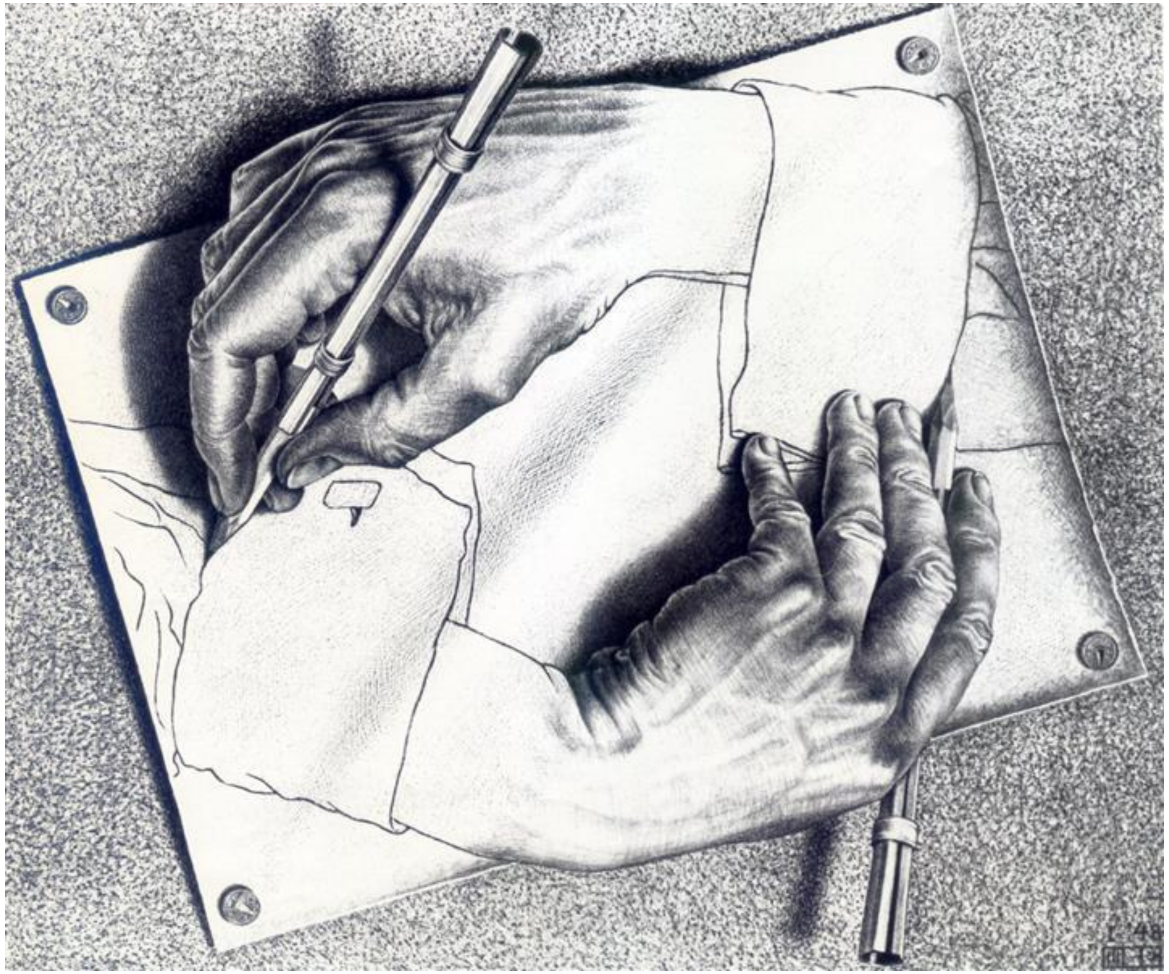
Covariant Formulation of 2D Chiral Scalars



