# CSL 101 DISCRRETE MATHEMATICS

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# COURSE INFORMATION

Class Schedule-
- Monday 9:30 AM
- Tuesday 8:30 AM
- Thursday 9:30 AM

Tutorial Wednesday 9:30 AM

Exams
- Midsem 25 %
- Endsem  35%
- Quiz 20 %
- Assignments 20 %

# INTRODUCTION

Why Discrete Mathematics

- Undestanding of Compuutability
- Uunderstanding efficiency of programs
- Help leaning techniques to prove correctness
- Learing mathematical tools for algorithm design and analysis

# TOPICS TO BE COVERED

1. Basics of Set theory
2. Functions and Relations
3. Basic proof systems
4. Counting and Combinatorics
5. Number Theory
6. Graph Theory
7. Logics

# BASICS OF SET THEORY

- What is a set?

An unordered collection of objects.

- Examples {apple, orange, banana}, the set of all horses in the earth,
- The set of tall students in the class
- The set of students  taller than Ashok in the class

This is a set as this precisely says who are the members of the set.

This is not a set as this does not precisely says who is tall and who is not.

# SET OPERATIONS

Union

Intersection

Complement

# SET OPERATIONS

Set Difference

Power set

# PROBLEM

If $a(t)$, $b(t)$, and $c(t)$ are the lengths of the three sides of a triangle t in non-decreasing order (i.e. $a(t) \leq b(t) \leq c(t)$), we define the sets:

$X := \{\text{Triangle } t : a(t) = b(t)\}$

$Y := \{\text{Triangle } t : b(t) = c(t)\}$

$T :=$ the set of all triangles

Using only set operations on these three sets, define:

(a)The set of all equilateral triangles (all sides equal)

(b) The set of all isosceles triangles (at least two sides equal)

(c) The set of all scalene triangles (no two sides equal)

# SIZE OF A SET

Size of a set means number of elements in the set

Example: Size of the set {a,b,c,d,e} is 5

How to find the size of a particular set?

Ans: Simple! Count one by one

What about infinite set?

# SIZE OF A SET

What is the size of N, the set of natural numbers?

Ans: $\infty$

What is the size of R, the set of Real numbers?

Ans: $\infty$

Does it mean N and R have equal number of elements?

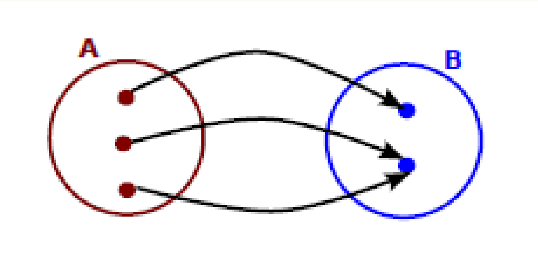
# SIZE OF A SET

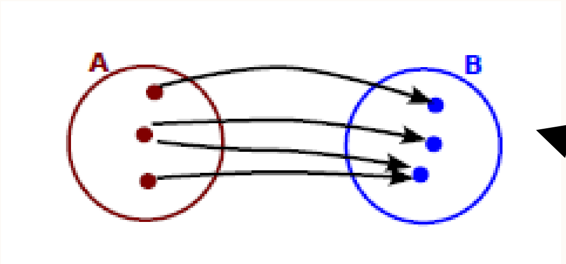
When can we say two sets are of equal size?

Before we answer this question, lets talk about another mathematical object Functions

# FUNCTION

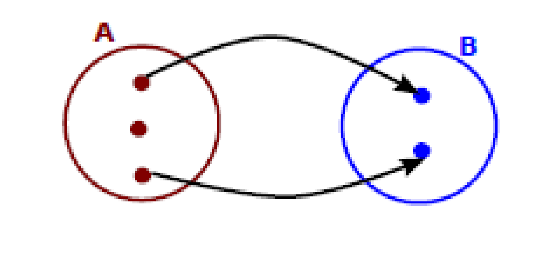
A function f from a set A to a set B assigns each element of A to exactly one element of B.



