positive relationship between firm productivity and its investment in innovation in Proposition 2 can explain why potential inventors are significantly less likely to file their first patent application at firms that pay lower wages (Table 2).

Proposition 3 (Heterogeneity in Returns to Innovation) *Assume the patent production by H-ability workers as a function of the investment chosen by firm and the effort chosen by worker, $h(\tau, e)$, satisfies: $h_1 > 0$, $h_2 > 0$, \$$\forall e \in \mathbb{R}^+ : \frac{\partial h}{\partial \tau} + \frac{\partial h}{\partial e} \times \frac{\partial e}{\partial \tau} > 0$, and $h_{12} < 0$ (τ, e are substitutes). We have:*

(a) The wage return to a new patent application at $t = 1$, γ_{1j}, decreases in λ at firms where $\frac{\partial \triangle_y v_{2j}^{(1)}}{\partial \lambda} < 0$.

(b) γ_{1j} decreases in firm productivity f_j if $\frac{dh_2}{d\tau} = h_{12} + h_{22}\frac{\partial e}{\partial \tau} \ll 0$. More precisely,

$$\frac{\partial \gamma_{1j}}{\partial f_j} < 0 \textit{ iff } \frac{\partial \ln\left(\frac{\mathbf{f_j}\theta + \beta_{\mathbf{J}} \triangle_{\mathbf{y}} \mathbf{v}_{\mathbf{2j}}^{(\mathbf{1})}}{\underline{\mathbf{w}}}\right)}{\partial \mathbf{f_j}} < \frac{-2\left(h_{12} + h_{22}\frac{\partial e}{\partial \tau}\right)\frac{(h - \pi h^2)}{h_2} + \left((1 - 2\pi h)\left(h_1 + h_2\frac{\partial e}{\partial \tau}\right)\right)}{h - \pi h^2 + \frac{\pi b}{c}(h_2)^2} \times \frac{\partial \tau}{\partial f_j} \tag{4.10}$$

(c) The wage return to a patent application for incumbent workers at at $t = 2$, $\gamma_{2j}^{(1)}$, decreases in market entry rate λ, whereas the wage return for new employees does not vary with λ.

(d) The wage return to a patent application at $t = 2$, γ_{2j}, decreases in firm productivity f_j for both the incumbent

and new employees iff

$$\frac{\partial}{\partial f_j} ln(f_j/\underline{w}) < \frac{\partial}{\partial f_j} ln\left(\frac{h - \pi h^2 + \frac{\pi b}{c}(h_2)^2}{(h_2)^2}\right) \tag{4.11}$$

At $t = 1$, the optimal effort a forward-looking worker chooses is increasing in both the wage return γ_{1j} and the change in option value the next period, $\triangle_y \Omega_j$ (see equation 4.5). Workers who believe they might be H-ability are motivated by the higher wages they can receive at $t = 2$ if there is a successful patent application at $t = 1$, and therefore put in more effort. That is, in a labor market with less frictions or higher market entry rate (λ), employers would not need to set a high reward for patent application to elicit efforts. On the other hand, in a labor market with limited job mobility (lower λ), a higher wage incentive is necessary for eliciting the same level of efforts from workers. In equilibrium we have the bonus γ_{1j} set by firms at $t = 1$ is decreasing in λ (Proposition 3(a)). We have not exploited any variation in λ within the Italian labor market to test this result. But it provides an explanation for the significantly positive wage increase when a worker produces a patent application despite limited mobility between firms.

Proposition 3(b) shows the condition under which the wage return to a patent application is higher at less-productive/lower-wage firms, a key result we have highlighted in Table 3. Under the assumption that $h_{12} < 0$, a firm's innovation investment τ and an employee's own effort e are substitutes. More productive firms set a higher τ (Proposition 2), conditional on which the marginal return to worker effort becomes lower. When h_{12} is sufficiently negative that (7.12) holds, we have the wage return γ_{1j} to be decreasing in firm productivity, which makes sense of the higher wage increases at lower-wage firms, especially in the bottom quartile (Table 3). In other words, setting a higher wage incentive γ can increase effort from workers and compensate for the lower investment on innovation made by firms. If instead, $h_{12} > 0$, firm investment τ and worker effort e are complements, we are more likely to see γ increasing in f_j, which contradicts our empirical results (Section 3.2).

Proposition 3(c) shows the relationship between the reward for a patent application and λ at $t = 2$, the last period in this model at which firms and workers make static decisions (4.3, 4.8). When λ is higher, a larger fraction of incumbent employees are expected to get on the market, and their employers have to set higher wages. The cost of rewarding incumbent workers for patent applications would increase, and in equilibrium, the proportional bonus $\gamma_{2j}^{(1)}$, decreases as $\lambda \uparrow$. The market entry rate λ does not enter the elasticity of labor supply from new workers, and as a result, affect neither the base wage nor $\gamma_{2j}^{(0)}$ for new employees.

Similar to what we have shown in 3(b), whether (τ, e) are complementary matters for the relationship between the wage return τ_{2j} and firm productivity f_j. If the marginal return to worker effort is decreasing in a firm's investment on innovation, $\frac{dh_2}{d\tau} \ll 0$, we will have $\frac{\partial \gamma_{2j}}{\partial f_j} < 0$ for both incumbent and new employees.

In summary, this 2-period model with dynamic decisions by firms and workers help reconcile the key empirical findings in Section 3:

1. Younger workers are less likely to become an inventor in the bottom quartile of firms than those in the top quartile.
2. Wage returns to a new patent application are significantly higher in the bottom quartile.
3. There is no significant differences between firms in mobility responses to a patent application.

5 Conclusion

This paper zooms in on the labor market for potential inventors in Italy to study the heterogeneity in talent discovery across firms. The lack of investment on employees, either in the form of training or taking some risks to learn about a worker's ability, can widen the wage gap between younger and older workers and further slow down the productivity growth. Italy is a particularly important country to study this issue, as the aging workforce and labor market reforms hurt the career prospect of younger workers and the overall labor productivity has been sluggish since the 1990s, lagging behind other advanced economies (Goldin et al. 2024). For policymakers, it is also meaningful to understand the heterogeneity across firms in the discovery of new inventors, and design R&D subsidy that can incentivize firms to increase investment on younger employees and reduce the gap between younger and older workers in innovation.

We find that younger potential inventors are 42% less likely to file their first patent application at an employer in the bottom quartile of wages than similar workers in the top quartile. The gap between low-wage and high-wage firms in patent applications disappear, however, among established inventors who have already applied for patents before. Further, there is a significant 3-4 log point increase in a person's annual wage when she files her first patent application, and the wage returns are significantly higher at firms that pay lower wages.

We build a dynamic framework of employer learning and incentive contract to reconcile these empirical findings. When firm investment and worker effort are substitutable with one another in the innovation production, we can have the wage return to a new application be decreasing in firm productivity, even if worker turnover remains low and similar across firms. Less productive firms under-invest in research by employees, but elicit effort from them using a higher bonus contingent on any successful innovation. This model contributes the employer learning literature by taking into account the dynamic incentives of firms and workers simultaneoulsy.

Our findings also have a policy implication on the design of R&D policy. A large number of younger workers who are capable of inventing but are not given a chance of doing so at lower-wage firms. Policy that subsidizes the promotion of younger inventors, which can be measured by the presence of younger inventors on a patent application, could encourage firms to invest more in talent discovery. Such a policy targeting young workers may generate a larger impact than traditional firm-level R&D on innovation productivity, and reduce the age wage gap.

References

Abowd, J. M., F. Kramarz, and D. N. Margolis (1999): "High Wage Workers and High Wage Firms," *Econometrica*, 67, 251–333.

Acemoglu, D. and J.-S. Pischke (1998): "Why Do Firms Train? Theory and Evidence," *The Quarterly Journal of Economics*, 113, 79–119.

Aghion, P., U. Akcigit, A. Hyytinen, and O. Toivanen (2018): "On the Returns to Invention within Firms: Evidence from Finland," *AEA Papers and Proceedings*, 108, 208–12.

Aghion, P., A. Bergeaud, and J. Van Reenen (2023): "The Impact of Regulation on Innovation," *American Economic Review*, 113, 2894–2936.

Akcigit, U. and N. Goldschlag (2023): "Where Have All the "Creative Talents" Gone? Employment Dynamics of US Inventors"," .

Altonji, J. G. and C. R. Pierret (2001): "Employer Learning and Statistical Discrimination," *The Quarterly Journal of Economics*, 116, 313–350.

Ashenfelter, O., D. Card, H. Farber, and M. R. Ransom (2022): "Monopsony in the Labor Market: New Empirical Results and New Public Policies," *Journal of Human Resources*, 57, S1–S10.

Becker, G. (1964): *Human Capital*, University of Chicago Press.

Bernhardt, D. (1995): "Strategic Promotion and Compensation," *Review of Economic Studies*, 62, 315–339.

Berry, S., J. Levinsohn, and A. Pakes (1995): "Automobile Prices in Market Equilibrium," *Econometrica*, 63, 841–890.

——— (1999): "Voluntary Export Restraints on Automobiles: Evaluating a Trade Policy," *American Economic Review*, 89, 400–430.

Bianchi, N., G. Bovini, J. Li, M. Paradisi, and M. Powell (2023): "Career Spillovers in Internal Labour Markets," *The Review of Economic Studies*, 90, 1800–1831.

Bianchi, N. and M. Paradisi (2023): "Countries of Old Men: An Analysis of the Wage Age Gap," Tech. rep., mimeo.

Boozer, M. (1994): *Wages and Turnover with Imperfect Information and Imperfect Competition*, Doctoral Dissertation, Princeton University.

Bresnahan, T. (1981): "Departures from marginal-cost pricing in the American automobile industry: Estimates for 1977-1978," *Journal of Econometrics*, 17, 201–227.

Burdett, K. and D. T. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39, 257–273.

Card, D., A. R. Cardoso, J. Heining, and P. Kline (2018): "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, 36, S13–S70.

Cederlöf, J., A. N. Peter Fredriksson, and D. Seim (2023): "Mandatory Notice of Layoff, Job Search, and Efficiency," *Working Paper*.

Cetrulo, A., V. Cirillo, and D. Guarascio (2019): "Weaker jobs, weaker innovation. Exploring the effects of temporary employment on new products," *Applied Economics*, 51, 6350–6375.

Daruich, D., S. Di Addario, and R. Saggio (2023): "The Effects of Partial Employment Protection Reforms: Evidence from Italy," *The Review of Economic Studies*, 90, 2880–2942.

Depalo, D. and S. L. Di Addario (2014): "Shedding Light on Inventors' Returns to Patents," Tech. Rep. 375, Development Studies Working Papers, Centro Studi Luca d'Agliano.

Depalo, D., S. L. Di Addario, and M. Sartori (2023): "Shedding Light on Inventors' Returns to Patents," Tech. rep., mimeo.

Dixit, A. K. and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *The American Economic Review*, 67, 297–308.

Farber, H. S. and R. Gibbons (1996): "Learning and Wage Dynamics," *The Quarterly Journal of Economics*, 111, 1007–1047.

Ferreira, D. and R. Nikolowa (2022): "Talent Discovery and Poaching Under Asymmetric Information," *The Economic Journal*, 133, 201–234.

Gambardella, A., D. Harhoff, and B. Verspagen (????): "The value of European patents," *European Management Review*, 5, 69–84.

Gibbons, R. and L. Katz (1991): "Layoffs and Lemons," *Journal of Labor Economics*, 9, 351–380.

——— (1992): "Does Unmeasured Ability Explain Inter-Industry Wage Differentials," *The Review of Economic Studies*, 59, 515–535.

Gibson, M. (2023): "Employer market power in Silicon Valley," *Working Paper*.

Goldberg, P. K. (1995): "Product Differentiation and Oligopoly in International Markets: The Case of the U.S. Automobile Industry," *Econometrica*, 63, 891–951.

Goldin, I., P. Koutroumpis, F. Lafond, and J. Winkler (2024): "Why Is Productivity Slowing Down?" *Journal of Economic Literature*, 62, 196–268.

Greenwald, B. C. (1986): "Adverse Selection in the Labour Market," *Review of Economic Studies*, 53, 325–347.

Haegele, I. (2022): "Talent hoarding in organizations," *arXiv 2206.15098*.

Hager, S., C. Schwarz, and F. Waldinger (2023): "Measuring Science: Performance Metrics and the Allocation of Talent," *Working Paper*.

Hall, B. H. and D. Harhoff (2012): "Recent Research on the Economics of Patents," *Annual Review of Economics*, 4, 541–565.

Hendricks, K. and R. H. Porter (1988): "An Empirical Study of an Auction with Asymmetric Information," *The American Economic Review*, 865–883.

Holmstrom, B. and P. Milgrom (1991): "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design," *Journal of Law, Economics, Organization*, 7, 24–52.

Imbens, G. and J. Wooldridge (2007): *What's New in Econometrics Lecture 11: Discrete Choice Models*, NBER.

Jones, B. F. (2009): "The Burden of Knowledge and the "Death of the Renaissance Man": Is Innovation Getting Harder?" *The Review of Economic Studies*, 76, 283–317.

Kahn, L. B. (2013): "Asymmetric Information between Employers," *American Economic Journal: Applied Economics*, 5, 165–205.

Kahn, L. B. and F. Lange (2014): "Employer Learning, Productivity, and the Earnings Distribution: Evidence from Performance Measures," *The Review of Economic Studies*, 81, 1575–1613.

Kirpalani, R. and E. Madsen (2023): "Strategic investment evaluation," *Theoretical Economics*, 18, 1141–1180.

Kline, P., N. Petkova, H. Williams, and O. Zidar (2019): "Who Profits from Patents? Rent-Sharing at Innovative Firms," *The Quarterly Journal of Economics*, 134, 1343–1404.

Krahé, M. (2023): "Understanding Italy's Stagnation," Tech. rep., Dezernat Zukunft.