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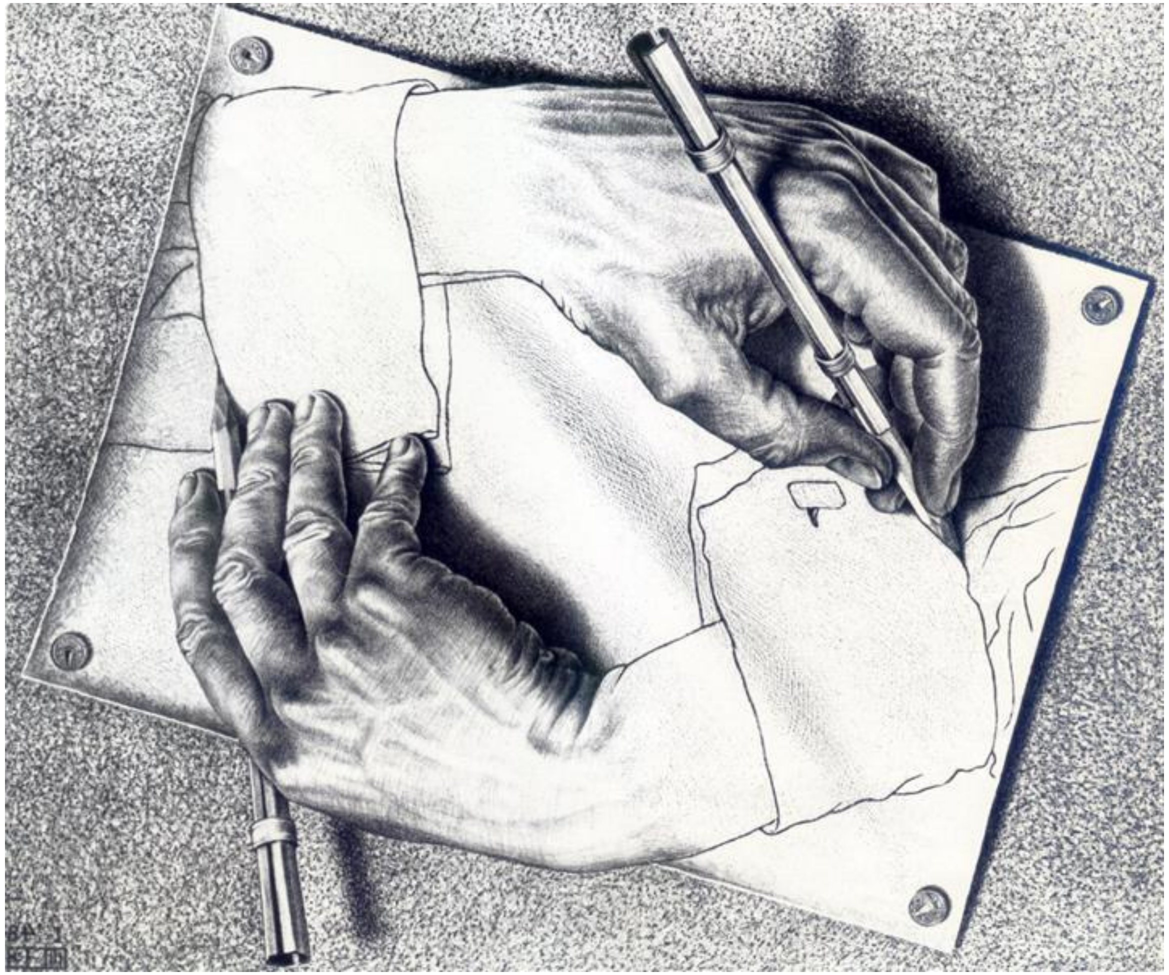
Covariant Formulation of 2D Chiral Scalars



based on 2501.16463 in collaboration with E. Joung, K. Mkrtchyan, and J. Yoon

Warm-Up: Maxwell Theory

Electromagnetic Duality

Consider the theory for gauge field $A \in \Omega^1(M)$ on a 4-manifold (M, g), described by the **Maxwell action**

$$S_{\text{Maxwell}} = \int_M -\frac{1}{2} F \wedge \star F, \quad F = \mathrm{d}A$$

The field strength obeys:

Bianchi Identity

$$\mathrm{d}^2 = 0 \longrightarrow \mathrm{d}F = 0$$

Equation of Motion

$$\delta S = 0 \longrightarrow \mathrm{d} \star F = 0$$

- **Electromagnetic Duality** refers to invariance of equations under

$$F \mapsto \star F, \quad \star F \mapsto -F$$

- Here: Enhancement to invariance of equations under $(S)O(2)$ rotations

$$\begin{pmatrix} F \\ \star F \end{pmatrix} \mapsto \begin{pmatrix} \cos(\beta) & \sin(\beta) \\ -\sin(\beta) & \cos(\beta) \end{pmatrix} \begin{pmatrix} F \\ \star F \end{pmatrix}$$

Off-Shell Duality Invariance

Switch to vector notation

$$E_i = F_{0i}, \quad B^i = \frac{1}{2}\epsilon^{ijk}F_{jk}$$

- Action takes the form

$$S_{\mathrm{Maxwell}} = \frac{1}{2}\int \mathrm{d}^4x \left(|\mathbf{E}|^2 - |\mathbf{B}|^2\right)$$

- **Duality rotation** is now

$$\begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix} \mapsto \begin{pmatrix}\cos(\beta) & \sin(\beta)\\ -\sin(\beta) & \cos(\beta)\end{pmatrix}\begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix}$$

→ Action only invariant under **hyperbolic** rotations?