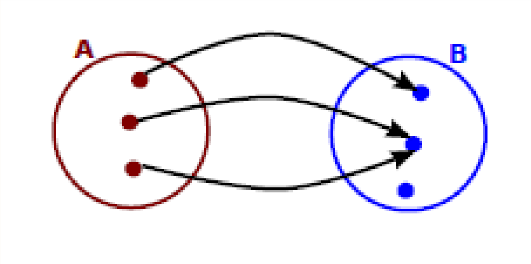




Not a function

# EXAMPLES



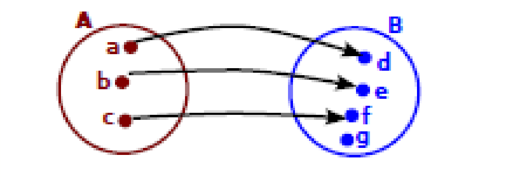

Not a function



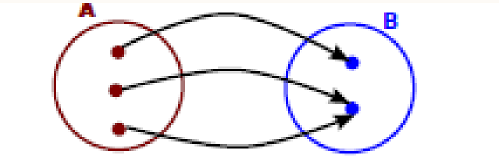

a function

# TYPE OF FUNCTIONS

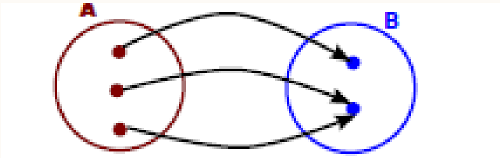
One to one function



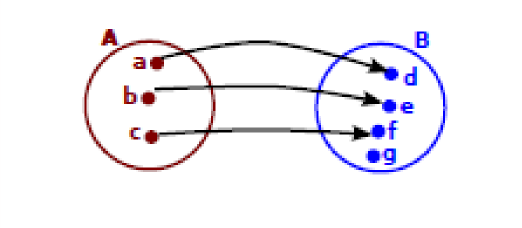

Is this one to one




Onto functions




Is this onto?

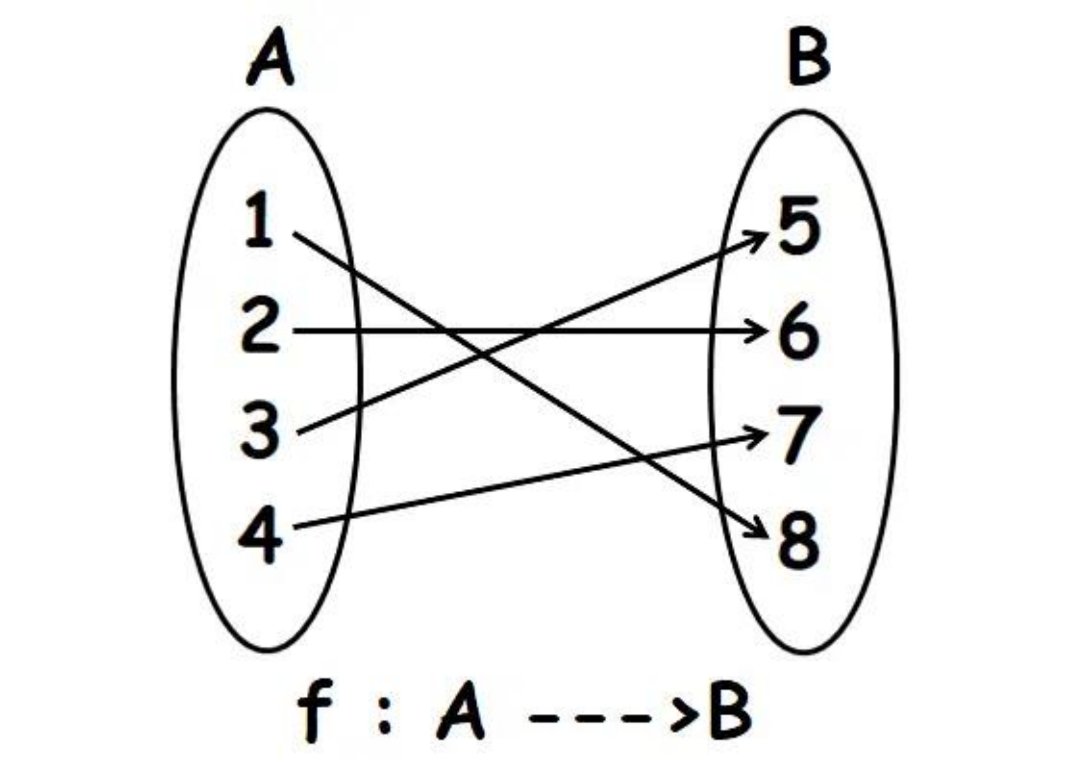


# EXAMPLES

- f:R->R such that $f(x)=x^2$
- f:R->N such that $f(x)= [x]$
- f:R->R such that $f(x)=\sin x$

