# Magnetic circuits

SI unit of magnetic field: A/m

$\boxed{H \cdot l = NI}$

H → magnetic field, l → mean path length

flux density

$B = \mu_0 \mu_r H$

$\boxed{B = \frac{\mu_0 \mu_r NI}{l}}$

flux

$\boxed{\phi = B \times A}$ → SI unit

$Tm^2 (Wb)$ SI unit

$\boxed{\lambda = N\phi}$ → flux linkage

$\boxed{L = \lambda / I}$ → inductance

H → magnetic field outside

B → magnetic field inside

$\phi = \frac{NI \mu_0 \mu_r A}{l} \Rightarrow \boxed{\phi = \frac{NI}{(l/\mu_0 \mu_r A)}}$

NI → magneto motive force

$(l/\mu_0 \mu_r A)$ → reluctance

EMF ⇔ MMF, Resistance ⇔ Reluctance, $I \Leftrightarrow \phi$

Q: A magnetic circuit has $A_c = 4 \times 4 cm^2$, path length = 40 cm and number of turns = 600. Taking $\mu_r = 6000$ for iron, calculate the exciting current for B = 1.2T. Calculate flux and inductance.

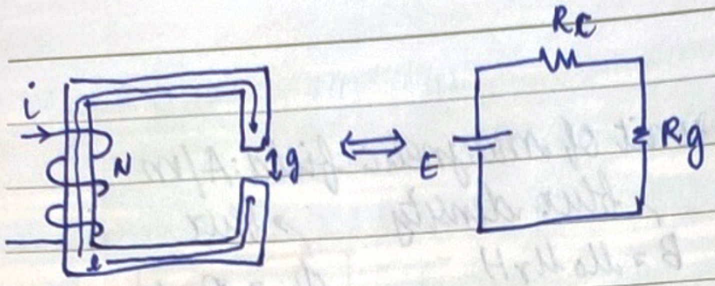
$A = 16 \times 10^{-4} m^2$, $l = 40 \times 10^{-2}$, $N = 600$

$\phi = BA = 1.2 \times 16 \times 10^{-4} = \underline{19.2 \times 10^{-4} \, Tm^2}$

$B = \frac{\mu_0 \mu_r NI}{l} \Rightarrow \frac{1.2 \times 40 \times 10^{-4}}{4\pi \times 10^{-7} \times 6 \times 10^3 \times 6 \times 10^2} = I$

$\therefore I = \underline{1.05A}$

$L = N\phi / I = \frac{600 \times 19.2 \times 10^{-4}}{1.05} = \underline{1.097H}$




$$\boxed{R_C = \frac{l}{\mu_0 \mu_r A}}$$

$$\boxed{R_g = \frac{g}{\mu_0 A}}$$ → length of free space.

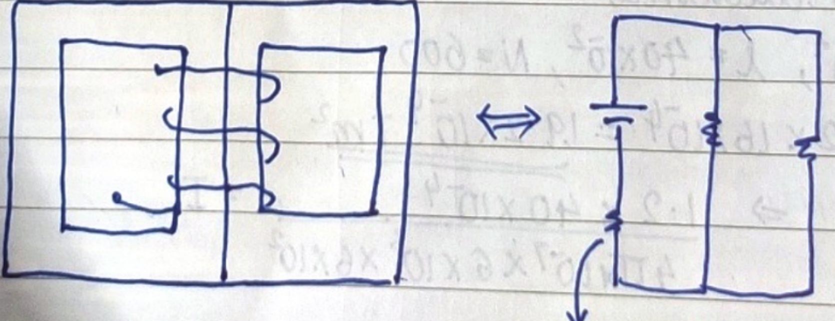
$$i = E/(R_c+R_g) \Leftrightarrow \phi = \frac{NI}{\left(\frac{l}{\mu_0 \mu_r A} + \frac{g}{\mu_0 A}\right)} \Rightarrow \boxed{\phi = \frac{NIA\mu_0}{\left(\frac{l}{\mu_r} + g\right)}}$$

Q: A magnetic circuit has $A_c = 4 \times 4\,cm^2$, path length = 40cm and no. of turns = 600. Taking $\mu_r = 6000$ for iron and an air gap of 6mm, calculate inductance.

$$L = \frac{N\phi}{I} = \frac{N^2 I A \mu_0}{(l/\mu_r + g) I} = \frac{N^2 A \mu_0}{\left(\frac{l}{\mu_r} + g\right)} = \frac{600 \times 600 \times 16 \times 10^{-4} \times 4\pi \times 10^{-7}}{\left(\frac{0.4}{6 \times 10^3} + 6 \times 10^{-3}\right)}$$

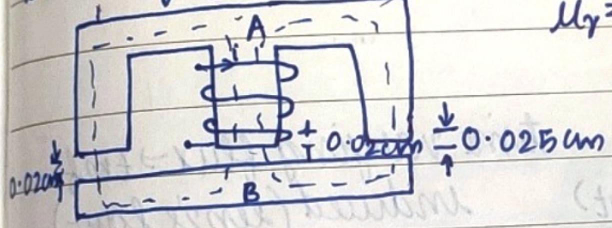
$$= \frac{36 \times 16 \times 4 \times 3.14 \times 10^{-4}}{6.06} = \underline{\underline{0.1193H}}$$

Parallel magnetic circuit



cancel out if area of cross section of both parts are equal.

(i) For the magnetic core shown below, take $\mu_r$ = infinite
Mean length from A-B through outer limb = 0.5m
Mean length from A-B through inner to limb = 0.2m
Area of CS of central limb = $2cm^2$, Area of CS of outer limb = $1cm^2$. It required to establish a flux of 0.75mWb in the air gap of central limb. Determine mmf of exciting coil.