Lange, F. (2007): "The Speed of Employer Learning," *Journal of Labor Economics*, 25, 1–35.

Li, J. (2013): "Job Mobility, Wage Dispersion, and Technological Change: An Asymmetric Information Perspective," *European Economic Review*, 60, 105–126.

Manning, A. (2003): *Human Capital and Training*, Princeton University Press, 301–322.

——— (2012): "Steady-state Equilibrium in a Model of Short-Term Wage-Posting," *CEP Discussion Paper No 1158*.

McFadden, D. (1973): "Conditional logit analysis of qualitative choice behavior," in *Fontiers in Econometrics*, ed. by P. Zarembka, New York: Academic press, 105–142.

Naticchioni, P., M. Raitano, and C. Vittori (2016): "La Meglio Gioventu': Earnings Gaps Across Generations and Skills in Italy," *Economia Politica*, 33, 233–264.

Pallais, A. (2014): "Inefficient Hiring in Entry-Level Labor Markets," *American Economic Review*, 104, 3565–99.

Pastorino, E. (2023): "Careers in Firms: The Role of Learning about Ability and Human Capital Acquisition," *Journal of Political Economy*, Forthcoming.

Postel-Vinay, F. and J.-M. Robin (2002): "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 70, 2295–2350.

Rosolia, A. and R. Torrini (2007): "The Generation Gap: Relative Earnings of Young and Old Workers in Italy," Tech. Rep. 639, Bank of Italy, Temi di discussione.

——— (2016): "The Generation Gap: A Cohort Analysis of Earning Levels, Dispersion and Initial Labor Market Conditions in Italy, 1974-2014," Tech. Rep. 366, Bank of Italy, Questioni di economia e finanza.

Schönberg, U. (2007): "Testing for Asymmetric Employer Learning," *Journal of Labor Economics*, 25, 651–691.

Stern, S. (2004): "Do Scientists Pay to Be Scientists?" *Management Science*, 50, 835–853.

Stevens, M. (1994): "A Theoretical Model of On-the-Job Training with Imperfect Competition," *Oxford Economic Papers*, 46, 537–62.

Strobl, G. and E. D. V. Wesep (2013): "Publicizing Performance," *Management Science*, 59, 918–932.

Tanzi, G. M. (2023): "Scars of Youth Non-employment and Labour Market Conditions," *Italian Economic Journal*, 9, 475–499.

Terviö, M. (2009): "Superstars and Mediocrities: Market Failure in the Discovery of Talent," *The Review of Economic Studies*, 76, 829–850.

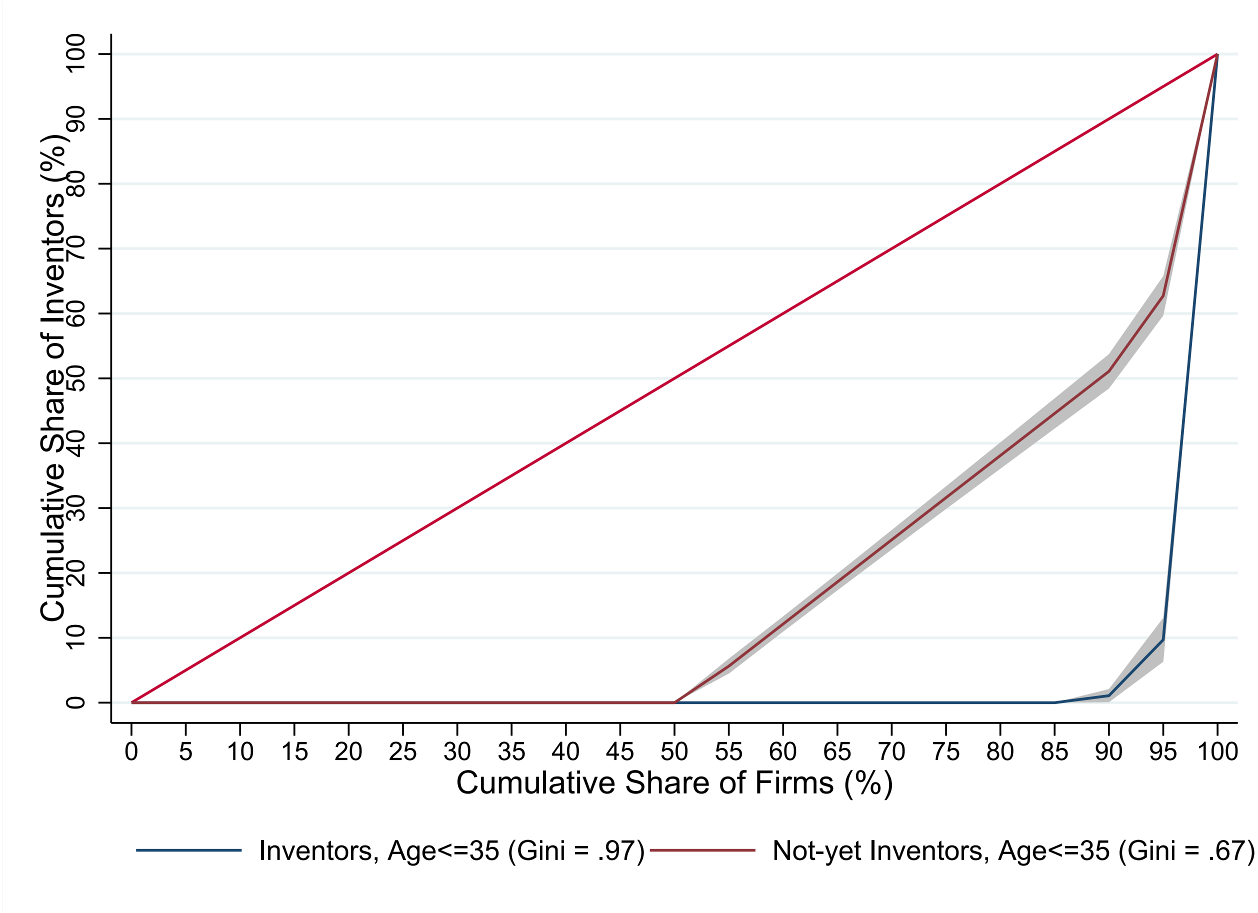
US, D. o. J. (2010): "Complaint, United States of America v. Adobe Systems," .

Waldman, M. (1984): "Job Assignments, Signalling, and Efficiency," *The RAND Journal of Economics*, 15, 255–267.

Wu, A. (2023): "Reveal or Conceal? Employer Learning in the Labor Market for Computer Scientists," *Working Paper*.

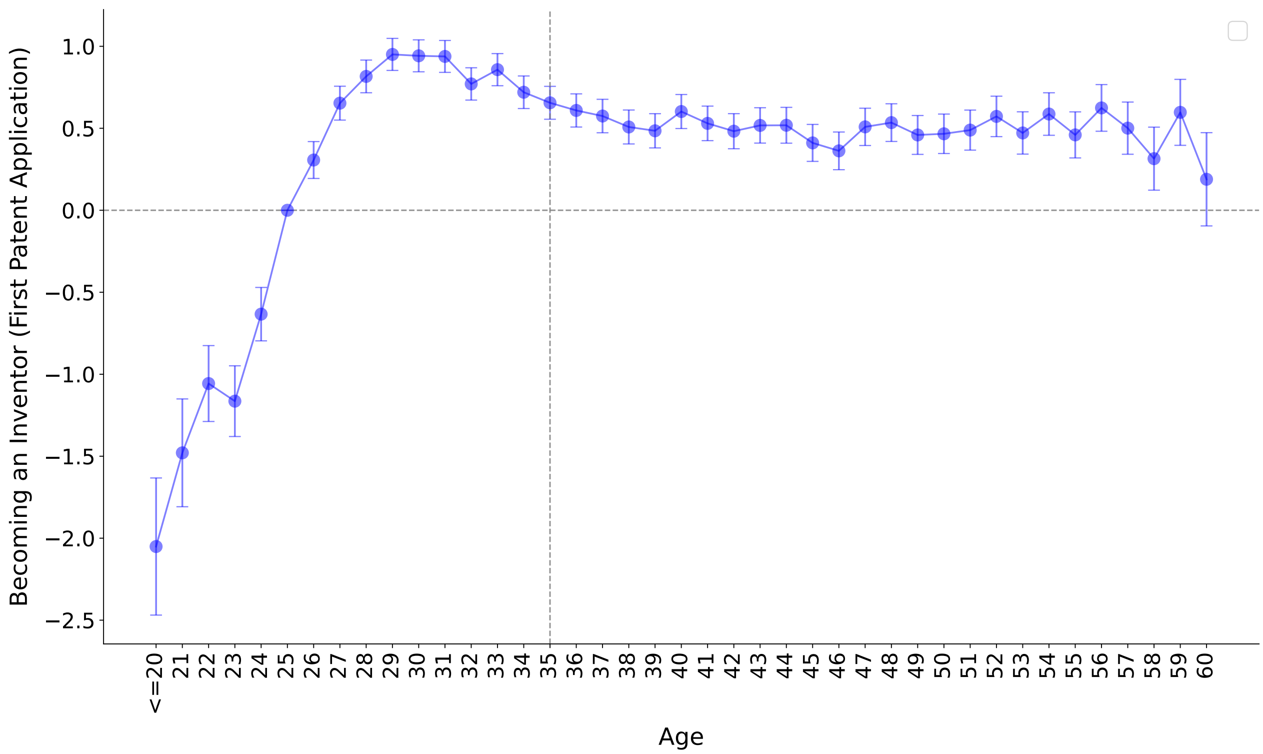
6 Figures

Figure 1: Lorenz Curves - Distribution of Younger Inventors Across Firms in Italy



Notes: This figure shows the distribution of younger inventors across firms. For each firm with at least one inventor matched in the INPS data, we compute the number of younger employees who apply for a patent at age ≤ 35, and the number of younger employees who have not applied for a patent but will do so at another firm in the future, referred to as "not-yet inventors". We require that the patent applications are assigned to an inventor's primary employer in the year of initial filing. Younger inventors are more concentrated at certain firms than younger workers who will become inventors elsewhere. About 90% of the younger inventors are employed by the top 5% or about 550 firms.

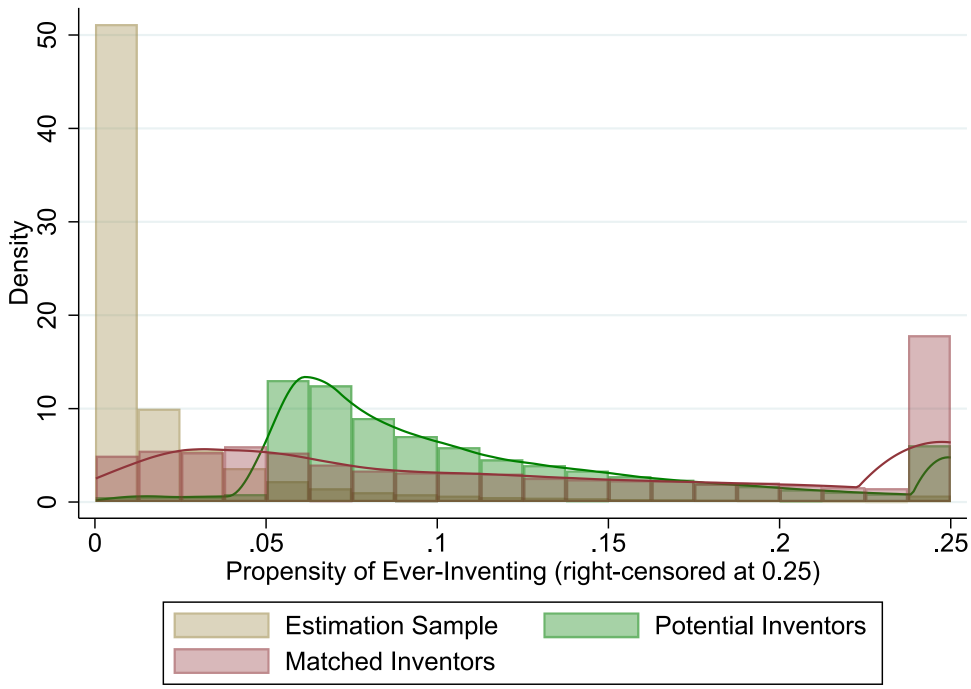
Figure 2: Becoming an Inventor by Age



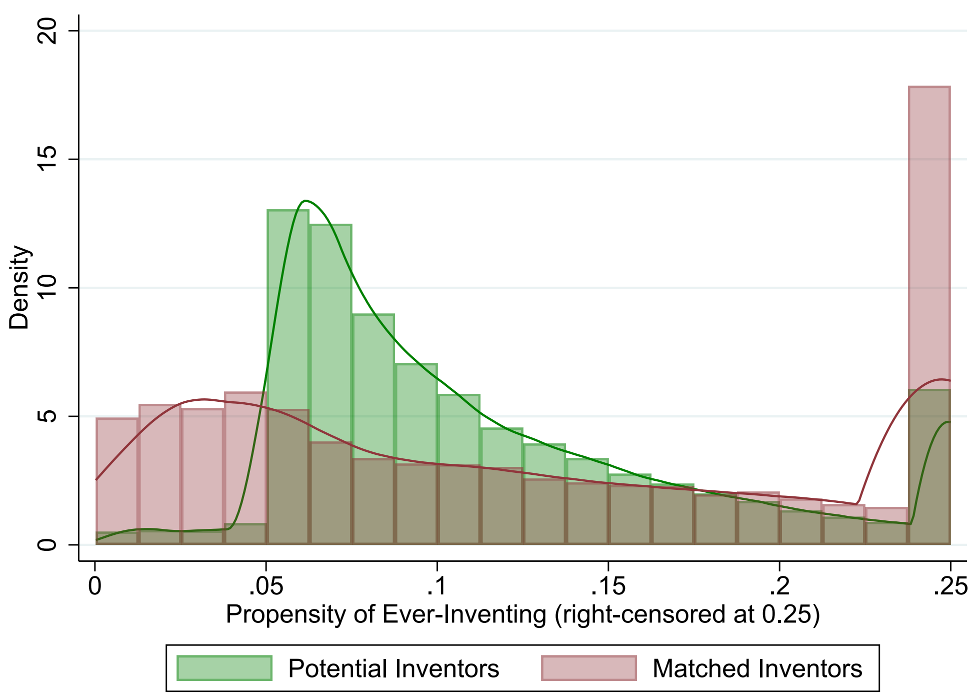
Notes: This figure plots the logit coefficients of becoming an inventor (first patent application) by age, relative to age 25. The estimation sample is at the (person, year) level, comprising the years in which a worker is aged between 18 and 60, has not applied for patents, or just submitted her first application. The logistic regression of becoming an inventor is estimated on a person $\times$ year panel that includes all potential inventors (see Section 2.2). We control for age dummies (age 18-20 are grouped together), calendar year fixed effects, and gender.