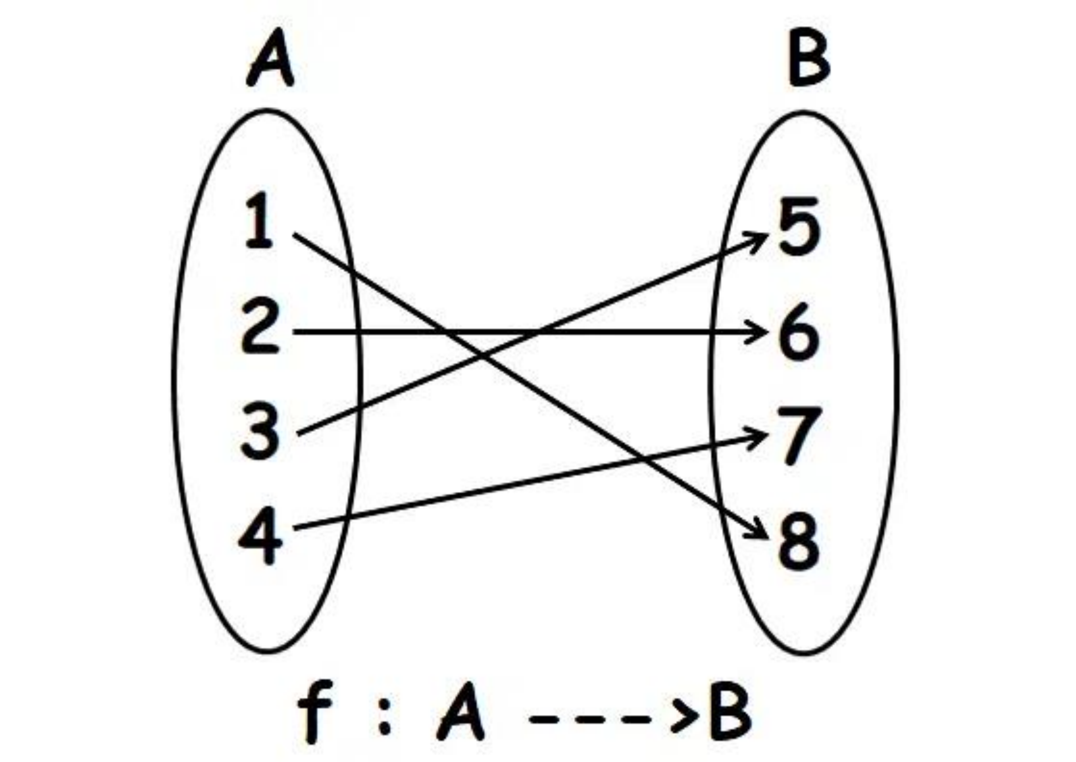
# BIJECTION

Both one to one and onto functions are called bijections



# BIJECTION VS SIZE OF SETS

Is it possible to have bijections between two sets whose sizes are not same?



# BIJECTION VS SIZE OF SETS

This gives a formal definitions of size of a set

Two sets A and B are of equal sizes if there exists a bijection from A to B

# SIZE OF A SET

What is the size of N, the set of natural numbers?

Ans: ∞

What is the size of R, the set of Real numbers?

Ans: ∞

Does it mean N and R have equal number of elements?

If yes, then there must exists a bijection between N and R

# COUNTABLE SETS

The countable sets are defined as follows

- Every set with finite number of elements is countable
- N is countable
- An infinite set G is countable if there exists a bijection from N to G.

# EXAMPLES

- Z, the set of all integers, positive and negative, including 0 is countable.

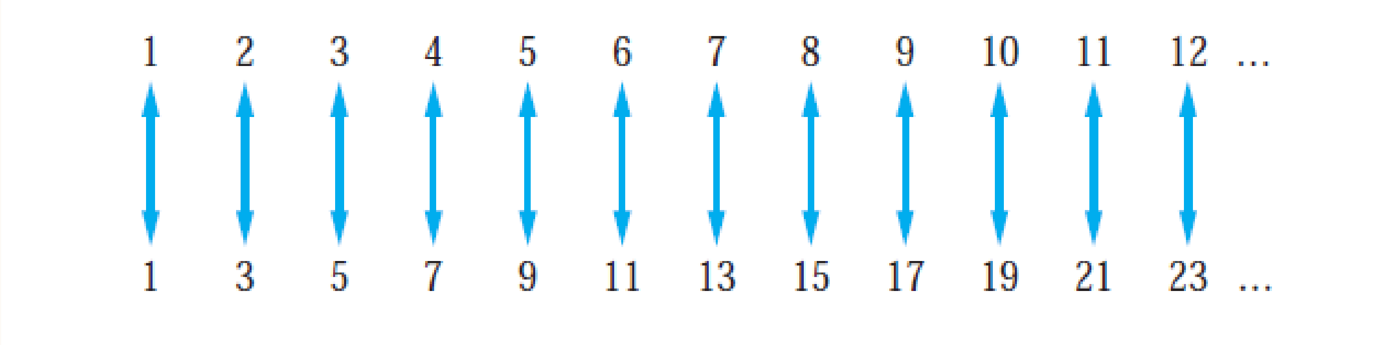
Consider the function
$f(1) = 0$
$f(n) = n/2$ if n is even
$f(n) = n/2$ if $n>1$ is odd.

Prove the above function is a bijection

# EXAMPLES

- The set of all odd positive integers, is countable.

Consider the function
$f(n) = 2n - 1$




Prove the above function is a bijection

# EXAMPLES

- The set of all even positive integers, is countable.

Consider the function
$f(n) = 2n$

Prove the above function is a bijection

# EXAMPLES

- If $S$ is countable, and $a$ is an element of $S$, then $S\setminus\{a\}$ is countable.

- *Given S is countable*
- *This implies there exists a bijection $f:N \rightarrow S$*
- *Since f is a bijection, there exists sone integer i such that $f(i)=a$*

- Consider the function
- $g(n) = f(n)$ for $n<i$
  $=f(n+1)$, *for* $n>i$

*Show that the function g is a bijection*

# EXAMPLES

- If S is countable, and a is not an element of S, then S U {a} is countable.

# EXAMPLES

- If S is countable, and F is a finite subset of S then S \ F is countable.

- If S is countable, and F is any finite set then S U F is countable.