Is EM duality only a "symmetry" of the **equations of motion**?

Key insight: *"Formal transformation [...] only meaningful [...] if it can be implemented at the level of the basic field variable [...]."*

[Deser & Teitelboim '76]

Off-Shell Duality Invariance

- In **first-order** form:

$$S_{\text{Maxwell}} = \frac{1}{2}\int \mathrm{d}^4x \left(|\dot{\mathbf{A}}^T|^2 - |\boldsymbol{\nabla}\times\mathbf{A}^T|^2 + |\dot{\mathbf{A}}^L - \boldsymbol{\nabla}A_0|^2 \right)$$

→ Third term is constraint, and e.o.m. is $\ddot{\mathbf{A}}^T = \nabla^2\mathbf{A}^T$.

Now consider following transformation

$$\delta\mathbf{A}^T = -\beta\nabla^{-2}\boldsymbol{\nabla}\times\dot{\mathbf{A}}^T$$

- This generates

$$\delta\mathbf{E} = \delta\dot{\mathbf{A}}^T = -\beta\nabla^{-2}\boldsymbol{\nabla}\times\ddot{\mathbf{A}}^T, \quad \delta\mathbf{B} = \boldsymbol{\nabla}\times\delta\mathbf{A}^T = \beta\mathbf{E}$$

→ **Duality rotation on-shell**.

- Changes action by **total time derivative**, and generated by (time-local) **conserved charge**.

→ Duality invariance is off-shell **symmetry,** BUT have to break **manifest covariance** to see this.

Twisted Self-Duality

Can make **duality** manifest using **doubled field content**.

- Introduce extra index $a \in \{1,2\}$ on gauge fields A^a. Then

$$F^a = \mathrm{d}A^a \qquad \longrightarrow \qquad \mathrm{d}F^a = 0$$

→ By construction, **invariant** under $SO(2)$ rotations in "extra directions".

- Would like to identify

$$F^1 = \star F, \quad F^2 = F$$

such that **EM duality** is equivalent to condition:

$$\star F^2 = F^1$$

→ This is **twisted self-duality relation**: Implies original equations of motion.

Manifest Duality

Duality invariance made manifest by following local action

[Schwarz & Sen ’93]
[Henneaux & Teitelboim ‘11]

$$S_{\text{Schwarz-Sen}} = -\frac{1}{2}\int \mathrm{d}^4x \left(\delta_{ab}\mathbf{B}^a \cdot \mathbf{B}^b - \varepsilon_{ab}\mathbf{E}^a \cdot \mathbf{B}^b\right)$$

- Equation of motion for $\mathbf{A}^2$ sets $\mathbf{B}^2 = \mathbf{E}^1$ (up to gauge transformations). Plugging back into action, recover

$$S_{\text{Schwarz-Sen}} = -\frac{1}{2}\int \mathrm{d}^4x \left(\mathbf{B}^1 \cdot \mathbf{B}^1 - \mathbf{E}^1 \cdot \mathbf{E}^1\right)$$

  → **Maxwell** action.

- Manifestly gauge invariant.

- Not manifestly covariant, but it is: Manifest invariance under spatial rotations but not **boosts** – latter changes action, but by boundary term.

Central tension between manifest duality and covariance

(Twisted) Self-Duality

Are there other examples of (twisted) self-duality relations?

Linearised Gravity

For example, consider **linearised gravity** in D dimensions.

- In vacuum, linearised Riemann tensor satisfies

Bianchi Identity

$$R_{\mu[\nu\rho\sigma]} = R_{\mu\nu[\rho\sigma,\lambda]} = 0$$

Equation of Motion

$$R_{\mu\nu} = 0$$

- Same relations satisfied by linearised **dual Riemann** tensor

$$\tilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}$$

- **Gravitational Duality**: $R \mapsto \tilde{R}$ (enhanced to full rotations in $D=4$) **off-shell symmetry** of theory – can phrase as twisted self-duality.

[Henneaux & Teitelboim '05; Bünster, Henneaux, and Hörtner '13]

→ Make manifest at expense of **covariance**.

- Similar holds for **(partially) massless higher-spin** fields.

- **Non-linearities**: Exchanges Schwarzschild mass with **Taub-NUT** parameter.

Form Fields

Most straightforward **generalisation** to Maxwell theory.