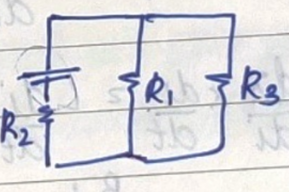

$\mu_r = \infty$, $R_{material} = 0$
Only consider reluctance of air gap

$R_1 = \frac{g_1}{\mu_0 A_1} = \frac{2\times10^{-4}}{\mu_0\times1\times10^{-4}} = \frac{2}{\mu_0}$

$R_3 = \frac{2.5\times10^{-4}}{\mu_0\times1\times10^{-4}} = \frac{2.5}{\mu_0}$

$R_2 = \frac{2\times10^{-4}}{\mu_0\times2\times10^{-4}} = \frac{1}{\mu_0}$

equivalent:

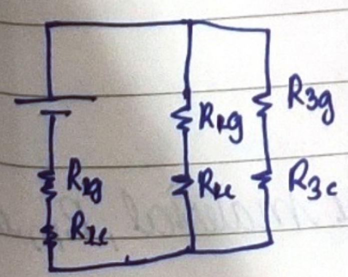



$R_{eq} = R_2 + R_1 // R_3 = \frac{1}{\mu_0} + \frac{\frac{2}{\mu_0}\times\frac{2.5}{\mu_0}}{\frac{4.5}{\mu_0}}$

$R_{eq} = \frac{19}{9\mu_0}$, mmf(NI) = $\phi \times R_{eq}$

$= \frac{0.75\times10^{-3}\times19}{4\pi\times10^{-7}\times9} = \underline{\underline{1.26\times10^3 A}}$

(ii) When $\mu_r$ = 5000
Same procedure but $R_{material} \neq 0$




$R_{eq} = R_1 // R_3 + R_2$

$= R_{2g} + R_{2c} + (R_{1g} + R_{1c}) // (R_{3g} + R_{3c})$

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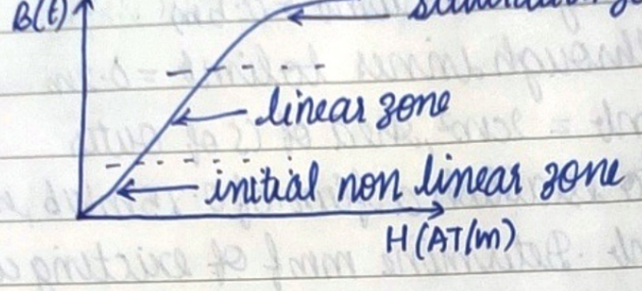
DC Magnetization Curve




$\mu_r$ is not constant

$B = \mu H = \mu_0 \mu_r H$

At saturation, $\mu_r = 0$

$L = \frac{\mu N^2 A}{l} = 0$

AC Magnetization

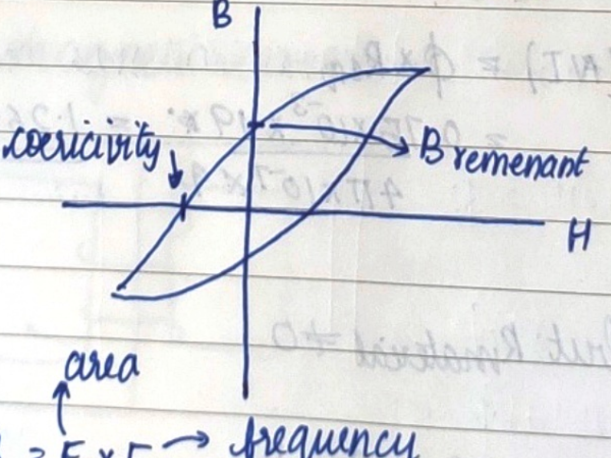
$\phi = \frac{\mu_0 \mu_r A N I}{l_g}$

time varying flux → Emf induced (Lenz's law)

$\phi_m \sin(\omega t) = \frac{\mu_0 \mu_r A N I_m \sin(\omega t)}{l_g}$

$E = \frac{d}{dt}(\text{flux linkage}) = \frac{d(N\phi(t))}{dt}$ [E=0 for DC current]

$E = \frac{d\lambda}{dt} = \frac{d\lambda}{di} \times \frac{di}{dt} = L\frac{di}{dt}$ as $L = \frac{d\lambda}{di}$ (flux linkage per unit current)




Area under this hysteresis loop is core loss/hysteresis loss (energy)

Power loss = E x F → frequency

$\boxed{P_h = K_h f B_m^n \ W/m^3}$

$K_h$ is a characteristic constant of the core material, $B_m$ is

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the maximum flux density and n, called Steinmetz exponent, may vary from 1.5 to 2.5 depending upon material and is often taken as 1.6.

Variation of B-H loop with F : frequency ↓ area ↓, f ↑ core becomes wide.

Types of B-H loop:
- Narrow for high frequency AC application
- Wide for permanent magnet application

AC core loss :
(i) Eddy current loss → due to induced EMF and circulating current. time varying flux → induced voltage → eddy current → loss.
eddy current loss → due to induced EMF and circulating current. Induced EMF proportional to frequency and amplitude of flux. Loss proportional to $EMF^2$. Eddy current loss proportional to $F^2$.

$$\boxed{P_e = K_e f^2 B^2 \ W/m^3}$$

Total core loss $= P_h + P_e = K_1 B^n f + K_2 B^2 f^2$
- eddy current is low if electrical resistance of core is high
- core with high resistivity (ferrite), core made with powdered iron. Material divided into thin laminations.

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Q: For a material, core loss = 1550W at 50Hz, 3000W at 75Hz (keeping $B_{peak}$ same). Calculate core loss at 100Hz.

core loss $= K_1 B^n f + K_2 B^2 f^2$

$$L = K_1 B^n f + K_2 B^2 f^2$$

$$\frac{L}{f} = \underbrace{K_1 B^n}_{A} + \underbrace{K_2 B^2}_{B} f$$

$$31 = K_1 B^n + K_2 B^2 \times 50$$

$$40 = K_1 B^n + K_2 B^2 \times 75$$

$$31 = A + 50B$$

$$40 = A + 75B$$

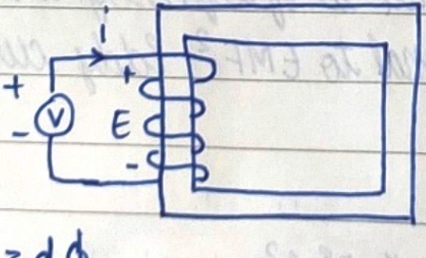
$$9 = 25B$$

$$B = \frac{9}{25}, \quad A = 13$$

$$L = 13 \times 100 + \frac{9}{25} \times 100 \times 100 = 1300 + 3600 = \underline{\underline{4900W}}$$

