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3) Sol^n:- # features (n) = 2
samples (N) = 4

$$\bar{x} = \frac{4+8+13+7}{4} = 8$$

$$\bar{y} = \frac{11+4+5+14}{4} = 8.5$$

$x-\bar{x}$	$y-\bar{y}$
-4	2.5
0	-4.5
5	-3.5
-1	5.5

Variance- Covariance Matrix

$$Cov(X,Y) = \frac{1}{N-1} \Sigma (x_i - \bar{x})(y_i - \bar{y})$$

$Cov(X,X) = Var(X) = 14$

$Cov(X,Y) = Cov(Y,X) = -11$

$Cov(Y,Y) = 23$

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$$S = \begin{bmatrix} 14 & -11 \\ -11 & 23 \end{bmatrix}$$

$$|S - \lambda I| = 0$$

$$\det\left(\begin{bmatrix} 14-\lambda & -11 \\ -11 & 23-\lambda \end{bmatrix}\right) = 0$$

$$\Rightarrow (14-\lambda)(23-\lambda) + 11^2 = 0$$

$$\Rightarrow \lambda^2 - 37\lambda + 201 = 0$$

$$\lambda = 30.3849,\ 6.6151$$

Eigen vector corresponding to $\lambda_1 = 30.3849$ (say)

$$(S - \lambda_1 I)U = 0$$

$$\begin{bmatrix} 14-\lambda_1 & -11 \\ -11 & 23-\lambda_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(14-\lambda_1)u_1 - 11u_2 = 0 \quad \text{--- } ①$$

$$-11u_1 + (23-\lambda_1)u_2 = 0 \quad \text{--- } ②$$

from ①
$$\frac{u_1}{11} = \frac{u_2}{14-\lambda_1} = t \text{ (say)}$$

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For $t=1$

$$u_1 = 11, \quad u_2 = 14 - \lambda_1 = 14 - 30.3849$$
$$= -16.3849$$

$$\therefore \ v = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 11 \\ -16.3849 \end{bmatrix}$$

$$U_{norm} = \frac{1}{\sqrt{11^2 + (-16.3849)^2}} \begin{bmatrix} 11 \\ -16.3849 \end{bmatrix}$$

$$U_{norm} = \begin{bmatrix} 0.5574 \\ -0.8303 \end{bmatrix} = e_1 \ (\text{say})$$

since ~~it~~ S is symmetric matrix & eigen vectors would be orthogonal, thus directly eigen vector corresponding to λ_2 is

$$V_{norm} = \begin{bmatrix} 0.8303 \\ 0.5574 \end{bmatrix} = e_2 \ (\text{say})$$

Projection of data set along the direction of maximum variance (for $\lambda_1 = 30.3849$)

First Principle Component (PC_1)	P_{11} $= -4.3052$	P_{12} $= 3.7361$	P_{13} $= 5.6928$	P_{14} $= -5.1238$