[Henneaux '85]

Theory of p-form gauge field $A \in \Omega^p(M)$ on manifold (M, g) with $\dim(M) = d$, described by action

$$S = \int_M -\frac{1}{2} F \wedge \star F, \quad F = \mathrm{d}A$$

- Numerous examples in **string theory**/**supergravity**: $p = 2$ in Kalb-Ramond sector, and $p = 1, 3$ and $p = 0, 2, 4$ in type-IIA and -IIB respectively.

- **Electromagnetic Duality**: $F \mapsto \star F$ (enhanced to full rotations in $d = 2p + 2$) **off-shell symmetry** – can phrase as twisted self-duality relation.

→ Make manifest at expense of **covariance**.

[Bünster & Henneaux '11]

(Anti-) Self-Duality

Can also have **(anti-) self-duality** relations!

Again, consider p-form $A \in \Omega^p(M)$ satisfying

$$\mathrm{d}A = \sigma \star \mathrm{d}A, \quad \sigma = \pm 1$$

- For consistency (in Lorentzian signature), need $d = 2(p+1), \quad p \in 2\mathbb{N}_{>0}$.
- Use null coordinates two-dimensional (Lorentzian) submanifold

$$\mathrm{d}A = \partial_- A \, \mathrm{d}x^- + \partial_+ A \, \mathrm{d}x^+$$

Then $\star \mathrm{d}x^{\pm} = \pm \mathrm{d}x^{\pm}$, so **(anti-)** **self-duality** amounts to

$$\partial_{\pm} A = 0$$

→ **Chirality** condition: Halves degrees of freedom!

- Notorious example: Self-dual 4-form in **type-IIB** supergravity.

(Twisted) Self-Duality

Twisted self-duality most general one-derivative eq. for bosonic fields.

- For particular (D, p) find (anti-) self-duality/EM duality: **Dimensional reduction** of chiral fields in $D = 4k + 2$ to $D = 4k$ gives EM duality.

- **Democratic actions**: Twisted self-duality as e.o.m. (from latter).

- Expect two-derivative eq. → difficult to find **covariant actions!**

    - For **gauge fields**: Actions with manifest EM duality.

    - For **chiral fields**: Describing right degrees of freedom. *E.g.* naïve action

$$S = \int_M F \wedge \star F + \lambda \wedge (F - \sigma \star F)$$

    wrong...

Democratic Actions

Americans for Democratic Action

ADA

Democratic Formulations

Fields obeying **(twisted) self-duality relations** clearly ubiquitous. For quantisation, convenient to have **actions**.

- Correct degrees of freedom, nothing manifest.

[Siegel ‘84; Tseytlin ’90; Floreanini & Jackiw ’87]

- **Duality invariance**, no covariance.

[Schwarz & Sen ‘94; Henneaux & Teitelboim ‘11]

- Manifest **duality** and **covariance**.

[Pasti, Sorokin, and Tonin ‘97; Sen ’15; Mkrtchyan ‘19]

Unifying picture: Descent from **topological field theory** in one higher dimension.

[Arvanitakis, Cole, Hulik, Sevrin, and Thompson ‘22]

- Description of EM, p-forms and other gauge theories from CS/BF theory.

→ Straightforward generalisation to **interactions**.

Example: Maxwell Theory

Consider $H_1, H_2 \in \Omega^2(M)$ with $\dim(M) = 5$ described by **BF-type action**

$$S = \int_M -H_2 \wedge \mathrm{d}H_1 + \mathrm{d}H_2 \wedge H_1 - \frac{1}{2}\int_{\partial M} H_a \wedge \star H_a$$

- Bulk equations of motion:

$$\mathrm{d}H_a = 0$$

→ **Pure gauge** (non-dynamical) in bulk.

- Boundary equation of motion:

$$H_1 = \star H_2$$

→ **Twisted self-duality!**

Idea: *Covariant reduction to the boundary should lead to covariant action of boundary degrees of freedom!*

Topological Reduction

Trick: Decompose

$$H_a = H_a^{\perp} + v \wedge H_a^{\parallel}$$

for closed, constant, and nowhere-null $v \in \Omega^1(M)$ — defines **foliation**.

- $H_a^{\parallel}$ acts as Lagrange multiplier for **constraint**

$$v \wedge \mathrm{d}H_a^{\perp} = 0$$

→ General solution

$$H_a^{\perp} = \mathrm{d}A_a + v \wedge S_a \longrightarrow H_a = \mathrm{d}A_a + v \wedge R_a$$

- Plug back into action: Obtain democratic action in **Mkrtchyan form**

[Mkrtchyan '19]

$$S = \int_{\partial M} \left[-\frac{1}{2}\delta^{ab} H_a \wedge \star H_b - \varepsilon^{ab} v \wedge R_a \wedge \mathrm{d}A_b \right]$$

2D chiral fields can be described as boundary degrees of freedom of 3D gravity

Goal: Do this!