Application of Magnetic Circuit
- transformer
- electric machines
- Inductors

Magnetic Circuits and Inductance




$$\phi = \frac{\mu_0 \mu_r N I}{l_g} \rightarrow f(t)$$

$\phi(t)$ : $E$ ~~mf~~ $= E = N\frac{d\phi}{dt}$

$$V = E = \frac{d\phi}{dt}$$

$$\phi(t) = \int V dt$$

$$V = L\frac{di}{dt}$$

$$V_m \sin\omega t = N\frac{d\phi}{dt}$$

$$i = \int \frac{V}{L} dt$$

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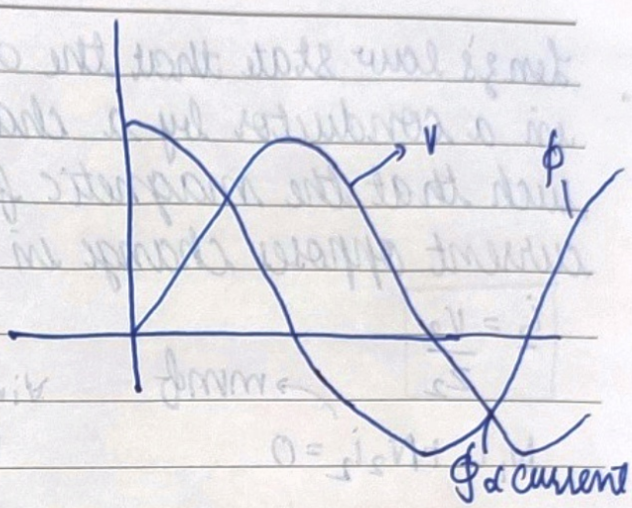
$V = iR + E$

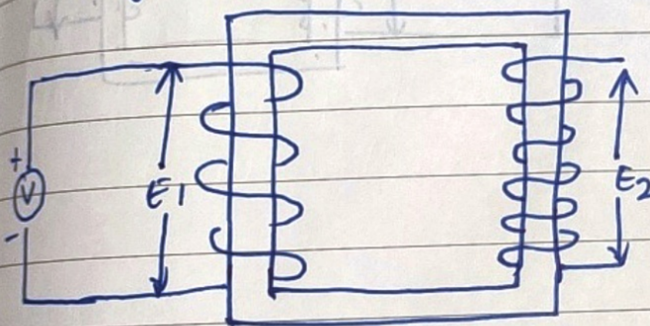
$= iR + N\frac{d\phi}{dt}$

$\phi = \frac{V_m}{\omega L}\cos\omega t$




$\boxed{V = 4.44\,\phi_m N f}$

$V$ rms voltage, $\phi_m$ peak flux




$E_1 = V_1 = N_1\, d\phi/dt$

$E_2 = N_2\, d\phi/dt$

$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}}$ (operating principal for a transformer)

$E_2 = \frac{N_2}{N_1} E_1$

AC magnetic circuit with two windings

$\boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1}}$ only when B is Not saturated

$\phi = -\frac{1}{N_1}\int V_{in}\,dt \qquad V_2 = -N_2\frac{d\phi}{dt}$

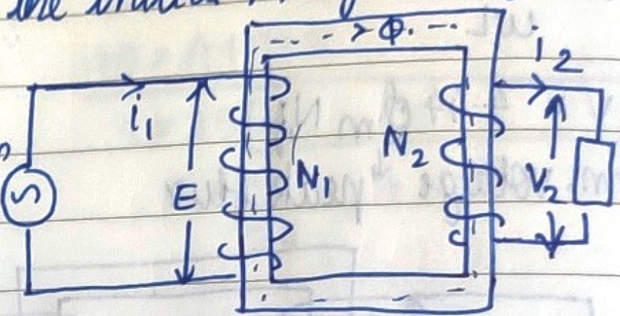
$V_1 = V_m \sin(2\pi f t)$, $V_{1,rms} = 4.44 f N_1 \phi_m$

$\frac{V_2 rms}{V_1 rms} = \frac{N_2}{N_1}$, $\boxed{i_1^0 = \frac{V_{in}}{\omega L}}$, $\boxed{L = \frac{\mu_0 \mu_r N_1^2 l_g}{A}}$

no load current

magnetizing current

Lenz's law state that the direction of the electric current in a conductor by a changing magnetic field is such that the magnetic field created by induced current opposes change in the initial magnetic field.




$$\boxed{i_2 = \frac{V_2}{Z_2}}$$

mmf

$$N_1 i_1 + N_2 i_2 = 0$$

$$\boxed{i_1 = \frac{N_2}{N_1}(-i_2)}$$

But $\phi = -\frac{1}{N_1}\int V_{in}\,dt$

Components of primary current

1. Load component $\rightarrow i_1 = -N_2 i_2 / N_1$
2. (a) no load magnetizing component (magnetizing current)
   (b) no load loss component

$i_1 = \frac{V_{in}}{\omega L}$ → core loss → Eddy, hysteresis

Depends only on voltage not on $Z_2$ or $i_2$

Reflected impedance:

Impedance connected to secondary, but current drawn on primary.

$$Z_2' = \frac{V_1}{i_2'} = \frac{V_1}{N_2/N_1 \times i_2} = \frac{V_1}{N_2/N_1 \times V_2/Z_2} = \frac{V_1}{N_2/N_1 \times N_2/N_1 (V_1/Z_2)}$$