Figure 3: Propensity Scores of Ever Inventing

(a) All

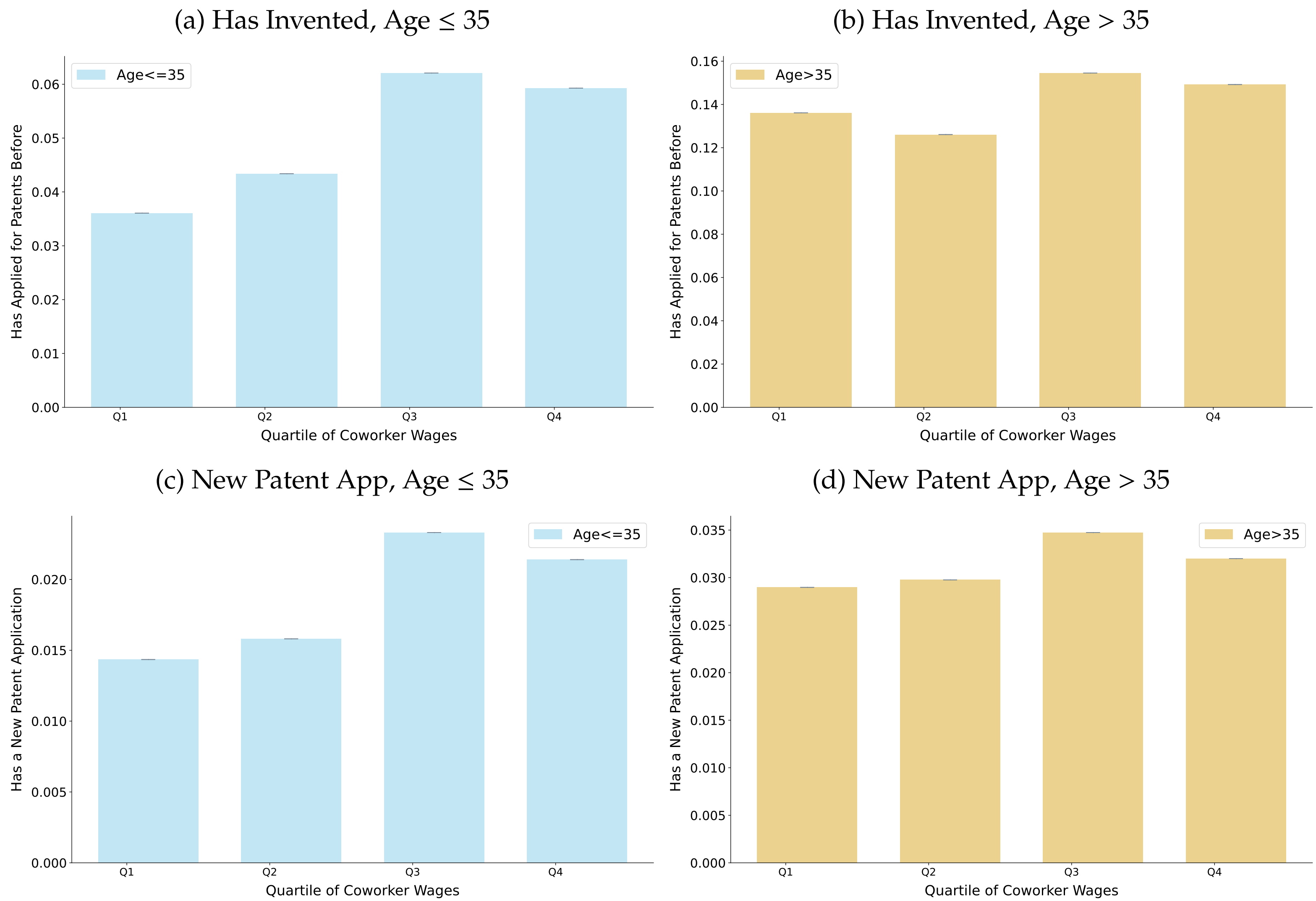


(b) Potential Inventors



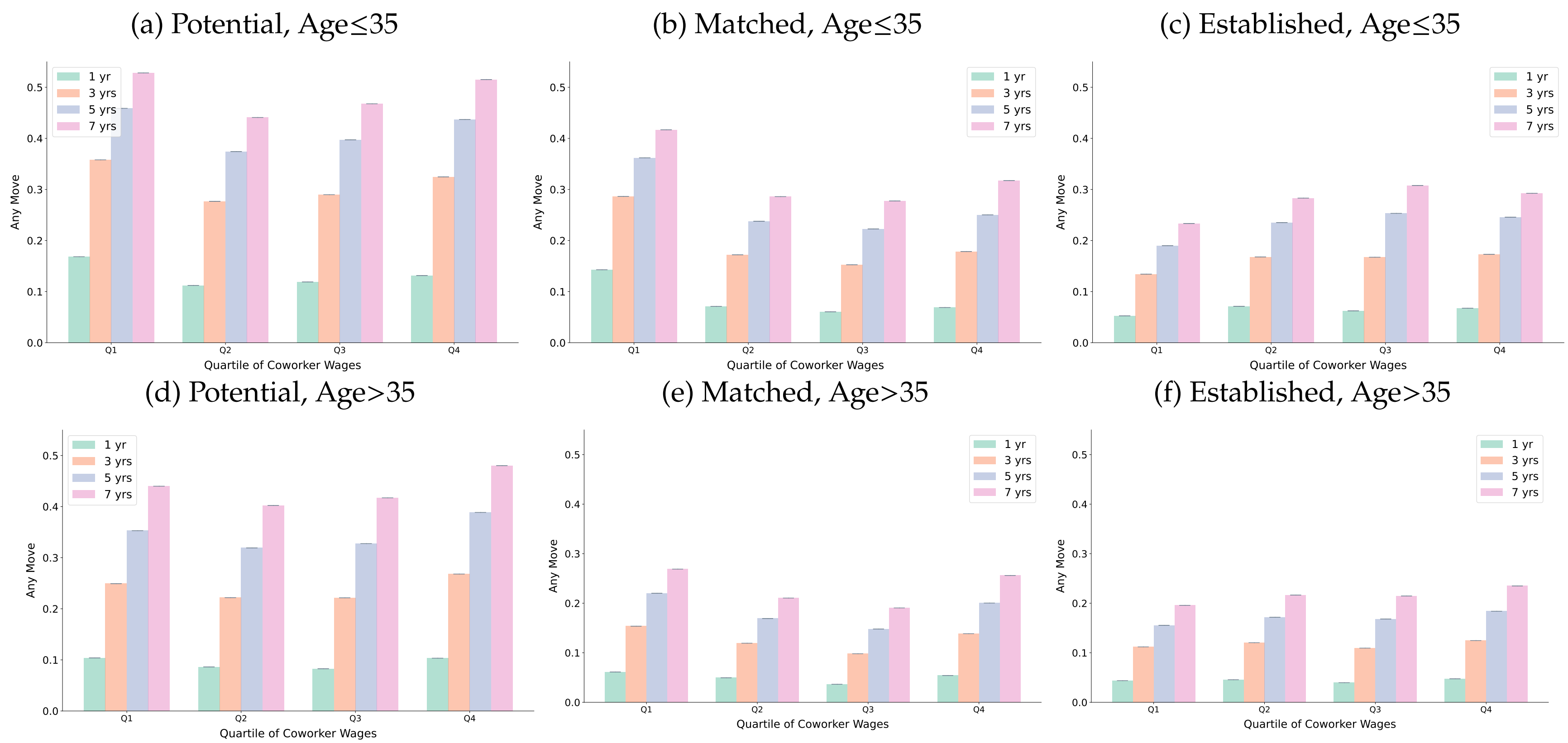
Notes: This figure presents histograms of the estimated probability of a worker ever inventing, as specified in Poisson regression (2.1). For illustration, the p-scores are right censored at 0.25. The estimation sample includes 1.5 million workers (see notes under Table 1). Matched inventors are workers with at least one patent application matched to their employment in INPS data 1987-2009. Potential inventors include all matched inventors and their coworkers whose estimated p-scores are above the median of the p-scores among matched inventors (Section 2.2).

Figure 4: Heterogeneity in Patenting by Age and Coworker Wages



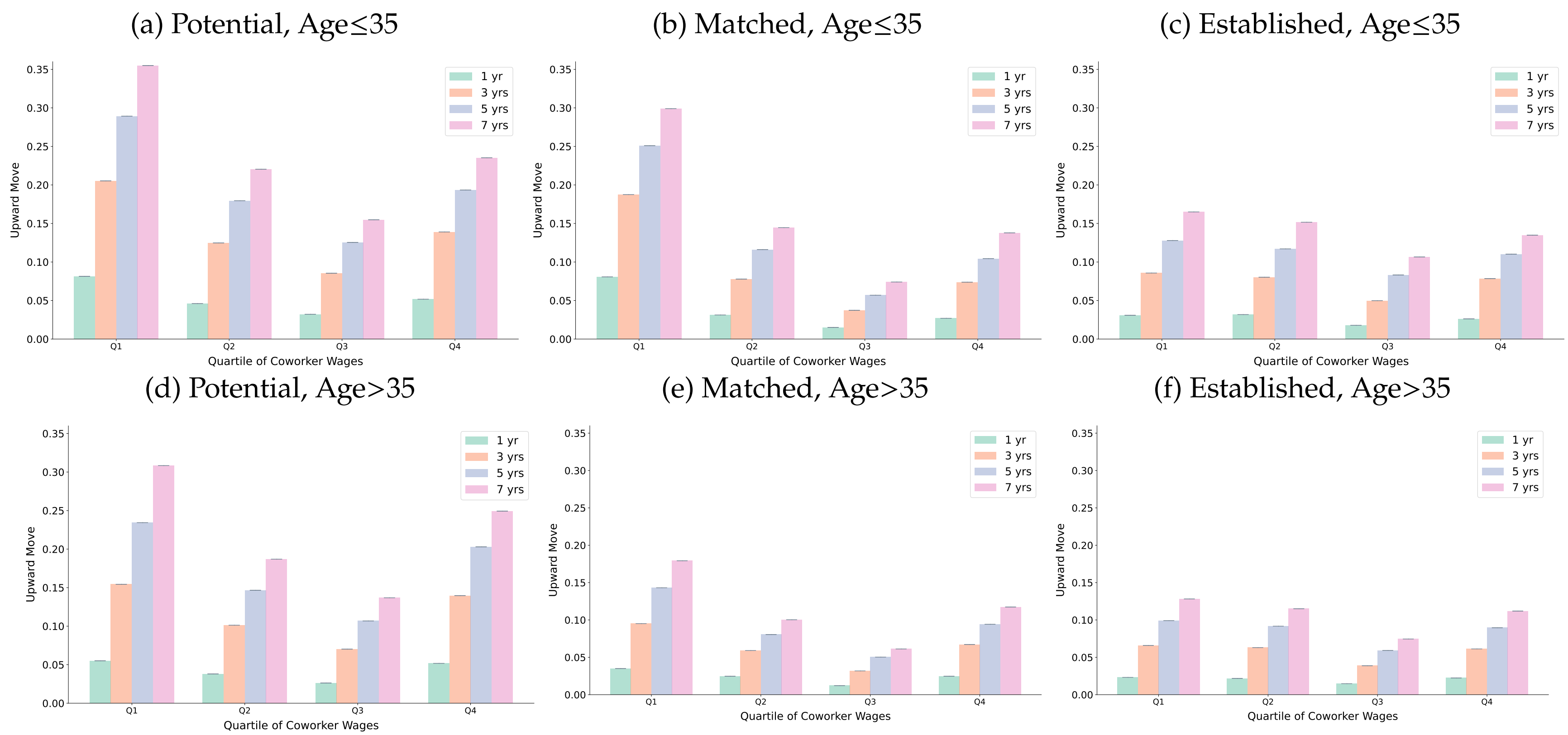
Notes: This figure shows the fraction of employees who have applied for a patent before, and who have a new patent application by age group, and by the quartile of firms (Q1=bottom 25% firms). Firms are ranked by mean coworker wages each year (leaving oneself out).

