CSL 252- Design and Analysis of Algorithms
Indian Institute of Technology Bhilai
Tutorial Sheet 2

Solve the following recurrences.

1. $T(1) = 1$, and for all $n \geq 2, T(n) = 3T(n-1) + 2$.
2. $T(1) = 8$, and for all $n \geq 2, T(n) = 3T(n-1) - 15$
3. $T(1) = 2$, and for all $n \geq 2, T(n) = T(n-1) + n - 1$
4. $T(1) = 3$, and for all $n \geq 2, T(n) = T(n-1) + 2n - 3$
5. $T(1) = 1$, and for all $n \geq 2, T(n) = 2T(n-1) + n - 1$
6. $T(1) = 1$, and for all $n \geq 2$ a power of 2 , $T(n) = 2T(\frac{n}{2}) + 6n - 1$
7. $T(1) = 4$, and for all $n \geq 2$ a power of 2 , $T(n) = 2T(\frac{n}{2}) + 3n + 2$
8. $T(1) = 1$, and for all $n \geq 2$ a power of 6 , $T(n) = 6T(\frac{n}{6}) + 2n + 3$
9. $T(1) = 3$, and for all $n \geq 2$ a power of 6 , $T(n) = 2T(\frac{n}{2}) + 3n - 1$
10. $T(1) = 3$, and for all $n \geq 2$ a power of 3 , $T(n) = 4T(\frac{n}{3}) + 2n - 1$
11. $T(1) = 2$, and for all $n \geq 2$ a power of 3 , $T(n) = 4T(\frac{n}{3}) + 3n - 5$
12. $T(1) = 1$, and for all $n \geq 2$ a power of 2, $T(n) = 3T(\frac{n}{2}) + n^2 - n$.
13. $T(1) = 4$, and for all $n \geq 2$ a power of 3, $T(n) = 4T(\frac{n}{3}) + n^2 - 7n + 5$.
14. $T(1) = 1$, and for all $n \geq 2$ a power of 3, $T(n) = 4T(\frac{n}{3}) + n^2$.
15. $T(1) = 1$, and for all $n \geq 4$ a power of 4, $T(n) = T(\frac{n}{4}) + \sqrt{n} + 1$.
16. $T(1) = 1, T(2) = 6$, and for all $n \geq 3$,
$$T(n) = T(n-2) + 3n + 4.$$
17. $T(0) = c, T(1) = d$, and for all $n > 1$,
$$T(n) = T(n-2) + n.$$
18. $T(1) = 1, T(2) = 6, T(3) = 13$, and for all $n \geq 4$,
$$T(n) = T(n-3) + 5n - 9.$$
19. $T(1) = 1$, and for all $n \geq 2$,
$$T(n) = \sum_{i=1}^{n-1} T(i) + 1$$

20. $T(1) = 1$, and for all $n \geq 2$,

$$T(n) = \sum_{i=1}^{n-1} T(i) + 7$$

21. $T(1) = 1$, and for all $n \geq 2$,

$$T(n) = \sum_{i=1}^{n-1} T(i) + n^2$$

22. $T(1) = 1$, and for all $n \geq 2$,

$$T(n) = 2\sum_{i=1}^{n-1} T(i) + 1$$

23. $T(1) = 1$, and for all $n \geq 2$,

$$T(n) = \sum_{i=1}^{n-1} T(n-i) + 1$$

24. Show that the solution $T(0) = 1, T(1) = 2$ and $T(n) = 5T(n-1) - 6T(n-2)$ for $n \geq 2$ is $\Theta(2^n)$
25. Show that the solution $T(n) = T(n-1) + n$ for $n \geq 2$ is $O(n^2)$
26. Show that the solution $T(n) = \sqrt{n}T(\sqrt{n}) + n$ for $n \geq 1$ and $T(1) = 1$ is $\Theta(n \log \log n)$
27. Show that the solution $T(n+1) = T(n) + 2n$ for $n \geq 1$ and $T(1) = 10$ is $O(n^2)$
28. Show that the solution of the recurrence is $O(n \log n)$
    $T(0) = 0$, and for all $n \geq 2$,

$$T(n) = \frac{2}{n}\sum_{i=1}^{n-1} T(i) + n - 1$$

29. Solve $nT(n) = n - 2T(n-1) + 2$ for for $n > 1$ with $T(1) = 1$
30. Show that the solution of the recurrence is $\Theta(n^2)$
    $T(1) = c$, and for all $n \geq 2$,

$$T(n) = 4T\left(\frac{n}{2}\right) + dn$$