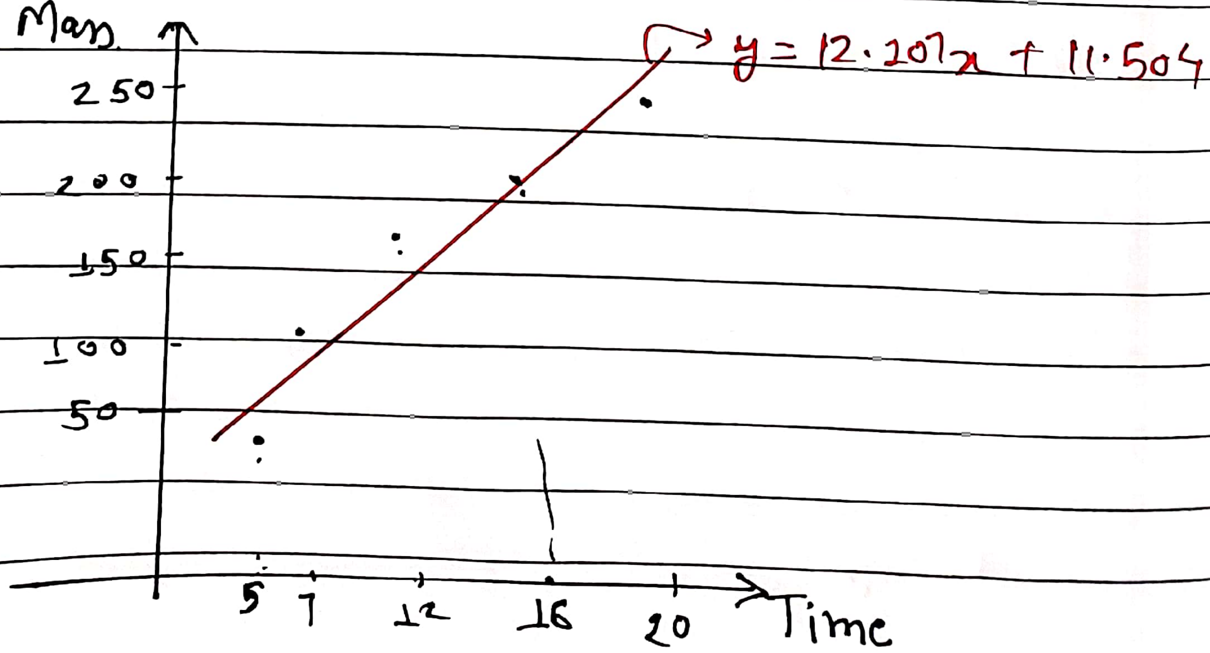
1) Sol^n: $\bar{x} = \frac{\Sigma x}{n} = 12$; $\bar{y} = \frac{\Sigma y}{n} = 158$

x_i	y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i-\bar{x})(y_i-\bar{y})$	$(x_i-\bar{x})^2$
5	40	-7	-118	826	49
7	120	-5	-38	190	25
12	180	0	22	0	0
16	210	4	52	208	16
20	240	8	82	656	64
				$\Sigma = 1880$	$\Sigma = 154$

$$m = \frac{\sum_{i=1}^{5} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{5} (x_i - \bar{x})^2} = \frac{1880}{154} = 12.207$$

$$c = \bar{y} - m\bar{x} = 158 - 12.208 \times 12 = 11.504$$

$$\hat{y} = 12.207x + 11.504$$



Sol^n:- $\because$ X and ϵ are independent.

$$Var(Y) = \beta_1^2 Var(X) + Var(\epsilon) \quad — ①$$

$$\hat{Y} - E[Y] = (\beta_0 + \beta_1 X) - (\beta_0 + \beta_1 E[X])$$

$$= \beta_1 (X - E[X]).$$

$$\therefore \quad E[(\hat{Y} - E[Y])^2] = \beta_1^2 Var(X)$$

Also, $E[(Y - E[Y])^2] = Var(Y)$, $E[(Y-\hat{Y})^2] = Var(\epsilon)$

from eq^n ①, we conclude,

$$E[(Y - E[Y])^2] = E[(\hat{Y} - E[Y])^2] + E[(Y-\hat{Y})^2] \quad \text{Ans}$$

3) Sol^n:- $f(x,y,z) = y^2 - 10z$; $g(x,y,z) = x^2+y^2+z^2-36$

$$l = f(x,y,z) + \lambda(g(x,y,z))$$

$$l = y^2 - 10z + \lambda(x^2+y^2+z^2-36)$$

$$\frac{\partial l}{\partial x} = 2\lambda x = 0 \Rightarrow x = 0 \text{ OR } \lambda = 0 \quad \text{--- } (a)$$