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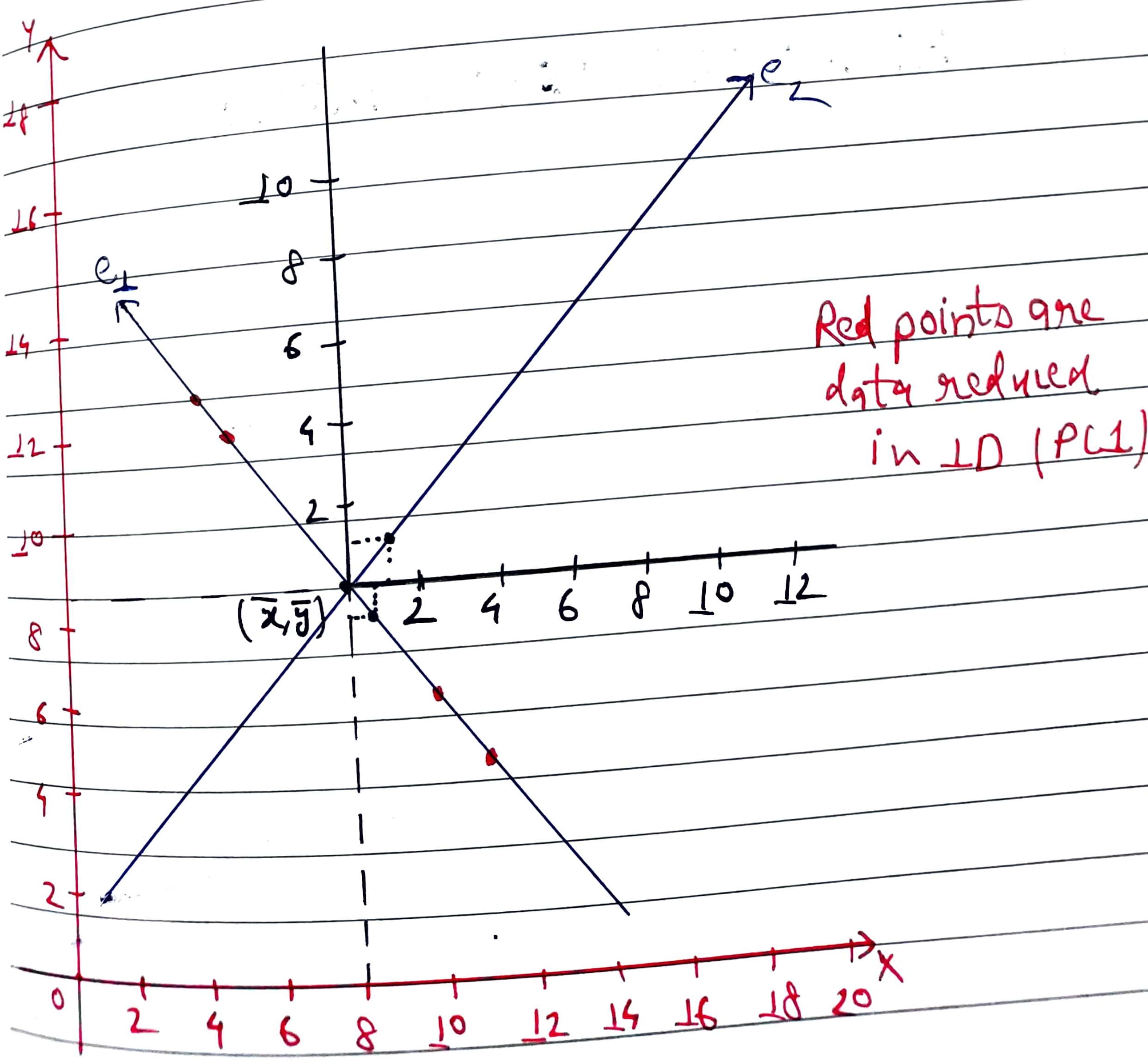
$$P_{11} = e_1^T \begin{bmatrix} x_1 - \bar{x} \\ y_1 - \bar{y} \end{bmatrix} = \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} -4 \\ 2.5 \end{bmatrix}$$

$$= -4.3052$$

$$P_{12} = e_1^T \begin{bmatrix} x_2 - \bar{x} \\ y_2 - \bar{y} \end{bmatrix} = 3.7361$$

$P_{13} = 5.6928$; $P_{14} = -5.1238$



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2) Solⁿ:- Given

$$X[k] = \sum_{n=0}^{N-1} x[n] \exp\left(-j \frac{2\pi}{N} nk\right)$$

Complex multiplication: $N \times N = N^2$ Ans

Complex addition: $N(N-1) = N^2 - N$ Ans

Real Multiplication: $4N^2$ Ans

Real addition: $2N^2 + (2N^2 - 2N) = 4N^2 - 2N$ Ans

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5) Sol^n:- $c_1 = \frac{\langle x, y_1 \rangle}{\langle y_1, y_1 \rangle}$; $c_2 = \frac{\langle x, y_2 \rangle}{\langle y_2, y_2 \rangle}$; $c_3 = \frac{\langle x, y_3 \rangle}{\langle y_3, y_3 \rangle}$

