

# **CSL 101 DISCRETE MATHEMATICS**

## **LECTURE 2**

Dr. Barun Gorain  
Department of CSE, IIT Bhilai  
Email: [barun@iitbhilai.ac.in](mailto:barun@iitbhilai.ac.in)

## SOME MORE FACT ON COUNTABILITY

**Theorem 1:** Let  $A$  and  $B$  be two countably infinite sets. Then  $A \cup B$  is countable.

- Given  $A$  is countable. This implies there exists a bijection from  $f: \mathbb{N} \rightarrow A$ .
- Given  $B$  is countable. This implies there exists a bijection from  $g: \mathbb{N} \rightarrow B$ .
- **To prove** there exists a bijection from  $h: \mathbb{N} \rightarrow A \cup B$

## SOME MORE FACT ON COUNTABILITY

**Theorem 2:** Let  $A$  be a countably infinite set, and  $B$  an infinite subset of  $A$ . Then  $B$  is countable.

### Exercise

## SOME MORE FACT ON COUNTABILITY

**Theorem:**  $\mathbb{N} \times \mathbb{N}$  is countable

- Define a function  $f: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$  as follows:
  - $f(m,n)=2^m3^n$

Now use the previous theorem to prove this

## SOME MORE FACT ON COUNTABILITY

**Theorem:** The set of rational number  $\mathbb{Q}$  is countable

Can we use now the previous theorem to prove this?

# UNCOUNTABLE SET

**Theorem:** The set of real numbers  $\mathbb{R}$  is uncountable.