Indian Institute of Technology Bhilai
CSL 202: Design and Analysis of Algorithms
Assignment

- Only pdf submission prepared in Latex will be accepted. The name of the pdf file should be rollno.pdf. For example, if your roll number is 12345, the the name should be 12345.pdf.
- Deadline of submisison is April 20, 2025, 11:59 PM.
- I will use plagerism software for all the submitted solutions. If it is found that the answers are copied between two students, both of them will be awarded -10 marks, irrespective of who copied from whom.

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1. For this question, you'll need the following definition: [5]
   Definition: A sunlet is a graph with $2n$ vertices that consists of a cycle of length n, and each vertex in the cycle is directly connected to exactly one node of degree one. An example of a sunlet with 12 vertices is shown below.

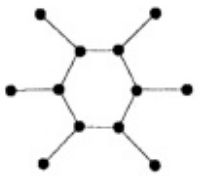

Figure 1: Example of sunlet

   Model: Input is an undirected graph G with $2n$ vertices. The algorithm can query an edge (i, j) and it will be told whether or not edge (i, j) is in graph G. Each query has cost 1.
   Problem: Output "Yes" if the input graph G is a sunlet, otherwise output "No".
   Your task: Prove that every correct algorithm for the problem has worst-case cost at least $\binom{2n}{2}$.

2. Consider the following model and problem:
   Model: For the range of numbers $1, 2, \ldots n$, there is a special threshold value $t \in \{0, 1, \ldots, n\}$. For all numbers $i > t$, the number i is considered "unsafe". All other numbers in the range $1, 2, \ldots n$ are considered "safe". The algorithm can query any number $i \in \{1, 2, \ldots, n\}$, and it will be told whether i is "safe" or "unsafe". However, if the algorithm ever queries an "unsafe" i, the system shuts down and no further queries are possible.
   Problem: Determine the exact value of t.
   Your task:
   (a) Prove that any algorithm that solves the problem must perform at least n queries in the worst case. [5]
   (b) Let's change the model a bit: suppose that one "unsafe" query is allowed. That is, the system shuts down after exactly two "unsafe" queries. Prove that any algorithm that determines the exact value of t must use $\Omega(\sqrt{n})$ queries in the worst case. [7]
   (c) Design an Algorrithm that uses $O(\sqrt{n})$ many queries for the problem in part (b). [3]