3D (Higher-Spin) Gravity

Focus on the gravitational sector for now

Frame-Like Formulation

For gravity in **first-order form**, fundamental fields: vielbein $\{e^a\}$ and connection $\{\omega^a{}_b\}$.

→ In $D = 3$, useful to dualise

$$\omega^a = \tfrac{1}{2}\epsilon^a{}_{bc}\omega^{bc}$$

- **Einstein-Hilbert action** with $\Lambda = -1/\ell^2 < 0$

$$S_{\mathrm{EH}} = \frac{1}{8\pi G}\int_M e^a \wedge \left[\mathrm{d}\omega^a + \frac{1}{2}\epsilon^a{}_{bc}\left(\omega^b \wedge \omega^c + \frac{1}{3\ell^2}e^b \wedge e^c\right)\right]$$

- Equations of motion are vanishing of torsion and curvature respectively:

$$\mathcal{T}^a := \mathrm{d}e^a + \omega^a{}_b \wedge e^b = 0,$$

$$\mathcal{R}^a{}_b := \mathrm{d}\omega^a + \epsilon^a{}_{bc}\left(\omega^b \wedge \omega^c + e^b \wedge e^c\right) = 0$$

→ Reproduce **Einstein's equation** with

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$$

Chern-Simons Formulation

Can write in Chern-Simons **formulation**.

- Define $\mathfrak{sl}\,(2,\mathbb{R})$-valued **gauge fields**

$$A^a = \omega^a + \frac{e^a}{\ell}, \qquad \tilde{A}^a = \omega^a - \frac{e^a}{\ell}$$

→ Useful basis of generators is

$$[L_m, L_n] = (m-n)L_{m+n} \longrightarrow A = \sum_{n=-1}^{+1} A^{(n)} L_n$$

Identifying $k = \ell/4G$,

$$S_{\mathrm{EH}} = S_{\mathrm{CS}}[A] - S_{\mathrm{CS}}[\tilde{A}], \qquad S_{\mathrm{CS}}[A] = \frac{k}{4\pi}\int_M \mathrm{Tr}\left(A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A\right)$$

- Einstein's equation equivalent to flatness condition on both field strengths.

→ (Classical) **equivalence** of AdS_3 gravity with $\mathfrak{so}(2,2) \sim \mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{sl}(2,\mathbb{R})$ CS theory.

[Achucarro & Townsend '86; Witten '88]

Aymptotically AdS Fluctuations

Put on **curved background**: Flat background gauge field $\underline{A}$, *i.e.* for which

$$\mathrm{d}\underline{A} + \underline{A} \wedge \underline{A} = 0$$

- Action for **fluctuations**: "Minimal coupling"

$$S[a] = \frac{k}{4\pi} \int_M \mathrm{Tr}\left(a \wedge \mathrm{D}a + \frac{2}{3} a \wedge a \wedge a \right), \qquad \mathrm{D}\cdot = \mathrm{d}\cdot + [\underline{A}, \cdot]$$

Most general asymptotically AdS_3 solution

[Bañados '99]

$$\mathrm{d}s^2 = \frac{\ell^2}{z^2}\left[\left(\mathrm{d}x^+ + z^2 \tilde{\mathcal{L}}(x^-)\mathrm{d}x^- \right) \left(\mathrm{d}x^- + z^2 \mathcal{L}(x^+)\mathrm{d}x^+ \right) + \mathrm{d}z^2 \right]$$

- Explicitly for chiral part:

$$\underline{A}^z = \frac{\mathrm{d}z}{z}, \quad \underline{A}^+ = 2\frac{\mathrm{d}x^+}{z}, \quad \underline{A}^- = 2z\mathcal{L}\left(x^+\right)\mathrm{d}x^+$$

Incorporate higher-spins now!

Higher-Spin Fields

Why **higher-spins**?

- Fields characterised by **mass** + **spin**: Higher-spin fields natural (in the free case).
- Natural context: **String theory** and **AdS/CFT** (*e.g.* $\mathcal{W}_N$-minimal model holography)
- Presence of $s > 2$ typically implies existence of spin-2: Higher-spin **gravity.**
- For interactions: Various **no-go "theorems"**!
  - *E.g.* in $D \geq 4$ need infinitely many higher spins, e.g. **Vasiliev HS gravity** (dual to vector model)

→ Caveat: In $D = 3$, **little group** does not admit arbitrary helicity massless rep. (no dynamical HS bulk degrees of freedom), but good **toy model**.

