[CSL202] **2024-25-M**

TUTORIAL IX

Date: **Nov 08, 2024**.

1. TRUE/FALSE: Let G be a graph with n vertices and m edges.

   (a) All its DFS forests (for traversals starting at different vertices) will have the same number of tree edges and the same number of back edges?

   (b) All its DFS forests will have the same number of trees?

2. Give a linear-time algorithm to check whether a given directed acyclic graph G contain a directed path that touches (contains) every vertex exactly once.

3. You are given a directed acyclic graph $G = (V, E)$ in which each node $u \in V$ has an associated price, denoted by $price(u)$, which is a positive integer. The cost of a node u, denoted by $cost(u)$, is defined to be the price of the cheapest node reachable from u (including u itself). Design an algorithm that computes $cost(u)$ for all $u \in V$. Write the pseudocode.

4. Recall that a directed graph G is strongly connected if, for any two vertices u and v, there is a path in G from u to v and a path in G from v to u. Describe an algorithm to determine, given an undirected connected graph G as input, whether it is possible to direct each edge of G so that the resulting directed graph is strongly connected.

5. Give a linear-time algorithm to find an odd-length cycle in a directed graph that is strongly connected.

6. Let G be a connected undirected graph. Suppose we start with two coins on two arbitrarily chosen vertices of G, and we want to move the coins so that they lie on the same vertex using as few moves as possible. At every step, each coin must move to an adjacent vertex.

   Describe and analyze an algorithm to compute the minimum number of steps to reach a configuration where both coins are on the same vertex, or to report correctly that no such configuration is reachable. The input to your algorithm consists of a graph $G = (V, E)$ and two vertices $u, v \in V$ (which may or may not be distinct).