$$\frac{\partial l}{\partial y} = 2y + 2y\lambda = 0 \Rightarrow 2y = -2\lambda y \quad \text{--- } (1)$$

$$\frac{\partial l}{\partial z} = -10 + 2\lambda z = 0 \Rightarrow 2\lambda z = 10 \Rightarrow \lambda z = 5 \quad \text{--- } (2)$$

$$\frac{\partial l}{\partial \lambda} = x^2+y^2+z^2-36 = 0$$

$$\frac{\text{Eq}^n (1)}{\text{Eq}^n (2)} \Rightarrow \frac{2y}{5} = \frac{-2\cancel{\lambda}y}{\cancel{\lambda}z} \Rightarrow \frac{\cancel{2y}}{5} = \frac{-\cancel{y}\times 2}{z}$$

$$\Rightarrow \boxed{z = -5}$$

$$x^2+y^2+z^2 = 36$$

From eq^n (a) let $x = 0$: $y^2 + (-5)^2 = 36$

$$\boxed{y = \pm\sqrt{11}}$$

eq^n (2)

$$\lambda z = 5 \Rightarrow \boxed{\lambda = -1}$$

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points $(0, \sqrt{11}, -5)$, $(0, -\sqrt{11}, -5)$

$f_{(0, \sqrt{11}, -5)} = (\sqrt{11})^2 - 10 \times (-5) = 11 + 50 = 61$

$f_{(0, -\sqrt{11}, -5)} = (-\sqrt{11})^2 - 10 \times (-5) = 61$

Maximum value $= 61$ Ans

Minimum value $= \times$

4) Sol^n:- $f(x,y,z) = 3x - y - 3z$

$g(x,y,z) = x + y - z$; $h(x,y,z) = x^2 + 2z^2 - 1$

$l(x,y,z,\lambda,\mu) = 3x - y - 3z + \lambda(x + y - z) + \mu(x^2 + 2z^2 - 1)$

$\frac{\partial l}{\partial x} = 3 + \lambda + 2\mu x = 0 \Rightarrow 3 + \lambda + 2\mu x = 0$ — (1)

$\frac{\partial l}{\partial y} = -1 + \lambda \cancel{+ \ldots} = 0 \Rightarrow \boxed{\lambda = 1}$ — (2)

$\frac{\partial l}{\partial z} = -3 - \lambda + 4\mu z = 0 \Rightarrow -3 - \lambda + 4\mu z = 0$ — (3)

$\frac{\partial l}{\partial \lambda} = x + y - z = 0$ — (4)

$\frac{\partial l}{\partial \mu} = x^2 + 2z^2 - 1 = 0$ — (5)

eq^n (1) : $3 + 1 + 2\mu x = 0 \Rightarrow 2\mu x = -4 \Rightarrow \boxed{\mu x = -2}$

eq^n (3) : $-3 - 1 + 4\mu z = 0 \Rightarrow 4\mu z = 4 \Rightarrow \mu z = 1$

$\Rightarrow \boxed{x = \frac{-2}{\mu}}$ $\boxed{z = \frac{1}{\mu}}$

eq^n (5) : $\left(-\frac{2}{\mu}\right)^2 + 2 \times \left(\frac{1}{\mu}\right)^2 - 1 = 0 \Rightarrow \frac{4}{\mu^2} + \frac{2}{\mu^2} - 1 = 0$

$\Rightarrow \cancel{4+2\mu-\mu^2=0} \Rightarrow \cancel{\mu^2-2\mu-4=0}$

$\Rightarrow -\mu^2+4+2=0 \Rightarrow \boxed{\mu = \pm\sqrt{6}}$

For $\mu = \sqrt{6}$

$\boxed{x = \frac{-2}{\sqrt{6}}}$; $\boxed{z = \frac{1}{\sqrt{6}}}$; $\cancel{\frac{-2}{\sqrt{6}} + y + \frac{1}{\sqrt{6}} = 0}$

$g(x,y,z) = x+y-z = \frac{-2}{\sqrt{6}} + y - \frac{1}{\sqrt{6}} = 0 \Rightarrow \boxed{y = \frac{3}{\sqrt{6}}}$

$(x,y,z) = \left(\frac{-2}{\sqrt{6}}, \frac{3}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

For $\mu = -\sqrt{6}$; $x = \frac{-2}{-\sqrt{6}} = \frac{2}{\sqrt{6}}$; $z = \frac{1}{-\sqrt{6}} = \frac{-1}{\sqrt{6}}$