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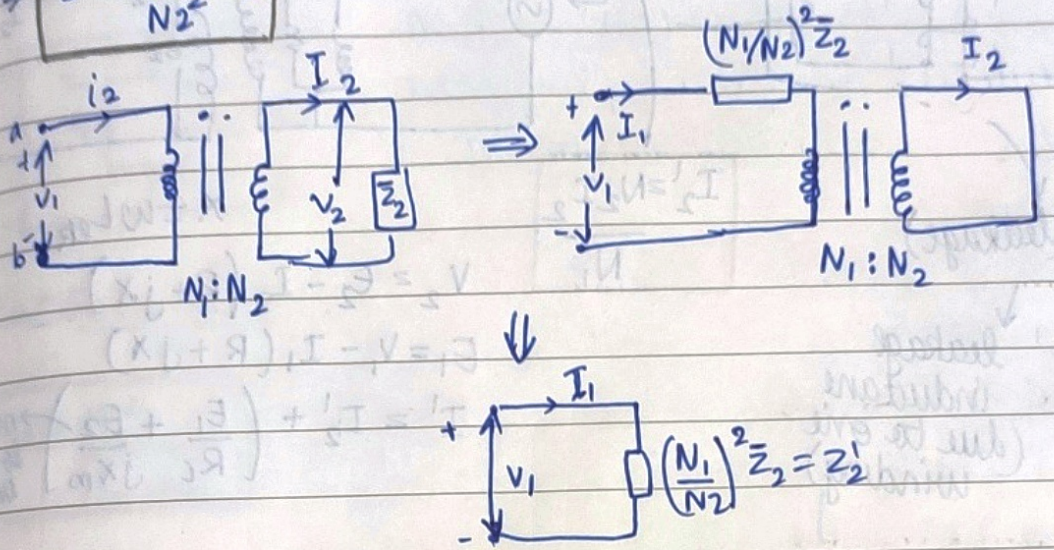
$$\Rightarrow \boxed{Z_2' = \frac{N_1^2}{N_2^2} Z_2}$$




Q: For a 150:75 ratio transformer, the primary is connected to 200 $V_{rms}$ source and secondary is connected to a load impedance of 8+6j. Calculate

(i) load impedance reflected to primary side

(ii) Secondary and primary current

(iii) Primary and secondary side real and reactive power

ans: (i) $z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2 = \left(\frac{150}{75}\right)^2 (8+6j) = \underline{\underline{32+24j}}$

(ii) $V_1 = 200V$, $V_2 = (N_2/N_1)V_1 = (75/150) \times 200 = 100V$

$I_2 = V_2 / z_2 = 100/(32+24j) = (2 - 3/2\,j)\,A$

$i_1 = -(N_2/N_1) \times i_2 = (2 - 3/2\,j\,/2) = (1 - 3/4\,j)\,A$

(iii) $P_2(\text{real}) = i_2^2 \, \text{Real}(z_2) \Rightarrow P_1(\text{real}) = i_1^2 \, \text{Real}(z_2')$ both are equal.

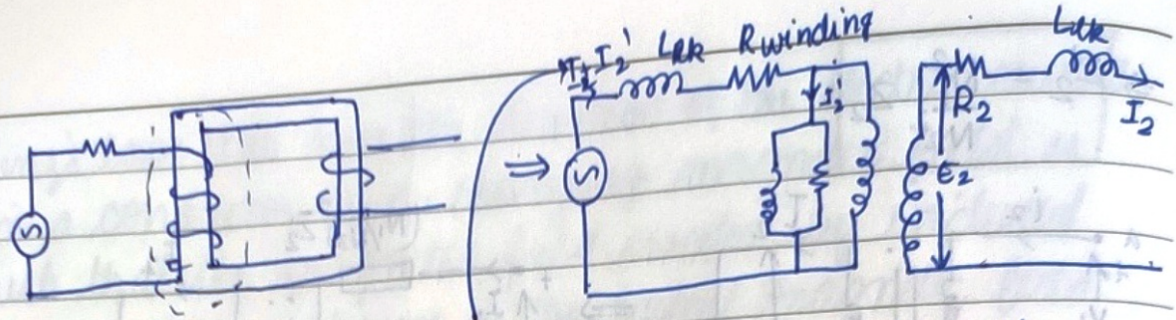
modulus values ← (rms value in formula)

50W    50W

reactive power

$Q_{reactive} = i_{rms}^2 X \rightarrow$ imaginary part of impedance

$Q_1 = 37.5W$    $Q_2 = 37.5W$




$$\boxed{I_2' = \frac{N_2 I_2}{N_1}}$$

flux (leakage)
- mutual inductance
- leakage inductance (due to one winding)

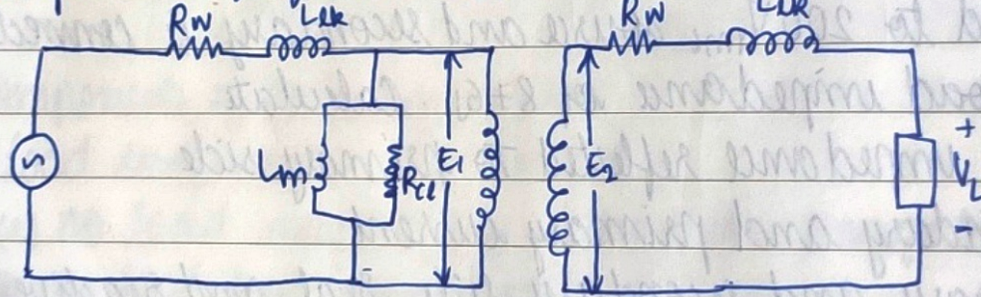
$X = \omega L_{lk}$

$V_2 = E_2 - I_2(R + jX)$

$E_1 = V_1 - I_1(R + jX)$

$I' = I_2' + \underbrace{\left(\frac{E_1}{R_c} + \frac{E_2}{jX_m}\right)}_{\text{no load current}}$

## Components of equivalent circuit




$R_W \rightarrow$ Winding resistances
$L_{lk} \rightarrow$ leakage inductances
$L_m \rightarrow$ Magnetizing inductances
$R_{cl} \rightarrow$ Core loss resistance

Ideal transformer:
- $i_1^0 = 0$
- $R = 0$
- $L_{lk} = 0$
- $L_m = \infty$
- $R_{cl} = \infty$

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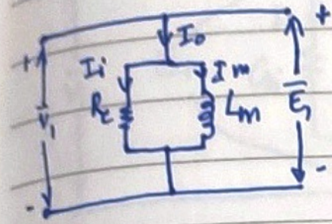
## Shunt Branch (No load current)




- No load current only depends on applied voltage and frequency.