Fronsdal Field

Massless **spin-s** on curved space described by symmetric rank-s tensor field obeying

$$\left(\Box - m_s^2\right) \phi_{\mu[s]} + \cdots = 0$$

$\alpha[n] = \alpha_1 \ldots \alpha_n$

- Invariance under **gauge transformation**

$$\delta\phi_{\mu[s]} = s\nabla_{(\mu_s}\epsilon_{\mu[s-1])}, \qquad \epsilon_{\mu[s-3]}{}^{\lambda}{}_{\lambda} = 0$$

Higher-spin analogues of vielbein and spin connection $\{e^{a_1\ldots a_{s-1}}, \omega^{a_1\ldots a_{s-1}}\}$.

- Fronsdal **action** around spin-2 background, schematically

$$S_{\text{Fronsdal}} \sim \int_M \{e \wedge [\mathrm{d}\omega + \underline{\omega} \wedge \omega] + \underline{e} \wedge [\omega \wedge \omega + e \wedge e]\}$$

  - E.o.m.: Vanishing of higher-spin **torsion** and **curvature** → Fronsdal equation.
  - Identification with Fronsdal field via

$$\varphi_{\mu[s]} = \underline{e}^{a_1}{}_{(\mu_1} \cdots \underline{e}^{a_{s-1}}{}_{(\mu_{s-1}} e_{\mu_s)a[s-1]}$$

Chern-Simons Formulation

For **CS formulations**, define $\mathfrak{sl}(N, \mathbb{R})$-valued gauge fields

$$a^{a_1 \ldots a_{s-1}} = \omega^{a_1 \ldots a_{s-1}} + \frac{1}{\ell} e^{a_1 \ldots a_{s-1}}, \qquad \tilde{a}^{a_1 \ldots a_{s-1}} = \omega^{a_1 \ldots a_{s-1}} - \frac{1}{\ell} e^{a_1 \ldots a_{s-1}}$$

- Useful **basis** of generators

$$[L_m, W_n^s] \sim W_{m+n}^s, \qquad \mathrm{Tr}\left(W_m^s W_n^{s'}\right) \sim \delta_{m,-n}\delta_{s,s'} \longrightarrow a = \sum_{n=-(s-1)}^{s-1} a^{(s,n)} W_n^{(s)}$$

- **Fronsdal action** now

$$S_{\text{Fronsdal}}[e, \omega] = S_{\text{CS}}^{(2)}[a] - S_{\text{CS}}^{(2)}[\tilde{a}]$$

$$S_{\text{CS}}^{(2)}[a] = \frac{k}{4\pi} \int_M \mathrm{Tr}\left(a \wedge \mathrm{D}a\right), \qquad \mathrm{D}a = \mathrm{d}a + \underline{A} \wedge a + a \wedge \underline{A}$$

Description of massless spin-s around on-shell **gravitational background**.

→ *E.g.* includes linearised gravity for spin-2!

Covariant Reduction to Boundary

Boundary Theory

Focus on the **linearised** theory.

For full reduction, careful about **boundary terms**.

- For any $\mathfrak{sl}(N, \mathbb{R})$

$$S[a, \tilde{a}] = S_L[a] - S_R[\tilde{a}]$$

  with <u>opposite sign</u>

$$S_L[a] = \frac{k}{4\pi}\left[\int_M \mathrm{Tr}\left(a \wedge \mathrm{D}a\right) - \frac{1}{2}\int_{\partial M} \mathrm{Tr}\left(a \wedge \star a\right)\right]$$
$$S_R[\tilde{a}] = \frac{k}{4\pi}\left[\int_M \mathrm{Tr}\left(\tilde{a} \wedge \mathrm{D}\tilde{a}\right) + \frac{1}{2}\int_{\partial M} \mathrm{Tr}\left(\tilde{a} \wedge \star \tilde{a}\right)\right]$$

- Correct **boundary equation**/condition: **Chirality condition**

$$(a - \star a)\Big|_{\partial M} = 0$$

Topological Reduction of Linearised Theory

Follow topological **boundary reduction** procedure from before.

- Trick: Decompose

$$a = b + v\,\psi$$

- Field ψ acts as Lagrange multiplier enforcing **constraint**

$$v \wedge \mathrm{D}b = 0 \qquad \longrightarrow \qquad a = \mathrm{D}\varphi + v\rho$$

- Plug back into action:

$$S_L = -\frac{k}{4\pi}\int_{\partial M} \mathrm{Tr}\left[-\frac{1}{2}\left(\mathrm{D}\varphi + v\rho\right) \wedge \star \left(\mathrm{D}\varphi + v\rho\right) - \mathrm{D}\phi \wedge v\rho\right]$$

→ **Covariant action**!