Figure 5: Mobility by Quartile of Firms: Any Move between Firms



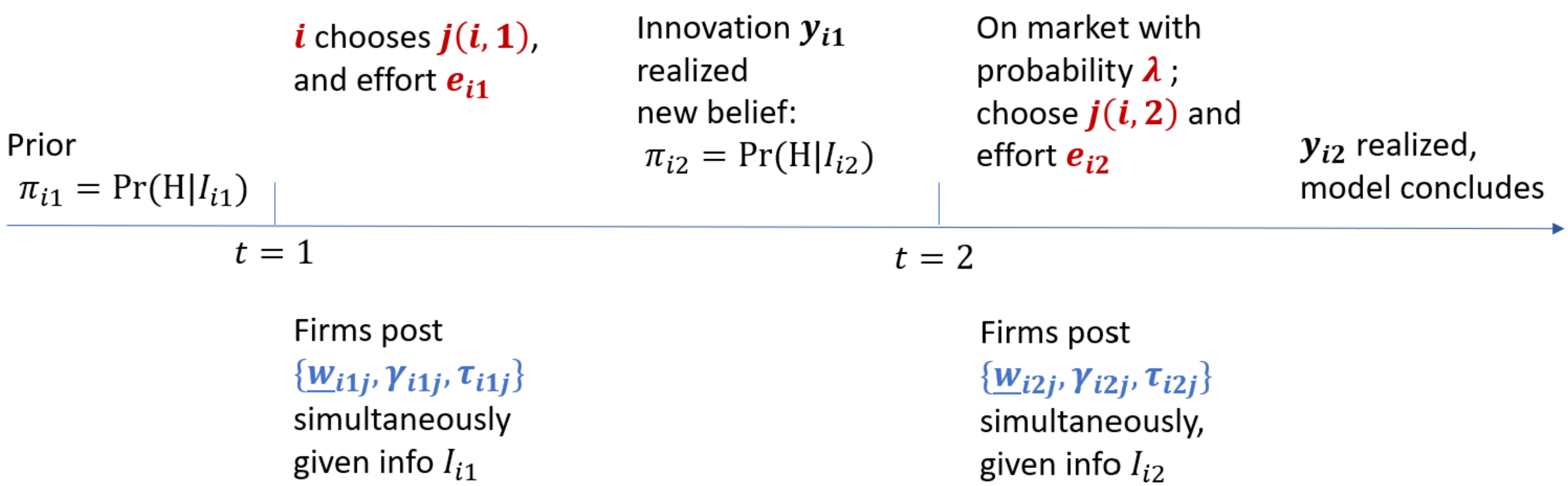
Notes: This figure shows the mean mobility of workers at each quartile of firms, ranked by leave-out coworker wages each year. (a)-(c) focus on workers younger than 35 as of year t, and show the fraction employed by a different firm, $j(i, t+k) \neq j(i, t)$, in $k \in \{1, 3, 5, 7\}$ years, while (d)-(f) shows the same for workers older than 35. Potential inventors refer to workers selected as in Section 2.2 who have not applied for any patent. Matched inventors are those who have not applied for a patent, but will eventually have a matched patent application matched with their employment records. And finally, established inventors are workers who have applied for a patent. The three groups are consistent with the definitions of estimation samples in Table 2.

Figure 6: Upward Mobility by Quartile of Firms: Any Move between Firms



Notes: This figure shows the mean upward mobility of workers at each quartile of firms, ranked by leave-out coworker wages each year. Upward mobility is defined as a worker moving into a different and higher-quartile firm, $j(i.t+k) \neq j(i,t)$ and $Q(j(i,t+k)) > Q(j(i,t))$, in $k \in \{1,3,5,7\}$ years. For workers starting at the top, a move between firms within $Q4$ is also coded as an upward move(a)-(c) focus on workers younger than 35 as of year t, and show the fraction employed by a different firm, $j(i,t+k) \neq j(i,t)$, in $k \in \{1,3,5,7\}$ years, while (d)-(f) shows the same for workers older than 35. Potential inventors refer to workers selected as in Section 2.2 who have not applied for any patent. Matched inventors are those who have not applied for a patent, but will eventually have a matched patent application matched with their employment records. And finally, established inventors are workers who have applied for a patent.

Figure 7: Model Timeline



Notes: This figure shows the model timeline. See Section 4 for details.

7 Tables

Table 1: Sample Overview - Person Characteristics

	Estimation Sample		Potential Inventors		Matched Inventors	
	mean	sd	mean	sd	mean	sd
Demographics						
Female	0.346	0.476	0.063	0.244	0.091	0.287
Yr of Birth	1960	11.923	1962	9.794	1958	10.836
INPS Sample (left-censored at 1987)						
First Yr in INPS	1991	5.162	1991	5.083	1990	4.886
Present in INPS in 1987	0.527	0.499	0.400	0.490	0.570	0.495
Patent Applications						
Any Patent App 1987-2009	0.010	0.100	0.138	0.344	1.000	0.000
Any Patent App Per Year	0.002	0.023	0.023	0.083	0.168	0.161
Any Patent App by Age 30	0.005	0.072	0.046	0.209	0.361	0.480
... by Age 35	0.009	0.095	0.081	0.273	0.586	0.493
... by Age 40	0.011	0.103	0.109	0.312	0.685	0.464
... by Age 45	0.011	0.106	0.145	0.352	0.760	0.427
... by Age 50	0.011	0.105	0.180	0.384	0.823	0.382
Job Characteristics						
Num. Employers (Firms)	2.089	1.337	2.726	1.492	2.150	1.414
Blue-Collar	0.055	0.123	0.049	0.115	0.018	0.067
White-Collar	0.935	0.145	0.949	0.118	0.980	0.070
Permananent Contracts	0.455	0.320	0.567	0.268	0.530	0.255
Temporary Contracts	0.101	0.224	0.061	0.146	0.036	0.105
Seasonal Contracts	0.002	0.025	0.001	0.011	0.000	0.007
Contract Type Missing	0.454	0.339	0.384	0.284	0.443	0.275
Wages						
Mean Log Wage	7.534	0.498	7.756	0.483	7.983	0.479
Log Wage at Age 30	7.376	0.442	7.515	0.337	7.585	0.294
... at Age 35	7.550	0.492	7.773	0.411	7.857	0.333
... at Age 40	7.678	0.499	7.960	0.493	8.072	0.424
... at Age 45	7.776	0.491	8.088	0.552	8.240	0.486
... at Age 50	7.850	0.493	8.171	0.593	8.384	0.543
Observations	1,537,000		112,000		15,000	

Notes: This table shows the summary statistics at person level, for workers who showed up in the INPS-PatStat matched data spanning 1987-2009 (Section 2.1) and are selected for identifying potential inventors. The estimation sample includes inventors and their coworkers who 1) first showed up in the INPS sample at age 14-55, 2) have at least five years of employment records in the sample between 1987 and 2009, and 3) worked in white-collar roles at least 50% of the time in the sample. We estimate the Poisson regression (2.1) that predicts ever-inventing on this sample. Potential inventors include any worker with a matched patent application (matched inventor), or any worker whose estimated propensity score of ever inventing is above the median p-score among matched inventors (Section 2.2). Job characteristics, wages, and patenting rates are computed from the person-year panel.

Table 2: First Patent Application at the Current Employer, by Quartile of Firms and Age Group

	First Patent Application at the Current Employer					
	Age ≤ 35			Age > 35		
	(1) Potential	(2) Matched	(3) Established	(4) Potential	(5) Matched	(6) Established
Quartile of Mean Coworker Wages						
Q1	-0.4244	-0.2213	0.0609	-0.0745	-0.1138	0.0318
	(0.0437)	(0.0380)	(0.1622)	(0.0456)	(0.0384)	(0.0662)
Q2	-0.1216	-0.1657	0.2708	-0.1050	-0.1013	0.1705
	(0.0402)	(0.0364)	(0.1184)	(0.0386)	(0.0338)	(0.0507)
Q3	0.0895	-0.1286	0.2559	-0.0583	-0.0856	0.2525
	(0.0392)	(0.0372)	(0.1160)	(0.0373)	(0.0336)	(0.0450)
(Base) Q4	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
Mean in Q4	.01037	.12639	.06899	.00712	.16361	.07337
N	644,238	52,843	5,869	744,670	34,735	33,170
Pseudo R2	.06588	.10866	.09064	.04000	.07296	.08350

Notes: This table shows the estimated Poisson regression (3.1) of 1 if a worker files her first patent application at her current employer on the quartile of her employer, ranked by mean coworker wages in a year leaving out her own wage. The estimation sample is at person × year level, and for each worker it includes years up to her first patent application at the current employer. All regressions control for sex, a cubic polynomial in age (relative to age 35), indicators for white/blue collar and permanent/temporary contract interacted with age, and fixed effects of the calendar year, 2-digit industry, and geographic region. Models (1) and (4) are estimated on potential inventors who have not applied for patents before. (2) and (5) are restricted to matched inventors who have not applied for patents before but will do so during the sample period. (3) and (6) are restricted to established inventors who have already applied for patents at former employers.

Table 3: Wage Returns to New Patent Application, by Quartile of Firms and Age Group

	Log Annual Wages					
	Age $\leq$ 35			Age > 35		
	(1) Potential	(2) Matched	(3) Established	(4) Potential	(5) Matched	(6) Established
Quartile of Mean Coworker Wages						
Q1	-0.2004	-0.1423	-0.1590	-0.1555	-0.0954	-0.1371
	(0.0016)	(0.0056)	(0.0097)	(0.0016)	(0.0057)	(0.0043)
Q2	-0.1124	-0.0568	-0.0904	-0.0877	-0.0513	-0.0813
	(0.0014)	(0.0047)	(0.0076)	(0.0011)	(0.0041)	(0.0029)
Q3	-0.0723	-0.0298	-0.0506	-0.0527	-0.0270	-0.0410
	(0.0013)	(0.0041)	(0.0059)	(0.0009)	(0.0036)	(0.0024)
(Base) Q4	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
Any New Patent Application						
y_{it}	0.0363	0.0330	0.0103	0.0295	0.0100	-0.0011
	(0.0064)	(0.0067)	(0.0076)	(0.0050)	(0.0052)	(0.0030)
Excess Returns (relative to Q4)						
$y_{it}\times$ Q1	0.0526	0.0279	0.0148	0.0261	0.0071	0.0170
	(0.0087)	(0.0089)	(0.0095)	(0.0084)	(0.0086)	(0.0050)
$y_{it}\times$ Q2	0.0202	-0.0102	-0.0037	0.0102	-0.0004	0.0104
	(0.0081)	(0.0084)	(0.0096)	(0.0067)	(0.0068)	(0.0043)
$y_{it}\times$ Q3	0.0133	-0.0107	0.0038	-0.0029	-0.0113	0.0085
	(0.0082)	(0.0086)	(0.0088)	(0.0066)	(0.0068)	(0.0039)
Constant	7.5771	7.5159	7.6485	8.0739	8.1070	8.2680
	(0.0470)	(0.0167)	(0.0527)	(0.0113)	(0.0447)	(0.0361)
Mean in Q4	7.75228	7.67659	7.83425	8.27278	8.25522	8.41744
N	646,123	52,411	25,602	740,741	34,054	117,178
Adjusted R2	.71654	.74704	.73993	.86675	.90300	.87285

Notes: This table shows the estimated OLS regression (3.2) of log annual wages on whether a worker applies for a new patent and its interactions with the quartile of her current employer, ranked by mean coworker wages in a year leaving out her own wage. The estimation sample is at person $\times$ year level. All models control for person fixed effects, in addition to the covariates noted under Table 2. Models (1) and (4) are estimated for all potential inventors. (2) and (5) are restricted to matched inventors, and (3) and (6) are restricted to established inventors who have already applied for patents at former employers.

Table 4: Between-firm Job Mobility in a Year, by Quartile of Firms and Age Group

	Move in 1 Year: $j(i,t+1) \neq j(i,t)$					
	Age $\leq$ 35			Age > 35		
	(1) Potential	(2) Matched	(3) Established	(4) Potential	(5) Matched	(6) Established
Quartile of Mean Coworker Wages						
Q1	0.09104	0.38981	-0.18411	-0.08443	-0.04075	-0.15446
	(0.01046)	(0.04971)	(0.09059)	(0.01191)	(0.07067)	(0.05003)
Q2	-0.08203	0.04595	0.07238	-0.16105	-0.09905	-0.03046
	(0.01103)	(0.05367)	(0.07965)	(0.01075)	(0.06952)	(0.04422)
Q3	-0.03129	-0.05954	0.00355	-0.16028	-0.28895	-0.04024
	(0.01131)	(0.05638)	(0.08292)	(0.01075)	(0.07262)	(0.04232)
(Base) Q4	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
Any New Patent Application						
y_{it}	-1.40165	-0.77698	-0.46142	-2.06911	-1.43029	-0.97013
	(0.16664)	(0.17151)	(0.13414)	(0.21178)	(0.22022)	(0.09018)
Excess Returns (relative to Q4)						
$y_{it}\times$ Q1	0.15357	-0.19993	-0.53764	-0.06517	-0.30673	-0.15991
	(0.20976)	(0.21430)	(0.21323)	(0.36692)	(0.37406)	(0.16968)
$y_{it}\times$ Q2	0.45979	0.32503	0.04950	0.96266	0.86208	0.28270
	(0.20722)	(0.21296)	(0.18536)	(0.26299)	(0.27059)	(0.13216)
$y_{it}\times$ Q3	0.11458	0.22278	0.14813	0.51973	0.73233	0.29595
	(0.21832)	(0.22456)	(0.18096)	(0.28059)	(0.28848)	(0.12545)
Constant	-3.16906	-2.59644	-1.65307	-2.43389	-2.40658	-1.37289
	(0.06985)	(0.22963)	(0.51133)	(0.08251)	(0.67805)	(0.51729)
Mean in Q4	.13147	.06901	.06767	.10333	.05447	.04744
N	633,858	53,060	25,046	671,293	34,578	105,222
Pseudo R2	.03972	.07648	.03464	.02724	.04449	.03476

Notes: This table shows the estimated Poisson regression (3.2) of any movement between firms in a year ($j(i,t+1) \neq j(i,t)$) on whether a worker applies for a new patent and its interactions with the quartile of her current employer, ranked by mean coworker wages in a year leaving out her own wage. The estimation sample is at person $\times$ year level. Models (1) and (4) are estimated for all potential inventors. (2) and (5) are restricted to matched inventors, and (3) and (6) are restricted to established inventors who have already applied for patents at former employers.