$$I_0 = I_m + I_i$$

$$I_0 = \frac{E_1}{j\omega L_m} + \frac{E_1}{R_c} \simeq \frac{V_1}{j\omega L_m} + \frac{V_1}{R_c}$$

$$L_m = \frac{\mu_0 \mu_r N_1^2 l_g}{A}$$

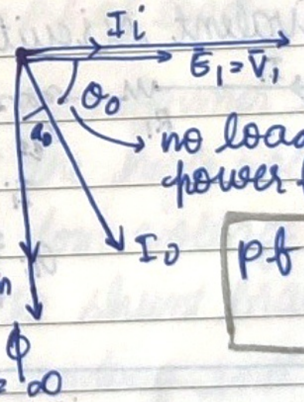
$$\boxed{R_c = \frac{V_1^2}{P_{ironloss}}}$$

- R is a function of applied voltage and frequency due to dependence of hysteresis and eddy current loss on B and F.
- Shunt branch also represented as shunt admittance, $G+jB$

$$G - jB = \frac{1}{R} + \frac{1}{j\omega L_m}$$

$$\vec{I_0} = \vec{I_i} + \vec{I_m}$$

$$\boxed{|I_0| = \sqrt{I_i^2 + I_m^2}}$$




$$\boxed{pf = \frac{I_i}{I_0}}$$

- for ideal transformer: $I_0 = 0$ as $R_c, L_m = \infty$

Q: A transformer on no load has a core loss of 50W, draws a current of 2A (rms) and has an induced emf of 230V (rms), 50 Hz. Determine the core-loss current, magnetizing current and no load power factor. Calculate the no load circuit parameters of the transformer ($R_c$ and $L_m$) (Neglect winding resistance and leakage flux).

$pf = 50/2\times230 = 0.108 \quad I_i = I_0 \times pf = \underline{0.217A}$

$I_m = I_0 \sin\theta = 2\times\sin(\cos^{-1}(pf)) = \underline{1.988A}$

no. load power factor: $pf = \underline{0.108}$

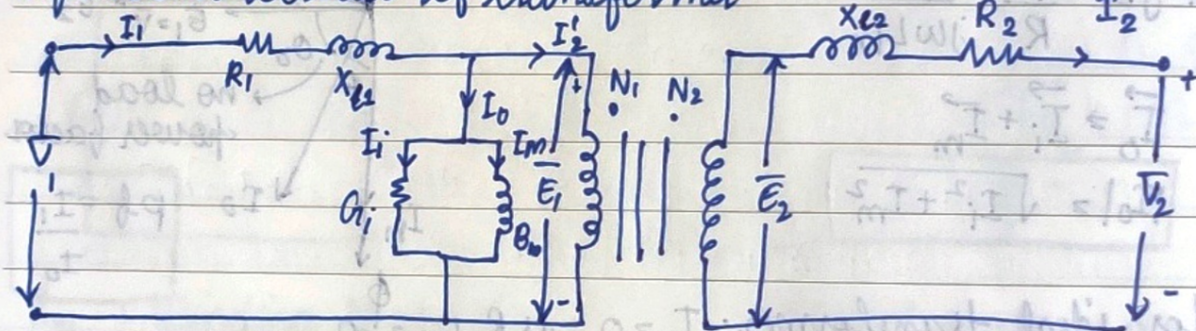
$R_C = \frac{V_1^2}{P_{ironloss}} = \frac{230\times230}{50} = \underline{1.058K\Omega}$

$B_m = \frac{I_m}{V_1} = \frac{1.988}{230} = 8.64\times10^{-3}\,\mho$

$B_m = \frac{1}{j\omega L_m} \Rightarrow L_m = \frac{1}{\omega B_m} = \frac{1\times230}{100\pi\times1.988} = \underline{0.368H}$

$R_c = 1.058k\Omega,\ B_m = 0.368H$

Equivalent circuit of transformer



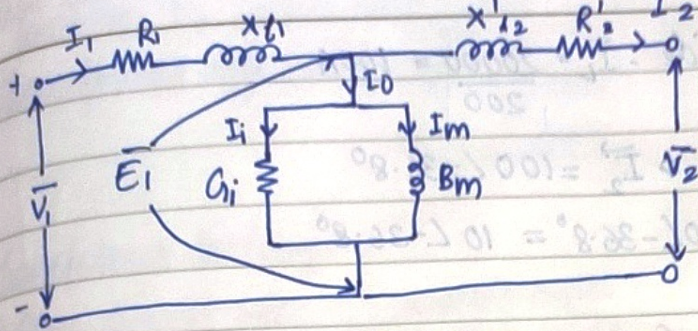

$\bar{V}_1 = \bar{E}_1 + \bar{I}_1 R_1 + j\bar{I}_1 \bar{X}_1 \rightarrow$ leakage resistance $\quad E_2 = \frac{N_2}{N_1}E_1$

$\bar{V}_2 = \bar{E}_2 - \bar{I}_1 R_2 - j I_2 X_2$

$I_2' = \frac{N_1}{N_2} I_1, \quad I_1 = \underbrace{I_2'}_{\text{load current}} + I_0$ - no load current

$R_2' + jX_{L_2}' = \left(\frac{N_1}{N_2}\right)^2 (R_2 + jX_{L_2})$

dashed quantities are reffered quantities.



equivalent circuit

Transformer ratings: for continous operation:

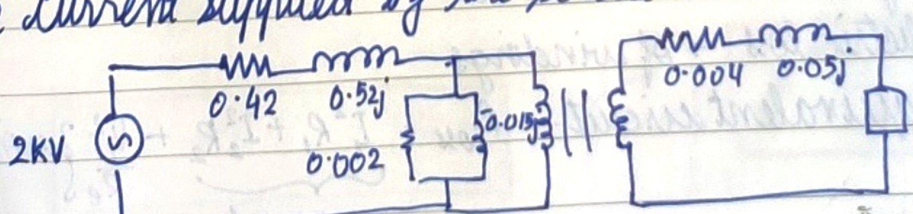
- voltage rating (depends on core)
- voltage ratio = 2000/200 = $\underline{\underline{10}}$ $\boxed{20KVA, 50Hz, 2000/200V}$ primary/secondary
- frequency rating
- current rating (depends on conductor) → for primary: $\frac{20k}{2000} = 10A$ sec: 100A : $\underline{10/100A}$
- VA rating → power but written VA as it doesn't depend on p.f of load

↓ V rating × I rating

Q: A 20KVA, 50Hz, 2000/200V distribution transformer has a leakage impedance of 0·42 + j0·52 in the high voltage (HV) winding and 0·004 + j0·05 in the low voltage (L-V) winding. When seen from HV side, the shunt branch admittance $Y_0$ is (0·002 − j0·015). If the sending-end voltage applied to the transformer is 2kV. The load is drawing rated transformer current at 0·8 pf lagging.