Is this the correct action?

Dynamical Content

Connect it to non-covariant formulations.

- Integrate auxiliary field ρ out:

$$S_L = -\frac{k}{4\pi}\int \mathrm{d}^2x \frac{1}{v_-}\mathrm{Tr}\left(\varepsilon^{\alpha\beta}v_\alpha \mathrm{D}_\beta\varphi \mathrm{D}_-\varphi\right)$$

→ **PST-type action** (covariant + duality symmetric, but non-polynomial)!

- Gauge fixing $v = \mathrm{d}x^0$

$$S_L = -\frac{k}{2\pi}\int \mathrm{d}^2x \,\mathrm{Tr}\left(\mathrm{D}_1\varphi \mathrm{D}_-\varphi\right)$$

→ **FJ-type action**: Up to gauge freedom, equation of motion reduces to:

$$\mathrm{D}_-\varphi = 0$$

as expected.

→ **<u>BUT</u> for right degrees of freedom, need to impose asymptotic boundary conditions.**

Boundary Conditions

Fluctuations should be subdominant to background.

- Gauge-fixing $v = \mathrm{d}x^0$, **asymptotic AdS conditions:**

$$a_1^{(s,n)} = \mathcal{O}\left(z^{1-n}\right), \quad \text{for} \quad n > -(s-1)$$

- Spin-1 reduction: **Floreanini-Jackiw**
- Spin-2 reduction: **(non-chiral) WZW/Liouville** or **Alekseev-Shatashvili**.

[Coussaert, Henneaux, van Driel ’95]
[Cotler & Jensen ‘18]

  - **Asymptotic symmetry** algebra: *Drinfeld-Sokolov* reduction of affine-$\mathfrak{sl}(2,\mathbb{R})$ to Virasoro (matching central charge!).

[Brown & Henneaux ’86]

- Higher-spin reduction: Toda or **higher-spin generalisation** of AS**.**
  - **Asymptotic symmetry** $\mathcal{W}_N$-algebra!

Higher-Order Chiral Scalar

Higher-spin AAdS conditions trivialised by

$$a^{(s,n)} = z^{-n}\mathrm{D}\phi^{(s,n)}, \quad \mathrm{D}_1\phi^{(s,n)}(0,x) = 0, \quad \text{for} \quad n > -(s-1)$$

→ Only need half of these.

- Gives **recurrence relation**

$$\phi^{(s,n)} = -\frac{\partial_1\phi^{(s,n+1)} + (s+n+1)\mathcal{L}\phi^{(s,n+2)}}{s-n-1}$$

→ Solve everything in terms of $\phi^{(s,s-1)}$!

- Plug back into FJ-type action: **HS generalisation of FJ/AS** describing **higher-order scalar**

$$S_L \sim \int_{\partial M} \mathrm{d}^2x\, \mathrm{D}_1\phi^{(s,1-s)}\partial_-\phi^{(s,s-1)} \sim \int_{\partial M} \mathrm{d}^2x\, \mathcal{D}_{\mathcal{L}}^{(2s-1)}\phi^{(s,s-1)}\partial_-\phi^{(s,s-1)}$$

Higher-Order Chiral Scalar

What is this operator $\mathcal{D}_{\mathcal{L}}^{(2s-1)}$?

- **Differential operator** of maximal order $2s-1$, defined by recurrence relation: *E.g.* for spin-2 and -3

$$\mathcal{D}_{\mathcal{L}}^{(3)} = \partial_1^3 - 2(\partial_1\,\mathcal{L} + \mathcal{L}\,\partial_1)\,,$$

$$\mathcal{D}_{\mathcal{L}}^{(5)} = \partial_1^5 - 2(2\,\partial_1^3\,\mathcal{L} + 3\,\partial_1^2\,\mathcal{L}\,\partial_1 + 3\,\partial_1\,\mathcal{L}\,\partial_1^2 + 2\,\mathcal{L}\,\partial_1^3) + 8(3\,\partial_1\,\mathcal{L}^2 + 2\mathcal{L}\,\partial_1\,\mathcal{L} + 3\,\mathcal{L}^2\,\partial_1)$$

- For constant $\mathcal{L}$: Operator **factorises**

$$\mathcal{D}_{\mathcal{L}}^{(2s-1)} = \partial_1 \prod_{n=1}^{s-1} \left(\partial_1^2 - 4\mathcal{L}n^2\right)$$

→ Coincides with **Bol operators**: Invariant under projective action of $\mathrm{SL}(2,\mathbb{R})$.

