CSL 101- Discrete Mathematics
Indian Institute of Technology Bhilai
Tutorial Sheet 2

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1. Let $f_1 : \mathbb{R} \to \mathbb{R}$ and $f_2 : \mathbb{R} \to \mathbb{R}$ be functions defined as follows:

$$f_1(y) = y^2 - 2$$

$$f_2(y) = y + 4$$

   Compute $f_1 \circ f_2$ and $f_2 \circ f_1$. Determine whether these functions are one-to-one , onto , one-one onto .

2. Let $f : A \to B$ and $g : B \to C$. If both f and g are onto, show that $g \circ f$ is also onto.Is $g \circ f$ one-to-one if both g and f are one-to-one?

3. Let $\mathbb{N}$ be the set of natural numbers including zero. Determine which of the following functions are one-to-one, onto, and which are both.

   1. $f : \mathbb{N} \to \mathbb{N},\ f(x) = x^2 + 2$
   2. $f : \mathbb{N} \to \mathbb{N},\ f(x) = x \mod 3$
   3. $f : \mathbb{N} \to \mathbb{N},\ f(x) = \left\{ \begin{array}{ll} 1, & \text{for x is even} \\ 0, & \text{for x is odd} \end{array} \right\}$

4. Let $f_1 : \mathbb{R}^2 \to \mathbb{R}$ and $f_2 : \mathbb{R} \to \mathbb{R}$ be functions defined as follows:

$$f_1(x, y) = w_1 \cdot x + w_2 \cdot y$$

$$f_2(x) = \frac{1}{1 + e^{-x}}$$

   Find $f_1 \circ f_2$.If $\sigma(\text{z}) = f_1 \circ f_2$ then write a derivative of $\sigma(\text{z})$ in in terms of $\sigma(\text{z})$ and draw the graph of $\sigma(\text{z})$. Try to draw some observations from $\sigma(\text{z})$.(Here w_1 and w_2 are real constant)

5. Consider these functions from set of students in a discrete mathematics class. Under what conditions is the function one-to-one if it assigns to a student his or her

   1. *mobile number*
   2. *student identity number*
   3. *final grade in the class*
   4. *home town*

6. Prove or disprove Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

   1. $\lceil x \rceil + \lceil y \rceil = \lceil x + y \rceil$
   2. $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$

7. Find out the domain,co-domain and range of the function

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   int function(float x){...}
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8. Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ that is:

   1. One-to-one but not onto.
   2. Onto but not one-to-one.
   3. Both onto and one-to-one (but different from the identity function).
   4. Neither one-to-one nor onto.

9. Suppose that f is a function from A to B, where A and B are finite sets with $|A| = |B|$. Show that f is one-to-one if and only if it is onto.