Table 5: Upward Mobility in a Year, by Quartile of Firms and Age Group

	Upward Move in 1 Year					
	Age ≤ 35			Age > 35		
	(1) Potential	(2) Matched	(3) Established	(4) Potential	(5) Matched	(6) Established
Quartile of Mean Coworker Wages						
Q1	0.62072	1.00693	0.31426	0.19444	0.28964	0.09279
	(0.01691)	(0.07624)	(0.13847)	(0.01731)	(0.09940)	(0.07238)
Q2	0.12605	0.25710	0.31704	-0.18266	0.12347	0.02660
	(0.01815)	(0.08475)	(0.12688)	(0.01630)	(0.10175)	(0.06571)
Q3	-0.31496	-0.52442	-0.23343	-0.55372	-0.54513	-0.24950
	(0.02091)	(0.10439)	(0.15004)	(0.01830)	(0.12132)	(0.06806)
(Base) Q4	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
Any New Patent Application						
y_{it}	-1.42371	-0.83458	-0.70814	-2.02244	-1.28762	-1.01158
	(0.27556)	(0.28452)	(0.23514)	(0.30068)	(0.31853)	(0.13289)
Excess Returns (relative to Q4)						
$y_{it}\times$ Q1	0.42581	-0.02449	0.16159	-0.83296	-1.17961	0.07819
	(0.31648)	(0.32447)	(0.29978)	(0.65004)	(0.66102)	(0.22454)
$y_{it}\times$ Q2	0.72674	0.57163	0.30795	0.76257	0.43080	0.41926
	(0.32355)	(0.33333)	(0.30503)	(0.39543)	(0.40654)	(0.18923)
$y_{it}\times$ Q3	0.28670	0.49060	0.52362	0.66010	0.78442	0.29967
	(0.37711)	(0.39043)	(0.32522)	(0.42535)	(0.44232)	(0.19699)
Constant	-4.26660	-3.69885	-3.21355	-3.45015	-2.99943	-1.81885
	(0.07779)	(0.36341)	(0.94528)	(0.13519)	(0.95439)	(0.81781)
Mean in Q4	.05173	.02705	.02633	.05177	.02506	.02247
N	620,048	52,445	24,728	661,388	34,169	104,230
Pseudo R2	.05160	.0974	.04192	.03584	.06634	.03750

Notes: This table shows the estimated Poisson regression (3.2) of upward mobility in 3 years ($Q(j(i,t+1)) > Q(j(i,t))$ or $Q(j(i,t+1)) = Q(j(i,t)) = 4$) on whether a worker applies for a new patent and its interactions with the quartile of her current employer, ranked by mean coworker wages in a year leaving out her own wage. The estimation sample is at person × year level. Models (1) and (4) are estimated for all potential inventors. (2) and (5) are restricted to matched inventors, and (3) and (6) are restricted to established inventors who have already applied for patents at former employers.

Appendix A: Model Details

A0. Model Timeline & Information Structure

There are two discrete periods in this model, as illustrated in Figure 7.

1. ($t = 1$) Employers start with zero employees. All workers are on the labor market looking for jobs.
   (a) Given initial information $\{I_{i1}\}$ about workers, employers share belief $\pi_{i1} = Pr(H|I_{i1})$ and post contracts $\{\underline{w}_{i1j}, \gamma_{i1j}\, \tau_{i1j}\}$ simultaneously, in which the base wage, the bonus for patenting, and investment on innovation $\underline{w}_{ij}$, $\gamma_{ij}\, \tau_{i1j} \geq 0$.
   (b) Each worker observes the wages posted by all firms, draws idiosyncratic preferences (4.2), and chooses the employer $j(i,1)$ that maximizes her utility (4.5) at $t = 1$. She will make effort e_{i1j} on innovation conditional on choosing $j = j(i,1)$.
   (c) Whether there is a public innovation, $y_{i1} \in \{0,1\}$, is realized by the end of $t = 1$. A worker receives a wage increase of $\gamma_{i1j} \times \underline{w}_{i1j}$ if $y_{i1} = 1$.
2. ($t = 2$) Let I_{i2} denote the information about i at the beginning of $t = 2$ (symmetric among all firms and workers).

$$\begin{aligned} y_{i1} = 1 &\rightarrow \text{public } I_{i2} = \{H\} \\ y_{i1} = 0 &\rightarrow \text{public } I_{i2} = I_{i1} \cup \{j(i,1), y_{i1} = 0\} \end{aligned} \tag{7.1}$$

   (a) Given information I_{i2}, firms post new contracts simultaneously by solving (**??**).
   (b) A worker reenters the job market with probability $\lambda \in [0,1]$. If she is on the market, she observes the new contracts, draws new preferences from (4.2), and chooses the employer that maximizes her expected utility (4.3). Otherwise, $j(i,2) = j(i,1)$ and she makes effort that maximizes her utility conditional on staying.
   (c) Repeat 1(c). The model concludes.

A1. Backward Induction

We clarify some notations that are used throughout the model.

$$\triangle_y f := E[f|y = 1] - E[f|y = 0] \tag{7.2}$$

Workers:

$$E[u_{i2j}|I_{i2}] = b \times E[ln(w_{i2j})|I_{i2}]$$

$$\overline{\Omega}(I_{i2}) = ln(\sum_j exp(E[u_{i2j}|I_{i2}])) \text{ value on market (Emax)}$$

$$p_{2j}(I_{i2}) = exp(E[u_{i2j}] - \overline{\Omega}(I_{i2})), \text{ incumbent: } p^{(1)}_{2j}(I_{i2}) = 1 - \lambda \times p_{2j}(I_{i2})$$

$$\Omega_j(I_{i2}) = (1 - \lambda) \times E[u_{i2j}] + \lambda \times \overline{\Omega}(I_{i2})$$

$$\triangle_y \Omega_j = \Omega_j(H) - \Omega_j(\pi(0)) = (1 - \lambda) \times b\, \triangle_y E[ln(w_{i2j})] + \lambda \times \triangle_y \overline{\Omega}$$

Firms:

$$v_{2j}^{(1)}(I_{i2}) = p_{2j}^{(1)}(I_{i2}) \times E[MP_j - w_{2j}|I_{i2}]$$
$$\text{“}EV2\text{''} = E[v_{2j}^{(1)}(\pi_{i2})|\pi_{i1}, \tau_{i1j}, e_{i1j}] = v_{2j}^{(1)}(\pi_{i2}(0)) + \pi_{i1} h(\tau_{i1j},\, e_{i1}) \times \, \triangle_y v_{2j}^{(1)}$$

We have stated the problems facing workers and firms in each period in Section 4.2. We complete the backward induction below.

Optimization at $t = 2$

At $t = 2$, workers who are on the job market solve (4.3), and the optimal effort at firm j satisfies:

$$\begin{aligned}
e_{i2j} &= argmax_e \; E[b\, ln(w_{i2j}) \,|\, \pi\, e, \tau] - c(e) \\
&= \pi_{i2} \underbrace{\frac{\partial h(\tau, e)}{\partial e}}_{\downarrow \text{as } \tau\uparrow \text{ if } h_{12}<0} \times \frac{b\, ln(1+\gamma_{i2j})}{c} \\
\frac{\partial e_{i2j}}{\partial \gamma} &= \pi \frac{\partial h(\tau, e)}{\partial e} \times \frac{b}{c} \times \frac{1}{1+\gamma} \\
\frac{\partial e_{i2j}}{\partial \tau} &= \pi \boldsymbol{h_{12}(\tau, e)} \times \frac{b\, ln(1+\gamma)}{c}
\end{aligned}$$

Firms solve (4.8) for incumbent ($\delta_{ij} = 1$) and new ($\delta_{ij} = 0$) employees, respectively. The first-order conditions for incumbent employees are:

$$\begin{aligned}
\frac{\partial}{\partial \underline{w}} &= \frac{\partial p_{2j}^{(1)}}{\partial \underline{w}} \times \left(MP_j - E_y[w_{2j}]\right) - p_{2j}^{(1)} \times (1 + \gamma \times \pi\, h(\tau, e)) \\
&= \lambda p_{2j} \times (1 - p_{2j}) \frac{b}{\underline{w}} \times \left(MP_j - \underline{w}(1 + \gamma \times \pi\, h(\tau, e))\right) - (1 - \lambda(1 - p_{2j})) \times (1 + \gamma \times \pi\, h(\tau, e)) = 0
\end{aligned}$$

$$\rightarrow \underline{w}_{2j}^{(1)}(\pi) = \frac{\xi_{2j}^{(1)}}{\xi_{2j}^{(1)} + 1} \frac{MP_j}{(1 + \gamma \times \pi\, h(\tau, e))} \text{ where elasticity } \boldsymbol{\xi_{2j}^{(1)} := \frac{\lambda p(1-p) \times b}{1 - \lambda(1-p)} = \frac{\lambda\, p(1-p) \times b}{p_{2j}^{(1)}}} \tag{7.3}$$

$$\text{and } MP_j - E_y[w_{2j}] = \frac{(1 + \gamma\pi\, h(\tau, e)) \times \underline{w}}{\xi_{2j}^{(1)}}$$

$$
\begin{aligned}
\frac{\partial}{\partial\gamma} &= \frac{\partial p_{2j}^{(1)}}{\partial\gamma}\times\left(MP_j - E_y[w_{2j}]\right) - p_{2j}^{(1)}\times\left(\underline{w}\pi\, h(\tau,e) + \frac{\partial MP_j - E[w_{2j}]}{\partial e}\frac{\partial e}{\partial\gamma}\right)\\
&= \lambda\left(\frac{\partial p_{2j}}{\partial\gamma} + \underbrace{\frac{\partial p_{2j}}{\partial e}\frac{\partial e}{\partial\gamma}}_{=0}\right)\times\left(MP_j - E_y[w_{2j}]\right) + p_{2j}^{(1)}\times\left(-\underline{w}\pi\, h(\tau,e) + \pi\frac{\partial h(\tau,e)}{\partial e}(f_j\theta-\gamma\underline{w})\frac{\partial e}{\partial\gamma}\right)\\
0 &= -\,\gamma\,\underline{w}h(\tau,e)\,\left(1-\pi h(\tau,e)\right) + \left(f_j\theta-\gamma\underline{w}\right)\pi\left(\frac{\partial h(\tau,e)}{\partial e}\right)^2\frac{b}{c}
\end{aligned}
\tag{7.4}
$$

$$
\begin{aligned}
\frac{\partial}{\partial\tau} &= \frac{\partial p_{2j}^{(1)}}{\partial\tau}\times\left(MP_j - E_y[w_{2j}]\right) + p_{2j}^{(1)}\times\left(\frac{\partial MP_j - E[w_{2j}]}{\partial\tau} + \frac{\partial MP_j - E[w_{2j}]}{\partial e}\frac{\partial e}{\partial\tau}\right)\\
&= \lambda\underbrace{\left(\frac{\partial p_{2j}}{\partial\tau} + \frac{\partial p_{2j}}{\partial e}\frac{\partial e}{\partial\tau}\right)}_{=0}\times\frac{(1+\gamma\pi\, h(\tau,e))\times\underline{w}}{\xi_{2j}^{(1)}} + p_{2j}^{(1)}\times\left(\pi\underbrace{\frac{\partial h(\tau,e)}{\partial\tau}}_{h_1}(f_j\theta-\gamma\underline{w}) - \zeta\tau + \frac{\partial MP_j - E[w_{2j}]}{\partial e}\frac{\partial e}{\partial\tau}\right)\\
\rightarrow \tau_{i2j}(\pi) &= \frac{\pi\,\left(f_j\theta-\gamma\underline{w}\right)}{\zeta}\times\left(h_1 + \pi\, h_2\, \boldsymbol{h_{12}}\,\frac{b\, ln(1+\gamma)}{c}\right)
\end{aligned}
\tag{7.5}
$$

Optimization at $t = 1$

At $t = 1$, workers take into account the option value at $t = 2$ if choosing a firm j, denoted by Ω_j. The optimal effort she would choose at firm j, as shown in (4.5) given the contracts firms posted, satisfies:[12]

$$
\begin{aligned}
e_{i1j} &= argmax_e\ \pi h(\tau_{i1j},e)\times b\, ln(1+\gamma_{i1j}) - c(e) + \beta_W E_y[\Omega_j(\pi_{i2})|\,\pi_{i1},\tau_{i1j},\, e]\\
&= \pi_{i1}\times\frac{\partial h(\tau_{i1j},e)}{\partial e}\times\frac{\left(b\, ln(1+\gamma_{i1j})\ + \beta_W\triangle_y\Omega_j\right)}{c}
\end{aligned}
\tag{7.6}
$$

$$
\begin{aligned}
\rightarrow \frac{\partial e_{i1j}}{\partial \underline{w}_{1j}} &= 0\\
\frac{\partial e_{i1j}}{\partial\gamma_{i1j}} &= \pi_{i1}h_2\times\frac{b/c}{1+\gamma}\approx\frac{e_{i1j}}{\gamma_{i1j}+(1+\gamma_{i1j})\beta_W/b\times\triangle_y\Omega_j}\\
\frac{\partial e_{i1j}}{\partial\tau_{i1j}} &= \pi_{i1}h_{12}\times\frac{\left(b\, ln(1+\gamma_{i1j})\ + \beta_W\triangle_y\Omega_j\right)}{c} = e_{i1j}\times\frac{h_{12}}{h_2},\ < 0 \text{ if } h_{12}<0
\end{aligned}
$$

Firms solve (4.9). Let $EV2$ denote the expected continuation value from incumbent employees who stay at $t = 2$: $EV2 := E[v_{2j}^{(1)}(\pi_{i2})|\pi_{i1},\tau_{i1j},e_{i1j}]$. The optimal contract set by firm j at $t = 1$ satisfy:

[12]Note worker effort e is more elastic w.r.t. wage incentive γ at $t = 2$ than $t = 1$: $\frac{\partial ln\, e_{i1j}}{\partial ln\,\gamma_{i1j}} < \frac{\partial ln\, e_{i2j}}{\partial ln\,\gamma_{i2j}} \approx 1$. In the first period, workers are motivated by an increase in their option values if they successfully innovate.

