Indian Institute of Technology Bhilai

Discrete Mathematics

Tutorial Sheet 5

• *Solve the following problems before the Tutorial.*

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1. Compute the number of zeros are there at the end of 100!?
2. Show that $\log_2 3$ is an irrational number.
3. Prove or disprove that there are three consecutive odd positive integers that are primes, that is, odd primes of the form p, $p+2$, and $p+4$.
4. The value of the Euler ϕ-function at the positive integer n is defined to be the number of positive integers less than or equal to n that are relatively prime to n.
   (a) Show that n is prime if and only if $\phi(n) = n - 1$.
   (b) What is the value of $\phi(p^k)$ when p is prime and k is a positive integer?
5. For two positive integers a and b, show that $ab = gcd(a, b) \cdot lcm(a, b)$.
6. Prove that
   (a) the square of any integer is of the form $3k$ or $3k + 1$.
   (b) the cube of any integer is of the form $9k$ or $9k + 1$ or $9k + 8$.
   (c) the fourth power of any integer is of the form $5k$ or $5k + 1$.
   (d) For any integer a, $3a^2 - 1$ is never a perfect square.