Pigeon Hole Principle

Example 1.1. Among 13 people there are two who have their birthdays in the same month.

Example 1.2. There are n married couples. How many of the $2n$ people must be selected in order to guarantee that one has selected a married couple?

Example 1.3. In any group of n people there are at least two persons having the same number friends. (It is assumed that if a person x is a friend of y then y is also a friend of x.)

Example 1.4. Given n integers $a_1, a_2, \ldots, a_n$, not necessarily distinct, there exist integers k and l with $0 \leq k < l \leq n$ such that the sum $a_{k+1} + a_{k+2} + \cdots + a_l$ is a multiple of n.

Example 1.5. A chess master who has 11 weeks to prepare for a tournament decides to play at least one game every day but, in order not to tire himself, he decides not to play more than 12 games during any calendar week. Show that there exists a succession of consecutive days during which the chess master will have played exactly 21 games.

Example 1.6. Given 101 integers from $1, 2, \ldots, 200$, there are at least two integers such that one of them is divisible by the other.