1) Find the voltage at the load-end of the transformer
2) Find the current supplied by the source.

⇒

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1) Rated current for LV side : $I_L = \frac{20000}{200} = 100A$

$\vec{I_2} = 100\angle(-\cos^{-1}(0.8))$ $\xrightarrow{}$ lagging $\Rightarrow \vec{I_2'} = 100\angle -36.8^\circ$

$I_2' = \frac{N_2 I_2}{N_1} = \frac{200}{2000} \times 100\angle -36.8^\circ = 10\angle -36.8^\circ$

$E_1 = V_1 - I_2' Z = 2000 - (0.42 + 0.52j) \times 10\angle -36.8^\circ$
$= 1993.5\angle 0.471^\circ$

$E_2 = \frac{N_2}{N_1} E_1 = 199.35\angle 0.471^\circ$

$V_2 = E_2 - I_2 Z_2 = 199.35\angle 0.471^\circ - (100\angle -36.8 \times (0.004 + 0.05j))$
$= \underline{\underline{196.07\angle -1.47^\circ}}$

2) $I_0 = E_1 \times Y_0 = 1993.5\angle 0.471^\circ (0.0002 - 0.0015j)$
$= 0.423 - 2.987j$

$I_1 = I_0 + I_2' = 0.423 - 2.987j + 8.007 - 5.99j$
$= 8.43 - 8.98j = \underline{12.315\angle -46.8^\circ}$

Transformer losses (decreases efficiency and increases temperature)
1. Core loss (Hysteresis & eddy current) → depends on induced voltage
2. Copper loss → 0 for no load (depends on current)
3. Stray loss → due to metals
4. Dielectric loss → of windings

In equivalent circuit : $\boxed{P_{loss} = \underbrace{I_1^2 R_1 + I_2^2 R_2}_{\text{copper loss}} + \frac{V_1^2}{R_e}}$ } iron loss

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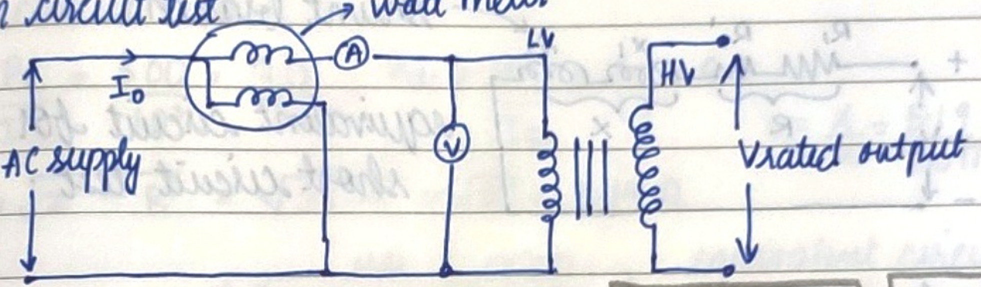
* $P_{load} = V_2 I_2 \cos(\phi_2) = S\cos(\phi_2)$, $V_2$ varies slightly with loading, so variation of $V_2$ with load can be neglected for efficiency calculation. Iron loss depends only on voltage, so it remains constant with loading. $I_2$ varies with load linearly. $\phi_2 \to$ load parameter.

Transformer efficiency: $\boxed{\eta = \frac{P_{output}}{P_{input}} = \frac{P_{load}}{P_{load} + losses}}$

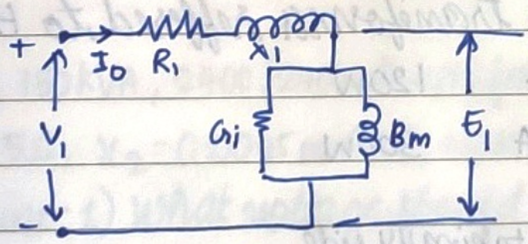
Transformer testing:

(i) Open circuit test

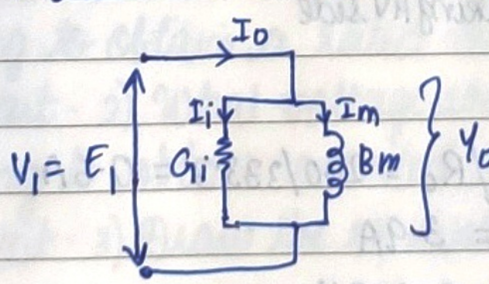



Ammeter reading: no load current : $\boxed{I_m = \sqrt{I_0^2 - I_i^2}}$, $\boxed{I_i = V_1/R_c}$

Watt meter reading: Iron loss (core loss) $\boxed{I_m = \frac{V_1}{\omega L_m}}$ $\boxed{R_c = \frac{V_1^2}{P_0}}$