(ignoring i for a worker in the subscripts):

$$\frac{\partial}{\partial \underline{w}} = \frac{\partial p_{1j}}{\partial \underline{w}} \times \left(MP_j(\pi_1) - \underline{w}\,(1 + \gamma \times \pi_1 h(\tau, e_{1j})) + \beta_J EV2\right) - p_{1j} \times (1 + \gamma \times \pi_1 h(\tau, e_{1j})) = 0$$

$$\rightarrow \underline{w}_{1j}(\pi) = \frac{b(1 - p_{1j})}{(1 + b(1 - p_{1j})) \times (1 + \gamma \times \pi_1 h(\tau, e_{1j}))} \times \left(f_j(1 - \tau) + f_j \theta \pi_1 h(\tau, e_{1j}) - \frac{\zeta}{2}\tau^2 + \beta_J EV2\right) \tag{7.7}$$

$$MP_j - E[w_{1j}] + \beta_J EV2 = \underline{w}_{1j} \times \frac{(1 + \gamma \times \pi_1 h(\tau, e_{1j}))}{b(1 - p_{1j})}$$

$$\frac{\partial}{\partial \gamma} = \frac{\partial p_{1j}}{\partial \gamma} \times \left(MP_j - E[w_{1j}] + \beta_J EV2\right) + p_{1j} \times \left(-\underline{w} \times \pi_1 h(\tau, e_{1j}) + \frac{\partial (MP_j - E[w_{1j}] + EV2)}{\partial e} \times \frac{\partial e_{1j}}{\partial \gamma}\right) = 0$$

$$\text{where } \frac{\partial p_{1j}}{\partial \gamma} = \frac{\partial p_{1j}}{\partial \gamma} + \frac{\partial p_{1j}}{\partial e}\frac{\partial e}{\partial \gamma} = p_{1j}(1 - p_{1j}) \times \left(\pi h(\tau, e)\frac{b}{1 + \gamma} + 0 \times \frac{\partial e}{\partial \gamma}\right)$$

$$\rightarrow 0 = -\,\gamma \times \underline{w}\; h(\tau, e)\,(1 - \pi_1 h(\tau, e)) + \left(f_j \theta - \gamma \underline{w} + \beta_J \triangle_y v_{2j}^{(1)}\right) \times \frac{\pi\, b}{c}\,(h_2)^2 \tag{7.8}$$

$$\frac{\partial}{\partial \tau} = \frac{\partial p_{1j}}{\partial \tau} \times \left(MP_j - E[w_{1j}] + \beta_J EV2\right) + p_{1j} \times \left(\frac{\partial (MP_j - E[w_{1j}] + EV2)}{\partial \tau} + \frac{\partial (MP_j - E[w_{1j}] + EV2)}{\partial e} \times \frac{\partial e_{1j}}{\partial \tau}\right)$$

$$\text{where } \frac{\partial p_{1j}}{\partial \tau} = p_{1j}(1 - p_{1j}) \times \left(\frac{\partial u_{1j}}{\partial \tau} + 0 \times \frac{\partial e}{\partial \tau}\right) = p_{1j}(1 - p_{1j}) \times\; \pi h_1\; \left(b\, ln(1 + \gamma) + \beta_W \triangle \Omega_j\right)$$

$$\rightarrow \tau_{1j}(\pi) = \frac{\pi}{\zeta} \times \left[\left(f_j \theta + \beta_J \triangle v_{2j}^{(1)} - \gamma \underline{w}\right)(h_1 + e h_{12}) + \underline{w}\,(1 + \gamma\, \pi h) h_1\; \left(ln(1 + \gamma) + \frac{\beta_W \triangle \Omega_j}{b}\right)\right] \tag{7.9}$$

A2. Proof of Propositions

Proof of Proposition 1: Base Wages Increasing in Firm Productivity

<u>Proof (sketch):</u>
From 7.3 and 7.7, we can see the base wages set by firms in equilibrium are strictly increasing in productivity f_j, as the expected marginal revenue product of labor MP_j increases faster in productivity f_j than equilibrium γ or the inverse of labor supply elasticity.

Proof of Proposition 2: Heterogeneity in Firm Investment on Innovation

<u>Proof (sketch):</u>
In an imperfectly competitive labor market, we have $f_j \theta - \gamma \underline{w}$ to be increasing in firm productivity f_j. The optimal investment on innovation at $t = 2$ is shown in (7.5), and it is increasing in f_j under the assumption that $h_1 + h_2 \frac{\partial e}{\partial \tau} = h_1 + \pi h_2\, h_{12} \frac{b\, ln(1+\gamma)}{c} > 0$. Similarly, (7.9) shows the optimal investment at $t = 1$, in which we have the each component to be increasing in f_j.

Proof of Proposition 3: Heterogeneity in Wage Returns to Innovation

(a)
From (7.8), we have

$$\gamma_{1j}(\pi) = \frac{\pi b}{c} \times \frac{\left(f_j\theta + \beta_J \triangle_y v_{2j}^{(1)}\right)}{\underline{w}} \times \frac{(h_2)^2}{h - \pi h^2 + \frac{\pi b}{c}(h_2)^2} \tag{7.10}$$

$$\frac{\partial \gamma_{1j}}{\partial \lambda} \propto \beta_J \frac{\partial}{\partial \lambda}\left(\frac{\triangle_y v_{2j}^{(1)}}{\underline{w}_{1j}}\right) < 0 \tag{7.11}$$

$$\text{note } \frac{\partial \triangle_y v_{2j}^{(1)}}{\partial \lambda} = -\triangle_y \left[(1-p_{2j})\left(MP_j - E[w_{2j}]\right)\right]$$

(b)
From the FOC at $t = 1$ w.r.t. γ (7.8), we have:

$$\begin{aligned}\frac{\partial ln(\gamma_{1j})}{\partial f_j} &= \frac{\partial\, ln\left(\frac{f_j\theta+\beta_J\triangle_y v_{2j}^{(1)}}{\underline{w}}\right)}{\partial f_j} + \frac{\partial}{\partial f_j} ln\left(\frac{(h_2)^2}{h-\pi h^2+\frac{\pi b}{c}(h_2)^2}\right) \\ &= \frac{\partial\, \mathbf{ln}\left(\frac{\mathbf{f_j}\theta+\beta_\mathbf{J}\triangle_\mathbf{y}\mathbf{v}_{\mathbf{2j}}^{(\mathbf{1})}}{\underline{\mathbf{w}}}\right)}{\partial \mathbf{f_j}} - \underbrace{\frac{\partial \tau_{1j}}{\partial f_j}}_{>0 \text{ Prop } 2} \times \frac{2\left(h_{12}+h_{22}\frac{\partial e}{\partial \tau}\right)\frac{(h-\pi h^2)}{h_2} - \left((1-2\pi h)\left(h_1+h_2\frac{\partial e}{\partial \tau}\right)\right)}{h-\pi h^2+\frac{\pi b}{c}(h_2)^2}\end{aligned} \tag{7.12}$$

We have $\frac{\partial \gamma_{1j}}{\partial f_j} < 0$ iff

$$\frac{\partial\, \mathbf{ln}\left(\frac{\mathbf{f_j}\theta+\beta_\mathbf{J}\triangle_\mathbf{y}\mathbf{v}_{\mathbf{2j}}^{(\mathbf{1})}}{\underline{\mathbf{w}}}\right)}{\partial \mathbf{f_j}} < \frac{\partial \tau}{\partial f_j} \times \frac{-2\left(h_{12}+h_{22}\frac{\partial e}{\partial \tau}\right)\frac{(h-\pi h^2)}{h_2} + \left((1-2\pi h)\left(h_1+h_2\frac{\partial e}{\partial \tau}\right)\right)}{h-\pi h^2+\frac{\pi b}{c}(h_2)^2}$$

(c)
From a firm's FOC w.r.t. γ at $t = 2$ (7.4), we have:

$$\gamma_{2j} = \frac{\pi b\theta}{c} \times \frac{f_j}{\underline{w}} \times \frac{(h_2)^2}{h-\pi h^2+\frac{\pi b}{c}(h_2)^2} \tag{7.13}$$

When $\lambda \uparrow$, the only object that varies on the RHS is $\underline{w}$, which increases according to (7.3). Hence we have:

$$\frac{\partial \gamma_{2j}}{\partial \lambda} < 0 \tag{7.14}$$

(d)

Differentiating both sides of (7.4) over firm productivity f_j, we have:

$$\frac{\partial ln(\gamma_{2j})}{\partial f_j} = \frac{\partial\, ln\left(\frac{f_j}{\underline{w}}\right)}{\partial f_j} + \frac{\partial}{\partial f_j} ln\left(\frac{(h_2)^2}{h - \pi h^2 + \frac{\pi b}{c}(h_2)^2}\right) \tag{7.15}$$
$$= \frac{1}{f_j} - \frac{\partial ln(\underline{w}_{2j})}{\partial f_j} - \underbrace{\frac{\partial \tau_{2j}}{\partial f_j}}_{>0 \text{ Prop } 2} \times \frac{2\left(h_{12} + h_{22}\frac{\partial e}{\partial \tau}\right)\frac{(h-\pi h^2)}{h_2} - \left((1-2\pi h)\left(h_1 + h_2\frac{\partial e}{\partial \tau}\right)\right)}{h - \pi h^2 + \frac{\pi b}{c}(h_2)^2}$$

$$\frac{\partial \gamma_{2j}}{\partial f_j} \propto \underbrace{\frac{\partial (f_j/\underline{w})}{\partial f_j}}_{>\mathbf{0}} \times \left(\frac{(h_2)^2}{h - \pi h^2 + \frac{\pi b}{c}(h_2)^2}\right) + \frac{f_j}{\underline{w}} \times \underbrace{\left[\frac{\partial}{\partial \tau} + \frac{\partial}{\partial e} \times \frac{\partial e}{\partial \tau}\right]}_{(***)} \times \frac{\partial \tau}{\partial f_j} \tag{7.16}$$

Hence we have:

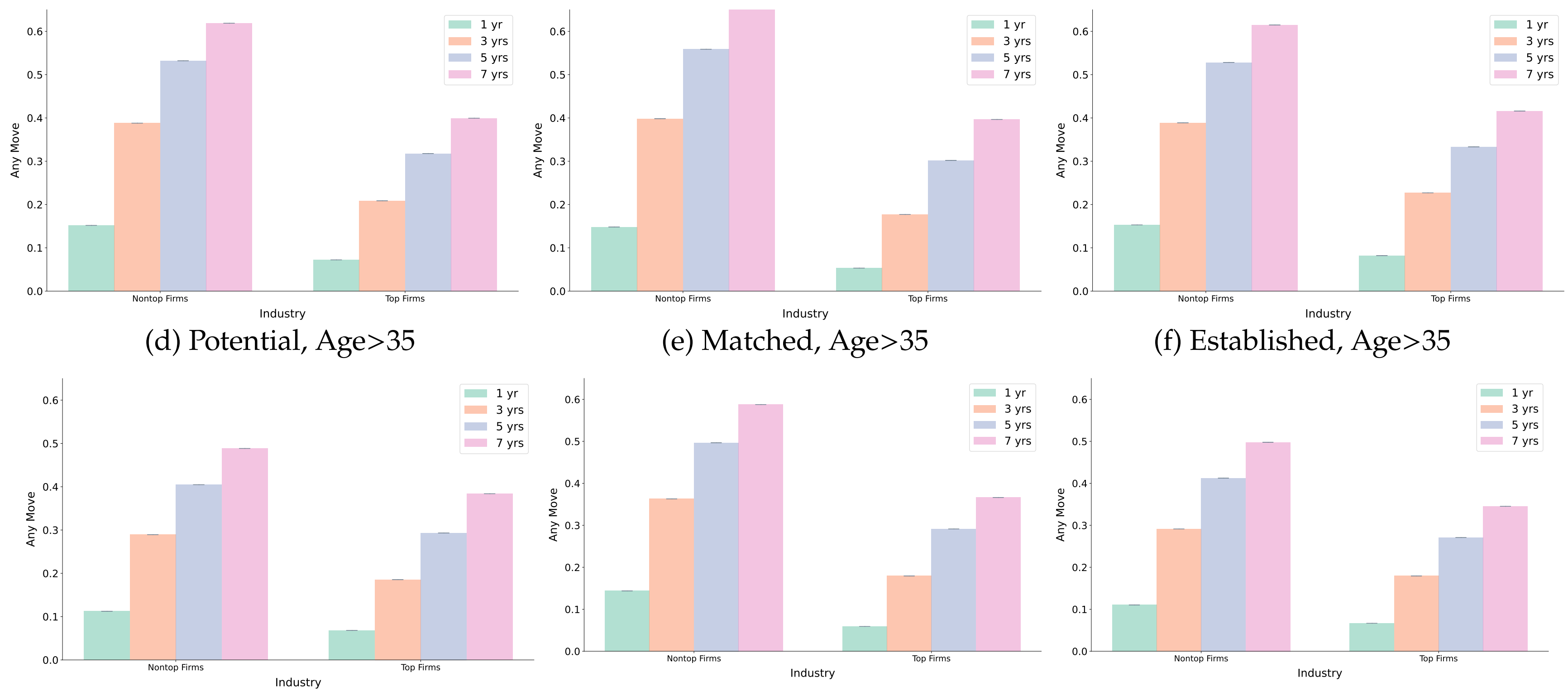
$$\frac{\partial ln \gamma_{2j}}{\partial f_j} < 0 \iff \underbrace{\frac{\partial}{\partial f_j} ln(f_j/\underline{w})}_{=\frac{1}{f_j} - \frac{\partial ln w}{\partial f_j}} < \frac{\partial}{\partial \tau} ln\left(\frac{h - \pi h^2 + \frac{\pi b}{c}(h_2)^2}{(h_2)^2}\right) \times \frac{\partial \tau_{2j}}{\partial f_j} \tag{7.17}$$

The condition is satisfied when (τ, e) are substitutes with $h_{12} \ll 0$, in which case

- High-f_j firms are setting lower γ to avoid excessive efforts from workers (diminishing return to effort as $h_{12} < 0$).
- Low-f_j firms on the other hand set lower τ but higher γ to elicit more efforts from workers, whose h_2 are higher when τ is low.