[Bol '49; Gieres & Theisen '94]

- Physical degrees of freedom are chiral scalar: Tachyonic (massive) scalars in $\mathrm{(A)dS}_2$ with definite-sign momentum (energy).

[Farnsworth, Hinterbichler, and Saha '24]

For non-linear reduction for spin-2 and -3 cf. our paper ☺

Conclusion

Concluding Remarks

Various fields obey **twisted self-duality relations** in theoretical physics.

- For gauge theories, **EM duality** symmetry.
- For chiral fields, (anti-) self-duality → **chirality**.

Long history of attempts at **covariant actions** with twisted self-duality relations as equations of motion.

→ Recent development: **Topological boundary reduction**.

- Applied this to higher-order chiral scalars: Boundary degrees of freedom in AdS_3 HS gravity.

Goal: Repeat for linearised (HS) gravity in $D = 4$.

Thanks for your attention!

Bonus Slides

Non-Linear HS Gravity

Non-linear **HS gravity** is $\mathfrak{sl}(N,\mathbb{R}) \oplus \mathfrak{sl}(N,\mathbb{R})$ CS theory: Massless fields up to spin-N with HS self-interactions.

- Including **boundary terms**:

$$S_{\mathrm{HSG}} = S_L[\mathcal{A}] - S_R[\tilde{\mathcal{A}}], \quad \mathcal{A} = \sum_{n=-1}^{1} A^{(n)} L_n + \sum_{s=3}^{N} \sum_{m=-(s-1)}^{s-1} a^{(s,n)} W_n^{(s)}$$

where

$$S_{L/R}[\mathcal{A}] = \int_M \mathrm{Tr}\left[\mathcal{A} \wedge \mathrm{d}\mathcal{A} + \frac{2}{3}\mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A}\right] \pm \int_{\partial M} \mathrm{Tr}\left[\mathcal{A} \wedge \star\mathcal{A}\right]$$

- **Bulk** equations of motion: **Flatness** condition, solved by

$$\mathcal{A} = G^{-1}\mathrm{d}G = g_\varphi^{-1} \underline{\mathcal{A}} g_\varphi + g_\varphi^{-1}\mathrm{d}g_\varphi$$

- **Boundary** equation of motion: **Chirality** condition

$$(\mathcal{A} - \star\mathcal{A})\big|_{\partial M} = G^{-1}\partial_- G = 0$$

Boundary Reduction

Once again interested in **covariant boundary reduction**.

- Action reduces to

$$S_L = \int_{\partial M} \mathrm{Tr}\left[-\frac{1}{2}\left(g^{-1}\mathrm{d}g + v\rho\right) \wedge \star \left(g^{-1}\mathrm{d}g + v\rho\right) + v \wedge (\rho + \lambda)\, g^{-1}\mathrm{d}g\right] + S_{\mathrm{WZW}}[g]$$

- Connect to **non-covariant formalisms** by integrating and fixing gauge:

$$S_L = -2\int_{\partial M} \mathrm{d}^2x\, \mathrm{Tr}\left[g^{-1}\partial_1 g\, g^{-1}\partial_- g\right] - \frac{1}{3}\int_M \mathrm{Tr}\left[\left(g^{-1}\mathrm{d}g\right)^3\right]$$

- Need to supply with **asymptotic conditions**

$$v \wedge \mathcal{A}^{(s,n)}\Big|_{\partial M} = v \wedge \underline{\mathcal{A}}^{(s,n)}, \quad \text{for} \quad n > 1 - s$$

→ In terms of group elements,

$$g^{-1}\partial_1 g = L_{+1}, \quad \text{for} \quad n > -(s-1)$$

Spin-2 Reduction

Consider *e.g.* reduction procedure for spin-2.

- **Gauss decomposition** of group element

$$g_\phi = e^{\varepsilon(L_1 - \mathcal{L}L_{-1})} e^{\sigma L_0} e^{f L_{-1}}$$

- In fund. rep., **asymptotic conditions** are

$$e^{-\sigma} = \partial_1 \chi, \quad f = -\frac{1}{2}\partial_1 \sigma$$

upon which action becomes

$$S_L = \int_{\partial M} \mathrm{d}^2 x \left(-\frac{\partial_1^2 \chi \partial_- \partial_1 \chi}{\left(\partial_1 \chi\right)^2} - 4\mathcal{L}\partial_1 \chi \partial_- \chi \right)$$

→ **Alekseev-Shatashvili action**.

For spin-3, possible but long expressions...