$\langle y_1, y_1 \rangle = \sum_{t=1}^{2} y_1(t)^2 = 1^2 + 1^2 = 2$

$\langle y_2, y_2 \rangle = \sum_{t=3}^{4} y_2(t)^2 = (2(3-3))^2 + (2(3.5-3))^2 + 2((4-3.5))^2 + (2(4-4))^2.$

$= 0 + 1 + 1 + 0 = 2$

$\langle y_3, y_3 \rangle = \sum_{t=5}^{7} y_3(t)^2 = (5-5)^2 + (6-5)^2 + (7-6)^2 + (7-7)^2$

$= 0 + 1 + 1 + 0 = 2$

$\langle x, y_1 \rangle = \sum_{t=1}^{2} x(t)\, y_1(t) = 1 \cdot 1 + 1 \cdot 1 = 2$

$\langle x, y_2 \rangle = \sum_{t=3}^{4} x(t)\, y_2(t) = 1 \cdot 1 + 1 \cdot 1 = 2$

$\langle x, y_3 \rangle = \sum_{t=5}^{7} x(t)\, y_3(t) = 1 \cdot 1 + 1 \cdot 1 = 2$

$c_1 = 1$; $c_2 = 1$; $c_3 = \cancel{2}\ 1$

$\hat{x}(t) = y_1(t) + y_2(t) + y_3(t)$

~~Since~~ ~~&~~

For $t<1$; $2<t<3$; $4<t<5$; and $t>7$:

$y_1(t)=0$, $y_2(t)=0$, $y_3(t)=0$

$\hat{x}(t) = 0+0+0=0$

Thus $\hat{x}(t)$ perfectly matches $x(t)$ over interval $0\leq t\leq 9$

$\therefore \varepsilon(t)=0$ i.e. $\underline{c_1 = c_2 = c_3 = 1}$ Ans.

4(i) $A = \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}$ $A^T = \begin{bmatrix} i & 1 \\ 1 & i \\ i & i \end{bmatrix}$

$$A^{*T} = \begin{bmatrix} -i & 1 \\ 1 & -i \\ -i & -i \end{bmatrix}$$

Now,

$$A^{*T} A = \begin{bmatrix} -i & 1 \\ 1 & -i \\ -i & -i \end{bmatrix} \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & i+1 \\ 0 & 2 & i+1 \\ 1-i & 1-i & 2 \end{bmatrix}$$

& $AA^{*T} = \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix} \begin{bmatrix} -i & 1 \\ 1 & -i \\ -i & -i \end{bmatrix}$

$$= \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

(ii) Since Q is unitary

$Q^{-1} = Q^H$

$\Rightarrow (Q^{-1})^H Q^{-1}$

$\Rightarrow (Q^H)^H Q^H = QQ^H = I$

$\therefore Q^{-1}$ is also unitary.

Now,

$$(QU)^H \times (QU) = I$$

$$\boxed{(QU)^H = U^H Q^H}$$

$$= U^H Q^H Q U$$

$$= U^H I U = I$$

$\therefore$ QU is also unitary

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1. (i) $$H = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$$

$$|H - \lambda I| = 0$$

$$\begin{bmatrix} 9-\lambda & -2 \\ -2 & 6-\lambda \end{bmatrix} = 0$$

Solving we get

$$\boxed{\lambda = 5, 10}$$

(ii) (a) eigen value $\rightarrow 2A$

(b) " " $\rightarrow m\lambda$

(c) " " $\rightarrow \lambda^2$

(d) " " $\rightarrow \lambda^n$

(e) " " $\rightarrow \frac{1}{\lambda}$

(f) " " $\rightarrow 2^\lambda$

(g) " " $\rightarrow 2\lambda + 3$