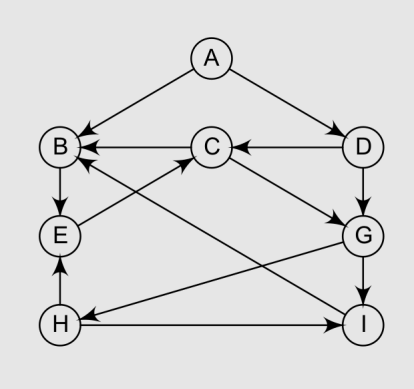
[CSL202] **2024-25-M**

TUTORIAL VIII

Date: **Oct 25, 2024**.

1. Find the strongly connected components of the following directed graph G using the (Kosaraju's) algorithm we have studied in the last lecture.



2. There are n rebels that are currently planning, well, a "party". A rebel v can send messages to only one other rebel, designated as $contact(v)$. Here, if a rebel is given a message, they will send it to their contact, and it would be propagated in this fashion to everyone reachable.

   (a) Describe an algorithm, as fast as possible, that computes a rebel v, such that if you send a message to v, the message gets propagated to all the rebels. If there is not such vertex v, the algorithm should output "no solution".

   (b) Describe an algorithm, as fast as possible, that computes the minimal number of rebels that needs to be sent directly a message, before the message can be propagated to all the rebels.

3. A directed graph G is *semi-connected* if, for every pair of vertices u and v, either u is reachable from v or v is reachable from u (or both).

   (a) Give an example of a directed acyclic graph that is NOT semi-connected.

   (b) Describe an algorithm to determine whether a given DIRECTED ACYCLIC GRAPH G is semi-connected. Analyze its running time.

(c) Describe an algorithm to determine whether an arbitrary directed graph G is semi-connected. Analyze its running time.

4. The police department in the city of Delhi has made every street in the city one-way. Despite widespread complaints from confused motorists, the mayor claims that it is possible to legally drive from any intersection in Delhi to any other intersection.

   (a) The city needs to either verify or refute the mayor's claim. Formalize this problem in terms of graphs, and then describe and analyze an algorithm to solve it.

   (b) After running your algorithm from part (a), the mayor reluctantly admits that she was misinformed.

   Suppose now that the mayor needs to show that, even though her original claim was false, something weaker does hold: If you start navigating one-way streets from airport, there is always a way to drive legally back to the airport. Formulate this weaker property as a graph-theoretic problem and show that this can also be solved in linear time.