

Discrete Mathematics
INDIAN INSTITUTE OF TECHNOLOGY BHILAI

(Quiz 1)

Write your answer only in the given space. The last page is blank. You can use it in case the given space is not sufficient

1. Answer the following questions.

(a) Write an infinite countable subset of the closed interval $[0, 1]$.

[1]

$$S = \left\{ \frac{1}{n} \mid n \in \mathbb{N} \right\}$$

(b) Define an one to one function from the set $\mathcal{N} \times \mathcal{N}$ to the set \mathcal{N} , where \mathcal{N} is the set of natural numbers.

[1]

$$f(m, n) = 2^m \cdot 3^n$$

(c) A licence plate number in Chhattisgarh contains 10 characters, CG XX YY ZZZZ, where XX is a two digit integer, YY is a string containing two capital letters from A, B, ..., Z, and ZZZZ is a 4 digit integer. Write the total numbers of different possible licence plate.

[1]

$$10^2 \cdot 26^2 \cdot 10^4$$

(d) Give a one to one mapping from the set of natural number \mathcal{N} to $P(\mathcal{N})$, where $P(A)$ denotes power set of a set A.

[1]

$$f(i) = \{i\}$$

(e) State the statement of well ordering property.

[1]

For any subset of non-negative integers, there is a minimum element.

2. Let A denote the set of all automobiles that are manufactured domestically. Let B denote the set of all imported automobiles. Let C denote the set of all automobiles manufactured before 1977. Let D denote the set of all automobiles with a current market value of less than 2000. Let E denote the set of all automobiles owned by students at the university. Express the following statements in set-theoretic notation:

- (a) The automobiles owned by students at the university are either domestically manufactured or imported. [1]

$$E \subset (A \cup B)$$

- (b) All domestic automobiles manufactured before 1977 have a market value of less than 2000. [1]

$$A \cap C \cap D$$

- (c) All imported automobiles manufactured after 1977 have a market value of more than 2000. [1]

$$B \cap \bar{C} \cap \bar{D}$$

3. There are 8 types of cookies available in a store. Count the number of ways

- (a) to pick 6 of them and arrange them in a line. [3]

Ans 8^6

(b) to pick 6 of them and place them into lines named A and B, with 3 in each.

[3]

Ans 8^6

4. Let S be the set of integers of at most 97 digits that contains only the digits 1 or 0. Prove that there exists non-zero integer in S which is divisible by 89.

[6]

consider the 90 integers 1, 11, 111, 1111, ...

At least two of them leaves the same remainder if divided by 89.

Therefore their subtraction is divisible by 89, which is a number that contains only 1's and 0's.