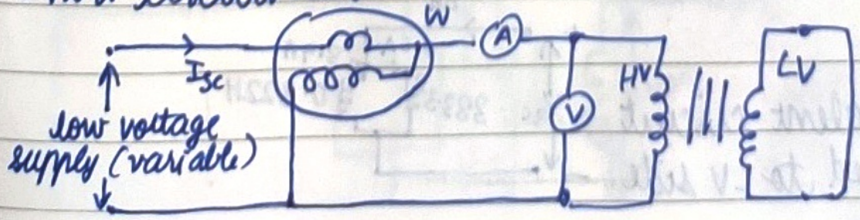



Current only flows through shunt part

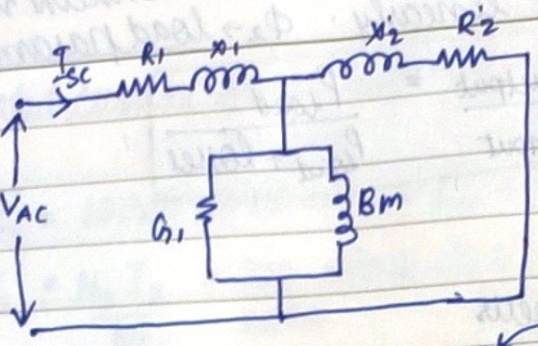



Equivalent circuit for open circuit test

(ii) Short circuit test



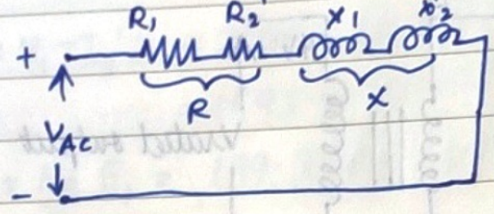

$\boxed{Z_{sc} = \frac{V_{sc}}{I_{sc}}}$ → voltmeter reading → Ammeter reading




$$\boxed{R_1 + R_2' = \frac{P_{sc}}{I^2_{sc}}} \quad (X = \omega L)$$

$$\boxed{X_{L_1} + X'_{L_2} = \sqrt{Z_{sc}^2 - (R_1 + R_2')^2}}$$

current in negelibilee in shunt branch




equivalent circuit for short circuit test.

Q: The following data were obtained on a 20KVA, 50Hz, 2000/200V distribution transforma. Draw the approximate equivalent circuit of the tranformer reffered to HV side

| | | | |
|---|---|---|---|
| OC test | 200 V | 4A | 120W |
| SC test | 60V | 10A | 300W |

Open circuit test:

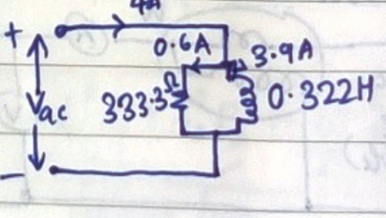
$$R_c = \frac{V_1^2}{P_0} = \frac{200 \times 200}{120}$$ → taking HV side

$R_c = \underline{333.3\,\Omega}$ , $I_i = V_1/R_c = 200/333.3 = 0.6A$

$I_m = \sqrt{I_0^2 - I_i^2} = \sqrt{4^2 - 0.6^2} = 3.9A$

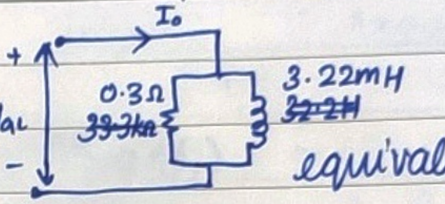
$$L_m = \frac{V_1}{\omega I_m} = \frac{200}{100\pi \times 3.9} = \underline{\underline{0.322H}}$$

equivalent circuit reffered to LV side



$R_{HV} = \left(\frac{N_1}{N_2}\right)^2 \times 333.3 = \cancel{33.3k\Omega}\ 0.3\Omega$

$L_{HV} = \left(\frac{N_1}{N_2}\right)^2 \times 0.322H = \cancel{32.2H}\ \cancel{0}\ 3.22\,mH$



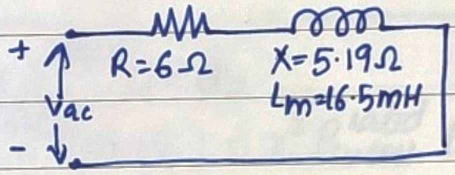

equivalent circuit

Short circuit test

$R = \frac{P_{sc}}{I_{sc}^2} = \frac{300}{10^2} = \underline{3\Omega}$ , $Z_{sc} = \frac{V_{sc}}{I_{sc}} = \frac{60}{10} = 6\Omega$

$L_m = \frac{X}{\omega} = \frac{5.19}{100\pi} = \underline{16.5mH}$

$X = \sqrt{Z^2 - R^2} = \sqrt{36-9} = \sqrt{27} = 5.19\,\Omega$

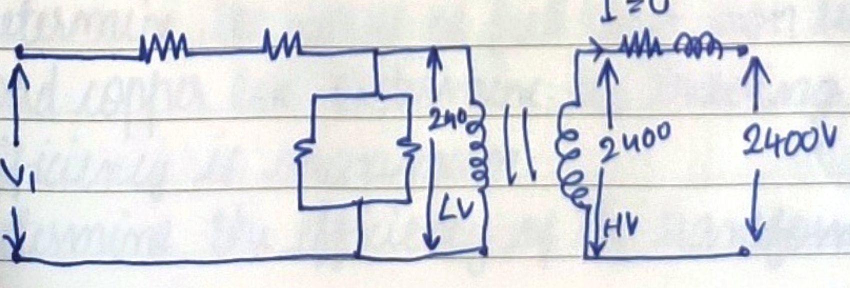



equivalent circuit for short circuit test.

Q: For a 150KVA, 2400/240V transformer, $R_1 = 0.2\Omega$, $R_2 = 0.002\cancel{m}\Omega$, $X_1 = 0.45\Omega$, $X_2 = 0.0045\cancel{m}\Omega$, $R_i = 10K\Omega$, $X_m = 1600\Omega$.

Determine: 1) What voltage should be applied at the LV winding to obtain a rated voltage at the HV terminals in OC test. 2) What voltage should be applied at the HV winding to obtain a rated current at the LV terminal in SC test. 3) Draw the equivalent circuit indicating values in PU.