$g(x,y,z) = x+y-z = 0 \Rightarrow \frac{2}{\sqrt{6}} + y + \frac{1}{\sqrt{6}} = 0 \quad y = \frac{-3}{\sqrt{6}}$

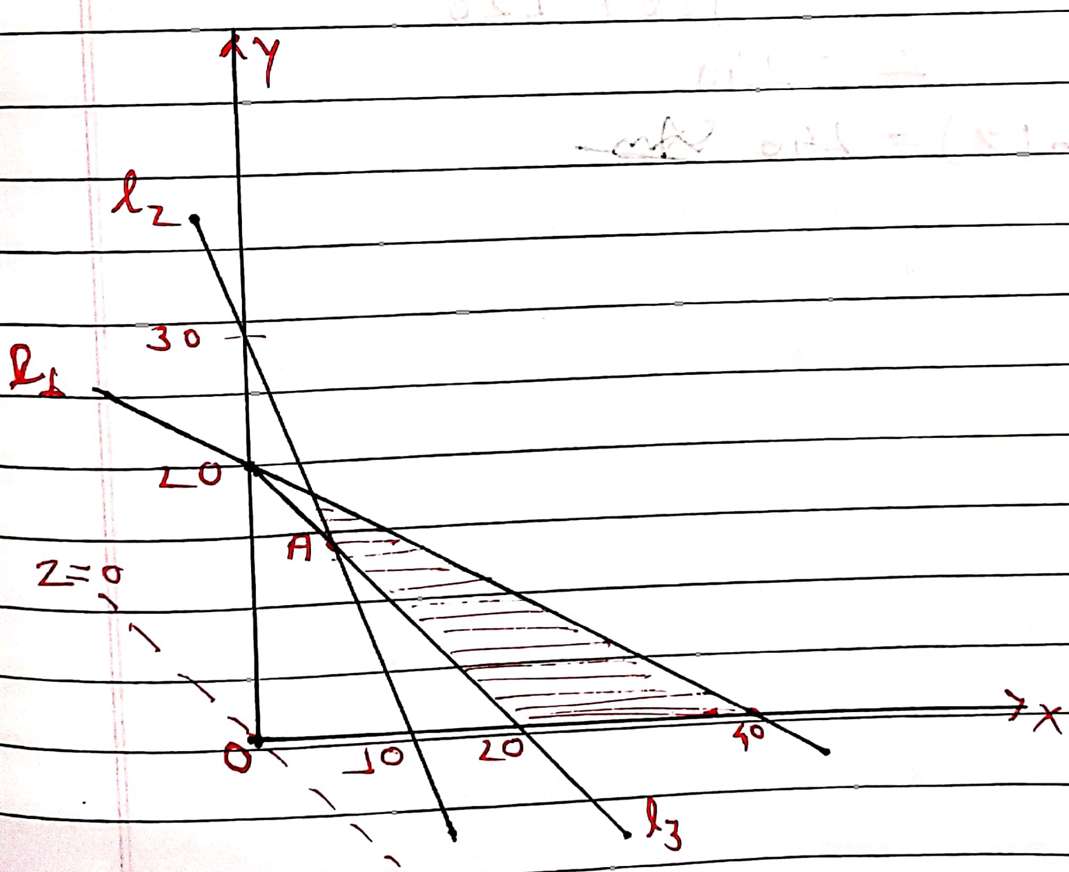
$(x,y,z) = \left(\frac{2}{\sqrt{6}}, \frac{-3}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$ Ans

5) Sol^n: $l_1: x+2y=40$ — ① $l_2: 3x+y=30$ — ②
$l_3: 4x+3y=60$ — ③

Take eq^n ① when $x=0, y=20$; $y=0, x=40$
~~② when x=~~
Points: $(0,20)$; ~~(0,40)~~ $(40,0)$

Eq^n ② : $x=0, y=30$; $y=0, x=10$; $(0,30)$; $(10,0)$

Eq^n ③ : $x=0, y=20$; $y=0, x=15$; $(0,20)$; $(15,0)$



Objective function: $Z = 20x + 10y$

assume $Z = 0$

$$20x + 10y = 0$$

$$x = -\frac{1}{2}y$$

$l_2: 3x + y = 30$
$l_3: 4x + 3y = 60$ $\Big\} \rightarrow$ on solving these eq^ns