Appendix B: Additional Empirical Results

Appendix Figure B1: Between-firm Mobility of Ph.D. Computer Scientists (Inventor = Has Applied for a Patent)

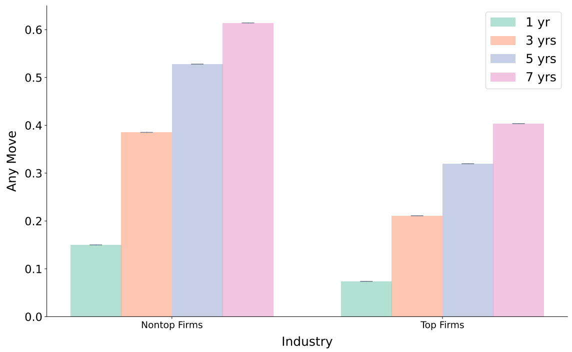


(a) Potential, Age≤35
(b) Matched, Age≤35
(c) Established, Age≤35
(d) Potential, Age>35
(e) Matched, Age>35
(f) Established, Age>35

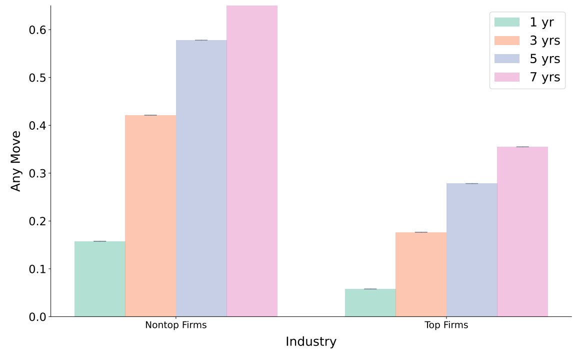
Notes: This figure replicates Figure 5 and shows the mean between-firm mobility of CS workers in the United States, using the matched LinkedIn-Publication-Patent database for over forty thousand Ph.D. computer scientists in Wu (2023). We show the fraction employed by a different firm, $j(i, t + k) \neq j(i, t)$, in $k \in \{1, 3, 5, 7\}$ years, separately by age group, whether a worker is employed by a top firm (Google/Meta/Amazon/Microsoft/Apple/IBM), and inventor status. To compare with the inventor status in Figure 5, we define potential inventors as computer scientists who have not applied for a patent. Matched inventors are those who have not applied for a patent, but will in the future. And established inventors are those who have already applied.

Appendix Figure B2: Between-firm Mobility of Ph.D. Computer Scientists (Inventor = Has Published a Paper)

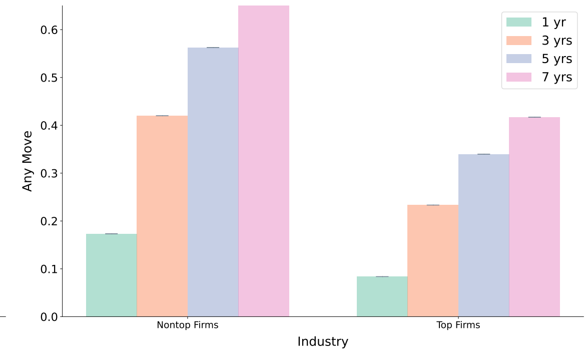
(a) Potential, Age≤35



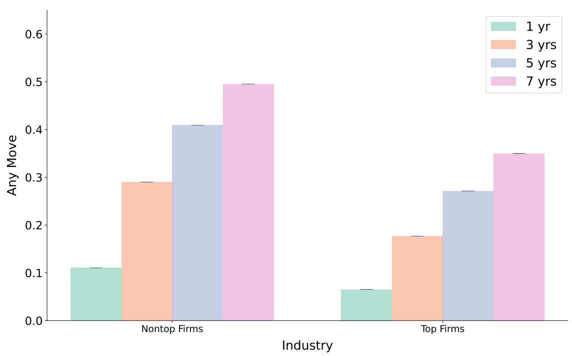
(b) Matched, Age≤35



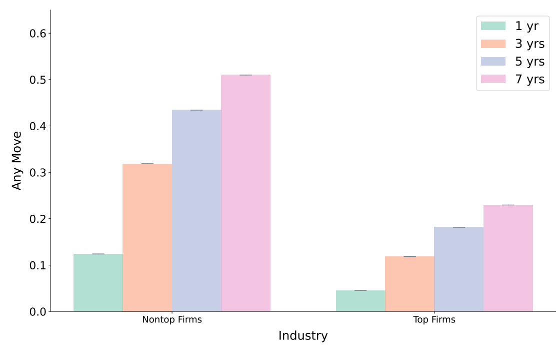
(c) Established, Age≤35



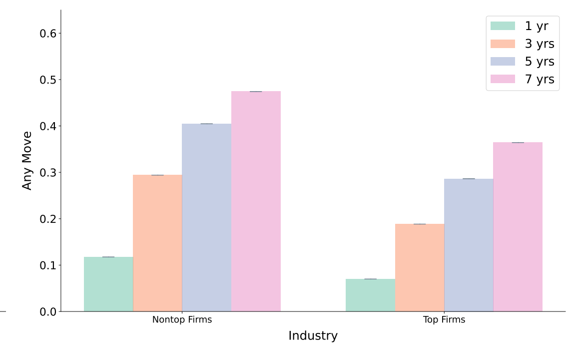
(d) Potential, Age>35



(e) Matched, Age>35

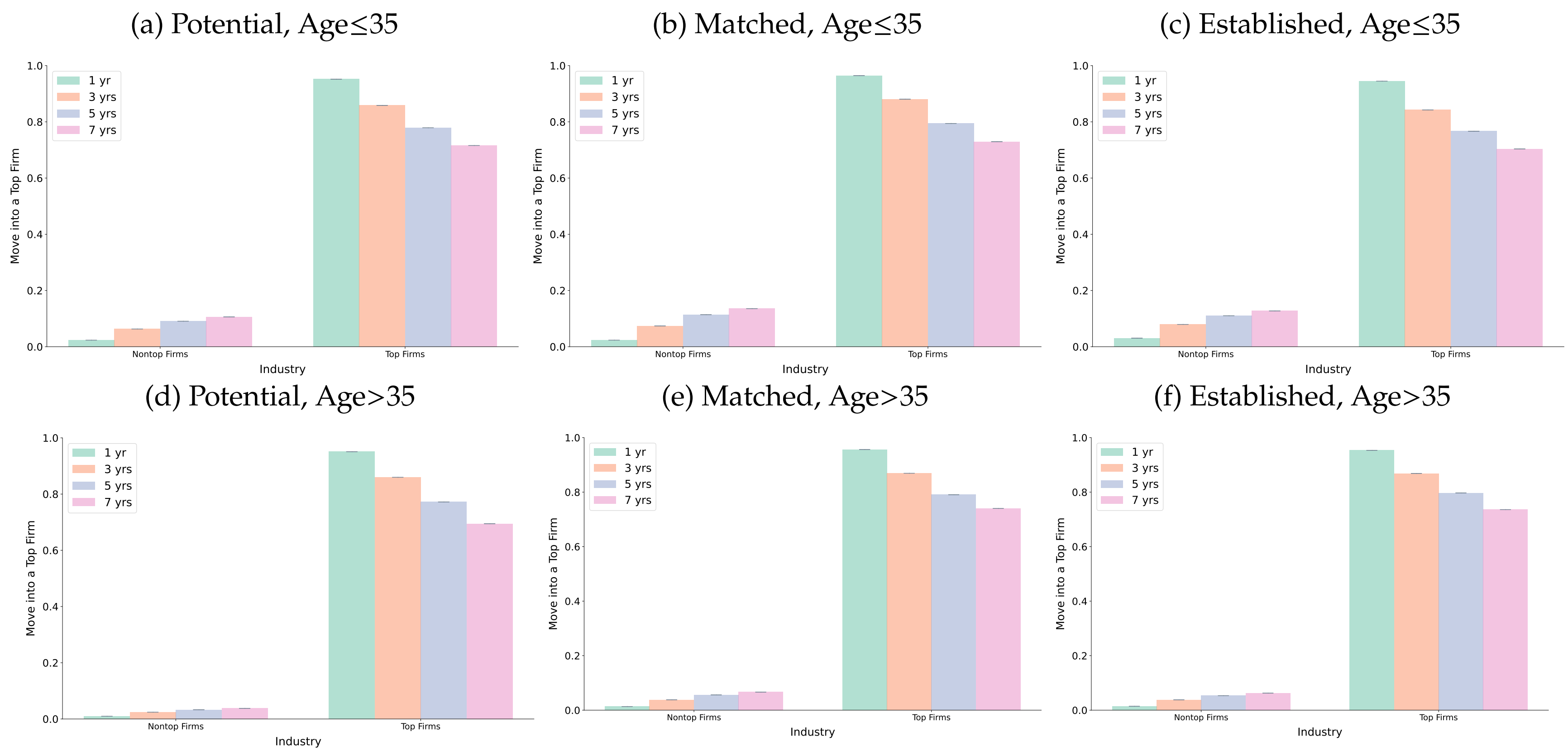


(f) Established, Age>35



Notes: This figure replicates Figure 5 and shows the mean between-firm mobility of CS workers in the United States, using the matched LinkedIn-Publication-Patent database for over forty thousand Ph.D. computer scientists in Wu (2023). We show the fraction employed by a different firm, $j(i, t+k) \neq j(i, t)$, in $k \in \{1, 3, 5, 7\}$ years, separately by age group, whether a worker is employed by a top firm (Google/Meta/Amazon/Microsoft/Apple/IBM), and inventor status. To compare with the inventor status in Figure 5, we define potential inventors as computer scientists who have not published a paper in CS conferences. Matched inventors are those who are yet to but will publish in the future. And established inventors are those who have already published.

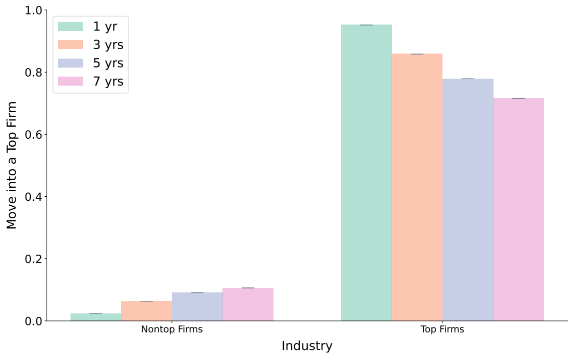
Appendix Figure B3: Upward Mobility of Ph.D. Computer Scientists (Inventor = Has Applied for a Patent)



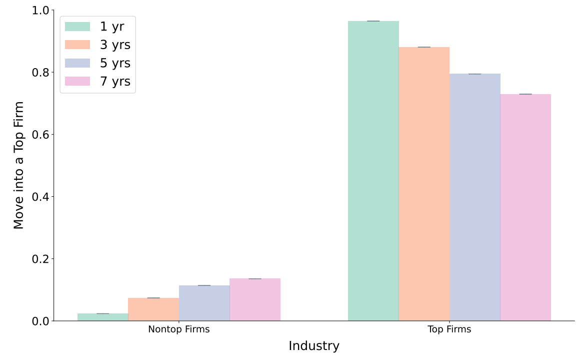
Notes: This figure replicates Figure 6 and shows the mean upward mobility of CS workers in the United States, using the matched LinkedIn-Publication-Patent database for over forty thousand Ph.D. computer scientists in Wu (2023). Upward mobility is defined as being employed by a top firm (Google/Meta/Amazon/Microsoft/Apple/IBM). To compare with the inventor status in Figure 6, we define potential inventors as computer scientists who have not applied for a patent. Matched inventors are those who have not applied for a patent, but will in the future. And established inventors are those who have already applied.

