[CSL202] **2024-25-M**

TUTORIAL IV

Date: **Sep 06, 2024**.

1. For the set of $\{2, 4, 6, 12, 16, 17, 21, 25\}$ of keys, draw (i) a MAX-HEAP and (ii) MIN-HEAP.
2. Is the array with values $[23, 17, 14, 6, 13, 10, 1, 5, 7, 12]$ a MAX-HEAP?
3. For the array $A = [4, 1, 3, 2, 16, 9, 10, 14, 8, 7]$, illustrate the operation of BUILD-MAX-HEAP.
4. Formally show that the height of an n element heap is $O(\log n)$.
5. Show that the running time of BUILD-MAX-HEAP is $O(n)$.
6. Given k-sorted arrays. Design an algorithm to merge the k arrays into a single sorted array. What is the running time?
7. Given an array A having n distinct elements and a positive integer k. Design an algorithm to find the kth smallest element of A. What is the running time?
8. The MEDIAN of a finite list of numbers is the "middle" number, when those numbers are listed in order from smallest to greatest.

   If the data set has an odd number of elements, the middle one is the median. If the data set has an even number of observations, there is no distinct middle value and the median is usually defined to be the average of the two middle values.

   Example: The median of $12, 5, 17, 8, 11$ is 11 and the median of $6, 4, 8, 10$ is $(6 + 8)/2 = 7$.

   Given a data stream of integers. Design an algorithm to find the median of elements read so far. What is the running time.

   Example: data stream $A = [12, 7, 11, 4, 3]$. Output should be $[12, 9.5, 11, 5, 5, 7]$

   Justification:

   (a) After reading 1st element, median =12.
   (b) After reading 2nd element, median =$(12 + 7)/2 = 9.5$.
   (c) After reading 3rd element, median =11.
   (d) After reading 4th element, median =$(7 + 4)/2 = 5.5$.
   (e) After reading 5th element, median =7.