$x = 6, \; y = 12$

pts. $(6, 12)$

$$Z = 10x + 10y = 20 \times 6 + 10 \times 12$$

$$= 120 + 120$$

$$Z = 240$$

$\min(Z) = 240$ Ans

6) Solⁿ :- $b = [4, 2, -1, 0, 0]$; $t = [-2, -1, 0, 1, 2]$

eqⁿ: $b = C + Dt$

$$A = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ 1 & t_3 \\ 1 & t_4 \\ 1 & t_5 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad x = \begin{bmatrix} C \\ D \end{bmatrix} \qquad b = \begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

Eqⁿ can be represented as $b = Ax$

least square solution to minimize $\|b - Ax\|^2$ is

$$x = (A^T A)^{-1} A^T b$$

$$A^T A = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \qquad A^T b = \begin{bmatrix} 5 \\ -30 \end{bmatrix}$$

$$(A^T A)^{-1} = \begin{bmatrix} 1/5 & 0 \\ 0 & 1/10 \end{bmatrix}$$

$$x = (A^T A)^{-1} \cdot A^T b = \begin{bmatrix} 1/5 & 0 \\ 0 & 1/10 \end{bmatrix} \begin{bmatrix} 5 \\ -30 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

best-fit line would be: $b = C + Dt$; $C = 1$ $D = -3$

Eqⁿ of best fit line: $b = 1 - 3t$ Ans

7) Sol^n:- $g = m^1x^1 + m^2x^2 + m^3x^3 + c_0$; $x^1 = 1.0$, $x^2 = 2.0$

$f(x^1, x^2, x^3) = f_0$ $x^3 = 3.0$

$\eta = 0.1$; $m^1, m^2, m^3 = 0$ (initial)

$g = m^1x^1 + m^2x^2 + m^3x^3 + c_0$

$g = 0 + 0 + 0 + 1$

$\boxed{g = 1.}$ (say $c_0 = 1$)

$L = \frac{1}{2}(f_0 - g)^2$

$\frac{\partial L}{\partial m^1} = -(f_0 - g)x^1$; $\frac{\partial L}{\partial m^2} = -(f_0 - g)x^2$; $\frac{\partial L}{\partial m^3} = -(f_0 - g)x^3$

- $\sum_{i=1}^{3} m_i$ $m^i \leftarrow m^i - \eta \frac{\partial L}{\partial m^i}$ $i = 1, 2, 3$

$\frac{\partial L}{\partial m^1} = -(f_0 - 1) \times 1 = -(f_0 - 1)$

$\frac{\partial L}{\partial m^2} = -(f_0 - 1) \times 2 = -2(f_0 - 1)$

$\frac{\partial L}{\partial m^3} = -(f_0 - 1) \times 3 = -3(f_0 - 1)$

$m^1 \leftarrow 0 - 0.1 \times (-(f_0 - 1)) = 0.1(f_0 - 1)$

$m^2 \leftarrow 0 - 0.1 \times (-2(f_0 - 1)) = 0.2(f_0 - 1)$

$m^3 \leftarrow 0 - 0.1 \times (-3(f_0 - 1)) = 0.3(f_0 - 1)$

$$y = m^1 x^1 + m^2 x^2 + m^3 x^3 + c_0$$

$$y = [0.1(f_0 - 1) \times 1 + 0.2(f_0 - 1) \times 2 + 0.3(f_0 - 1) \times 3] + 1$$

$$y = [0.1(f_0 - 1) + 0.4(f_0 - 1) + 0.9(f_0 - 1)] + 1$$

$$y = 1.4(f_0 - 1) + 1$$

$$\text{error} = f_0 - y = f_0 - [1.4(f_0 - 1) + 1] = f_0 - 1.4 f_0 + 1.4$$
$$= -0.4 f_0 + 2.4$$

$$\frac{\partial L}{\partial m^1} = -(-0.4 f_0 + 2.4) \cdot 1 = 0.4 f_0 - 2.4$$

$$\frac{\partial L}{\partial m^2} = -(-0.4 f_0 + 2.4) \times 2 = 0.8 f_0 - 4.8$$

$$\frac{\partial L}{\partial m^3} = -(-0.4 f_0 + 2.4) \times 3 = 1.2 f_0 - 7.2$$

$$m^1 \leftarrow 0.1(f_0 - 1) - 0.1(0.4 f_0 - 2.4)$$
$$= 0.1(f_0 - 1) - 0.04 f_0 + 0.24$$

$$m^2 \leftarrow 0.2(f_0 - 1) - 0.1(0.8 f_0 - 4.8)$$
$$= 0.2(f_0 - 1) - 0.08 f_0 + 0.48$$

$$m^3 \leftarrow 0.3(f_0 - 1) - 0.1(1.2 f_0 - 7.2)$$
$$= 0.3(f_0 - 1) - 0.12 f_0 - 0.72$$