Appendix Figure B4: Upward Mobility of Ph.D. Computer Scientists (Inventor = Has Applied for a Patent)

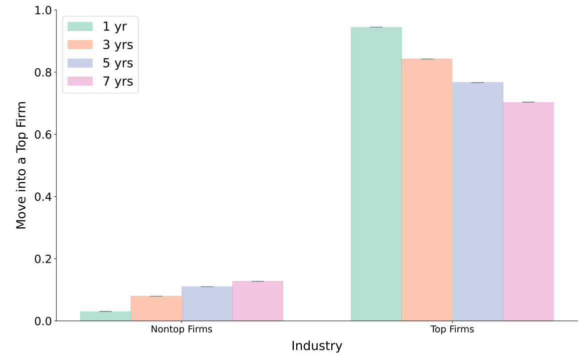
(a) Potential, Age≤35



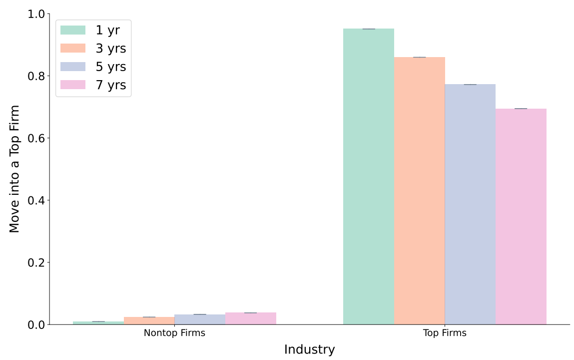
(b) Matched, Age≤35



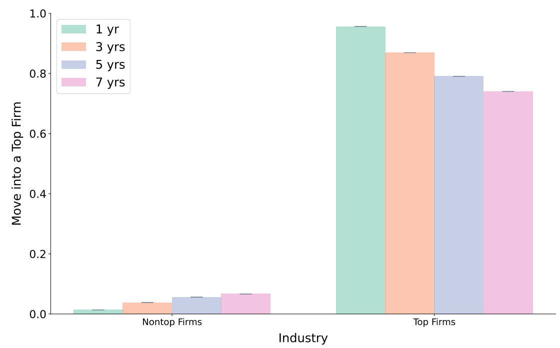
(c) Established, Age≤35



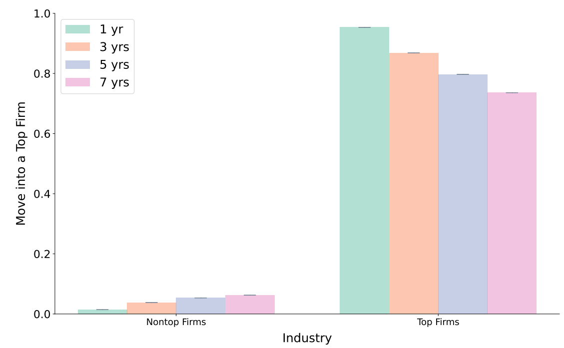
(d) Potential, Age>35



(e) Matched, Age>35



(f) Established, Age>35



Notes: This figure replicates Figure 6 and shows the mean upward mobility of CS workers in the United States, using the matched LinkedIn-Publication-Patent database for over forty thousand Ph.D. computer scientists in Wu (2023). Upward mobility is defined as being employed by a top firm (Google/Meta/Amazon/Microsoft/Apple/IBM). To compare with the inventor status in Figure 6, we define potential inventors as computer scientists who have not published a paper in CS conferences. Matched inventors are those who are yet to but will publish in the future. And established inventors are those who have already published.

Appendix Table B1: Poisson Regressions of Ever Inventing

	Full Sample	
	(1)	(2)
Demographics		
Female	-1.29809	-1.22436
	(0.00849)	(0.00857)
Age (normalized at 35)	-1.18215	-0.49511
	(0.06283)	(0.06022)
Age^2, Age^3	X	X
Job Characteristics		
Tenure	0.16152	0.26883
	(0.03540)	(0.03514)
$Tenure^2$, $Tenure^3$	X	X
Blue Collar	-0.49502	-0.80953
	(0.03692)	(0.04304)
White Collar	1.25696	1.10266
	(0.04116)	(0.04033)
Blue/White × Age	X	X
Permanent Contract	0.45127	0.05251
	(0.03263)	(0.03210)
Temporary Contract	0.03128	0.01116
	(0.02861)	(0.02793))
Seasonal Contract	-1.58927	-1.19540
	(0.29588)	(0.34350)
Contract Type × Age	X	X
INPS Sample (1987-2009)		
Min(Yr | INPS)=1987	-0.17688	-0.15301
	(0.00974)	(0.00946)
Min(Age | INPS)	0.66709	0.50071
	(0.00971)	(0.00966)
(Min(Yr)=1987) × Min(Age)	0.02812	0.01559
	(0.00834)	(0.00809)
Constant	-4.74880	-3.51388
	(0.04737)	(0.04647)
Fixed Effects		
Year	X	X
Industry and Region	X	
Firm		X
N	1.25e+07	8,434,000
Pseudo R2	0.10132	0.22189

Notes: This table shows the estimated Poisson regression (2.1) of $Inv_i = 1$ if person i has any patent application matched to INPS on observable characteristics above. There are about 1.5 million workers who have ≥ 5 years of employment in the INPS data between 1987 and 2009, who entered the sample between age 14 and age 55, and who have worked in more white-collar jobs than blue-collar (see summary statistics in Column 1 of Table 1). The estimation sample is at the person× year level. We use the estimated p-scores from columns (2) conditional on firm fixed effects to select potential inventors (Section 2.2).

Appendix Table B2: Any New Patent Application, by Quartile of Firms and Age Group

	Any New Patent Application					
	Age ≤ 35			Age > 35		
	(1) Potential	(2) Matched	(3) Established	(4) Potential	(5) Matched	(6) Established
Quartile of Mean Coworker Wages						
Q1	-0.2720	-0.0635	0.1538	0.0644	0.0752	0.1550
	(0.0291)	(0.0265)	(0.0369)	(0.0193)	(0.0180)	(0.0201)
Q2	-0.1514	-0.0646	0.0329	-0.0938	0.1030	0.1451
	(0.0277)	(0.0255)	(0.0350)	(0.0168)	(0.0156)	(0.0175)
Q3	0.0619	-0.0195	0.0815	0.0193	0.1056	0.1465
	(0.0267)	(0.0257)	(0.0347)	(0.0159)	(0.0149)	(0.0165)
(Base) Q4	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
Mean in Q4	.02140	.16778	.23374	.03207	.17734	.18112
N	670,589	79,195	26,291	862,921	152,548	117,741
Pseudo R2	.07587	.06551	.02496	.03149	.01380	.01927

Notes: This table shows the estimated Poisson regression (3.1) of whether a worker produces a new patent application on the quartile of a worker's current employer, ranked by mean coworker wages in a year leaving out her own wage. The estimation sample is at person × year level. See notes under Table 2 for covariates and fixed effects. Models (1) and (4) are estimated for all potential inventors in sample. (2) and (5) are restricted to matched inventors who have at least one patent application matched with their INPS employment records, and (3) and (6) are restricted to established inventors who have already applied for patents at former employers.

Appendix Table B3: Wage Returns to New Patent Application, by Quartile of Firms and Age Group

	Log Annual Wages					
	Age ≤ 35			Age > 35		
	(1) Potential	(2) Matched	(3) Established	(4) Potential	(5) Matched	(6) Established
Quartile of Mean Coworker Wages						
Q1	-0.3200	-0.2203	-0.2084	-0.4845	-0.2834	-0.3126
	(0.0013)	(0.0044)	(0.0072)	(0.0018)	(0.0084)	(0.0050)
Q2	-0.1811	-0.1087	-0.1256	-0.3296	-0.1873	-0.1960
	(0.0013)	(0.0040)	(0.0064)	(0.0016)	(0.0071)	(0.0044)
Q3	-0.1193	-0.0814	-0.0900	-0.2290	-0.1365	-0.1471
	(0.0013)	(0.0041)	(0.0063)	(0.0016)	(0.0070)	(0.0042)
(Base) Q4	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
Any New Patent Application						
y_{it}	-0.0118	0.0167	0.0306	0.0478	0.0008	0.0552
	(0.0085)	(0.0088)	(0.0093)	(0.0122)	(0.0130)	(0.0067)
Excess Returns (relative to Q4)						
$y_{it}\times$ Q1	0.1694	0.0716	0.0359	0.2167	0.0244	0.0705
	(0.0105)	(0.0110)	(0.0117)	(0.0195)	(0.0207)	(0.0104)
$y_{it}\times$ Q2	0.1001	0.0175	0.0336	0.1723	0.0495	0.0867
	(0.0105)	(0.0108)	(0.0120)	(0.0163)	(0.0172)	(0.0097)
$y_{it}\times$ Q3	0.0738	0.0176	0.0138	0.0571	0.0057	0.0471
	(0.0104)	(0.0107)	(0.0113)	(0.0160)	(0.0168)	(0.0091)
Constant	7.6911	7.8622	7.7834	7.6578	7.9718	7.8818
	(0.0111)	(0.0249)	(0.0669)	(0.0147)	(0.0789)	(0.0654)
Mean in Q4	7.75271	7.67763	7.83742	8.27185	8.25162	8.41673
N	649,892	53,108	26,376	746,369	34,750	117,834
Adjusted R2	.44019	.46108	.26083	.23313	.22173	.22607

Notes: This table shows the estimated OLS regression (3.2) of log annual wages on whether a worker applies for a new patent and its interactions with the quartile of her current employer, ranked by mean coworker wages in a year leaving out her own wage. The estimation sample is at person × year level. All models control for the covariates noted under Table 2, including year/region/2-digit ateco (industry) fixed effects, but no person effects as in Table 3. Models (1) and (4) are estimated for all potential inventors. (2) and (5) are restricted to matched inventors, and (3) and (6) are restricted to established inventors who have already applied for patents at former employers.

Appendix Table B4: Between-Firm Job Mobility in 3 Years, by Quartile of Firms and Age Group

	Move in 3 Years: $j(i, t+3) \neq j(i, t)$					
	Age $\leq$ 35			Age > 35		
	(1) Potential	(2) Matched	(3) Established	(4) Potential	(5) Matched	(6) Established
Quartile of Mean Coworker Wages						
Q1	0.00132	0.24340	-0.20798	-0.10931	-0.04373	-0.15025
	(0.00612)	(0.02982)	(0.05861)	(0.00764)	(0.04312)	(0.03410)
Q2	-0.09275	-0.01675	0.00595	-0.15138	-0.11388	-0.00013
	(0.00638)	(0.03192)	(0.05101)	(0.00677)	(0.04286)	(0.02967)
Q3	-0.04848	-0.07650	0.01924	-0.12431	-0.21944	-0.02723
	(0.00654)	(0.03335)	(0.05065)	(0.00664)	(0.04385)	(0.02806)
Any New Patent Application						
y_{it}	-0.73730	-0.11588	-0.33907	-1.26176	-0.54376	-0.63110
	(0.07335)	(0.07705)	(0.07909)	(0.09118)	(0.09523)	(0.05160)
Excess Returns (relative to Q4)						
$y_{it}\times$ Q1	-0.21736	-0.49549	-0.20240	0.17715	-0.07985	-0.18399
	(0.10279)	(0.10666)	(0.12178)	(0.14905)	(0.15491)	(0.10005)
$y_{it}\times$ Q2	0.15591	0.08217	-0.03744	0.41359	0.33146	0.14275
	(0.09669)	(0.10134)	(0.11500)	(0.12753)	(0.13339)	(0.07888)
$y_{it}\times$ Q3	0.09074	0.19324	0.15924	0.22008	0.38380	0.23596
	(0.09688)	(0.10187)	(0.10743)	(0.12842)	(0.13447)	(0.07277)
Constant	-2.55223	-2.03242	-1.01033	-1.96823	-1.35456	-0.55204
	(0.04311)	(0.14379)	(0.30888)	(0.05902)	(0.44214)	(0.34562)
Mean in Q4	.32467	.17861	.17338	.26807	.13914	.12477
N	588,284	52,714	21,648	532,345	33,837	81,894
Pseudo R2	.03224	.05580	.02932	.02727	.03819	.03225

Notes: This table shows the estimated Poisson regression (3.2) of any movement between firms in 3 years ($j(i, t+3) \neq j(i, t)$) on whether a worker applies for a new patent and its interactions with the quartile of her current employer, ranked by mean coworker wages in a year leaving out her own wage. The estimation sample is at person × year level. Models (1) and (4) are estimated for all potential inventors. (2) and (5) are restricted to matched inventors, and (3) and (6) are restricted to established inventors who have already applied for patents at former employers.

Appendix Table B5: Upward Mobility in 3 Years, by Quartile of Firms and Age Group

	Upward Move in 3 Years					
	Age ≤ 35			Age > 35		
	(1) Potential	(2) Matched	(3) Established	(4) Potential	(5) Matched	(6) Established
Quartile of Mean Coworker Wages						
Q1	0.57308	0.91552	0.29004	0.23726	0.28519	0.12952
	(0.01024)	(0.04631)	(0.08634)	(0.01113)	(0.06040)	(0.04855)
Q2	0.12022	0.20392	0.11095	-0.17854	-0.03292	0.10389
	(0.01093)	(0.05141)	(0.07958)	(0.01075)	(0.06407)	(0.04349)
Q3	-0.33204	-0.54420	-0.34776	-0.55231	-0.61994	-0.35700
	(0.01274)	(0.06373)	(0.09112)	(0.01209)	(0.07646)	(0.04685)
(Base) Q4	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
Any New Patent Application						
y_{it}	-0.77767	-0.16809	-0.59389	-1.25810	-0.47312	-0.71296
	(0.12283)	(0.12868)	(0.13337)	(0.13332)	(0.13988)	(0.07763)
Excess Returns (relative to Q4)						
$y_{it}\times$ Q1	-0.05445	-0.38882	0.07970	-0.13127	-0.39407	0.02381
	(0.15181)	(0.15845)	(0.17803)	(0.22331)	(0.23296)	(0.13265)
$y_{it}\times$ Q2	0.24734	0.20402	0.27936	0.63350	0.45444	0.21458
	(0.15524)	(0.16315)	(0.18131)	(0.18039)	(0.19025)	(0.11467)
$y_{it}\times$ Q3	-0.03911	0.24782	0.34541	0.49179	0.65269	0.39985
	(0.18426)	(0.19370)	(0.19579)	(0.19716)	(0.20947)	(0.11598)
Cobstant	-3.61368	-3.11895	-1.76196	-2.66503	-2.12179	-0.59834
	(0.04929)	(0.22134)	(0.53307)	(0.09520)	(0.65245)	(0.55830)
Mean in Q4	.13907	.07400	.07876	.13968	.06800	.06136
N	567,938	51,977	21,322	518,663	33,292	80,667
Pseudo R2	.05332	.09371	.03875	.04045	.05874	.03550

Notes: This table shows the estimated Poisson regression (3.2) of upward mobility in 3 years ($Q(j(i,t+3)) > Q(j(i,t))$ or $Q(j(i,t+3)) = Q(j(i,t)) = 4$) on whether a worker applies for a new patent and its interactions with the quartile of her current employer, ranked by mean coworker wages in a year leaving out her own wage. The estimation sample is at person × year level. Models (1) and (4) are estimated for all potential inventors. (2) and (5) are restricted to matched inventors, and (3) and (6) are restricted to established inventors who have already applied for patents at former employers.