Open circuit:



## Transformer efficiency:

At x times rated load:

$P_{load}$ at (x times rated load) $= xS^{rated}\cos(\phi_2)$

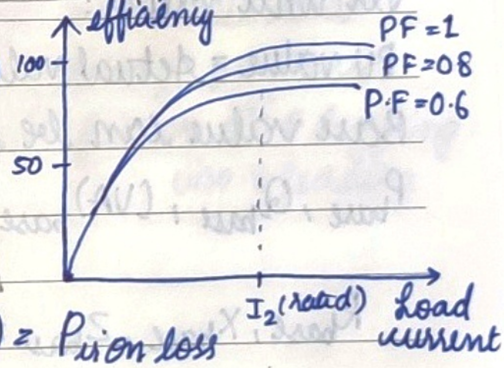
$I_2$ at (x times rated load) $= xI_2^{rated}$

$P_{cu\,loss}$ at (x times rated load) $= x^2 P_{cu\,loss}^{rated}$

$$\eta(x) = \frac{P_{load}(x)}{P_{load}(x) + P_{cu\,loss}(x) + P_{iron\,loss}(x)}$$

$$\boxed{\eta(x) = \frac{xS^{rated}\cos(\phi_2)}{xS^{rated}\cos(\phi_2) + x^2 P_{cu\,loss}^{rated} + P_{iron\,loss}^{rated}}}$$

for maximum efficiency, $\frac{d(\eta(x))}{dx} = 0$




$$\boxed{x = \sqrt{\frac{P_{iron\,loss}^{rated}}{P_{cu\,loss}^{rated}}} = x_{max\text{-}efficiency}}$$

$P_{cu\,loss}$ at ($x_{max\text{-}eff}$ times rated load) $= P_{iron\,loss}$

Q: A 500KVA transformer has an efficiency of 95% at full load and also at 60% full load; both at upf

1. Determine the values of full load iron loss and full load copper loss. Determine the loading at which efficiency is maximum.
2. Determine the efficiency of the transformer at 3/4th full load.

1) $\eta = \frac{500\times 1}{500\times 1 + P_i + \cancel{0.36}P_c} = 0.95$ $\quad 0.95 = \frac{500\times 0.6}{500\times 0.6 + P_i + 0.36P_c}$

$475 + 0.95P_i + 0.95P_c = 500$ $\quad 285 + 0.95P_i + 0.342P_c = 300$

$P_i + P_c = 25/0.95$ $\quad 0.95P_i + 0.342P_c = 15$

$\Rightarrow 0.608P_c = 10 \quad P_c = 10/0.608 = \underline{\underline{16.45\,KW}}$

$\Rightarrow P_i = 25/0.95 - 10/0.608 = \underline{\underline{9.87\,KW}}$

2) $\eta = \frac{500\times 0.75}{500\times 0.75 + 9.87 + 16.45\times(0.75)^2}$

$= \frac{375}{375 + 9.87 + 9.253125} = 0.9515 = \underline{\underline{95.15\%}}$

Per unit values:

PU value = Actual value / Base value

Base value can be taken as rated value

$P_{base}, Q_{base}, (VA)_{base} = V_{base} I_{base}$

$R_{base}, X_{base}, Z_{base} = \frac{V_{base}}{I_{base}}, \frac{V^2_{base}}{(VA)_{base}}$

Voltage regulation

~~% voltage regulation = $\frac{|V_2(\text{no load})| - |V_2(\text{full load})|}{|V_2(\text{no load})|} \times 100$~~

$$\boxed{\%\text{ voltage regulation} = \frac{V_2(\text{no load}) - V_2(\text{full load})}{V_2(\text{full load})} \times 100}$$

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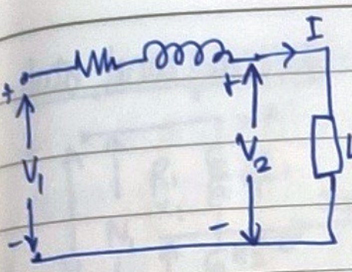
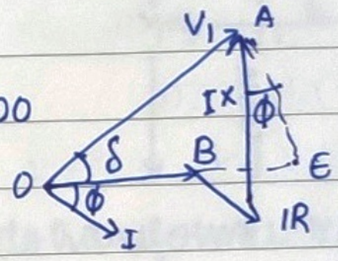



$V_{20} = V_1$

$V_{20} - V_2 = I(R\cos\phi \pm \sin\phi)$

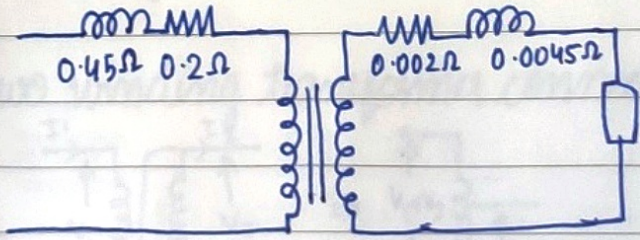
($V_{20} \to V_2$ when $I$ is 0)



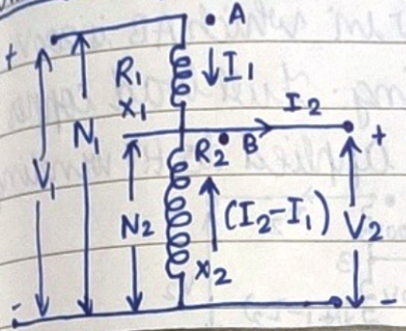

% voltage regulation, $Reg = \frac{V_{20} - V_2}{V_2} \times 100$

$$\boxed{\Rightarrow Reg = \frac{V_{20} - V_2}{V_2} \times 100 = \frac{I(R\cos\phi \pm X\sin\phi)}{V_2} \times 100}$$

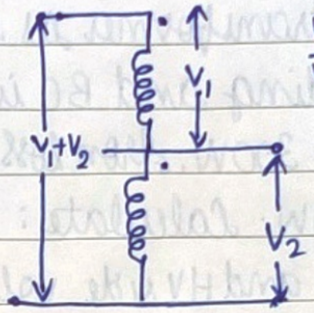
Q: For a 150 KVA, 2400/240V transformer, $R_1 = 0.2\Omega$, $R_2 = 0.002\Omega$, $X_1 = 0.45\Omega$, $X_2 = 0.0045\Omega$ with full-load on the LV side at rated voltage, calculate the excitation voltage on HV side and voltage regulation. The load power factor is (i) 0.8 lagging. (ii) 0.8 leading



## Auto transformer




$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$




$$\frac{V_{in}}{V_{out}} = \frac{V_1+V_2}{V_2}$$

$$V_1 I_1 = V_2 I_2$$

$$\boxed{\begin{aligned}(VA)_{upper} &= (V_1-V_2)\,I_1 \\ (VA)_{lower} &= V_2\,(I_2-I_1)\end{aligned}}$$

for auto transformer VA rating is reduced

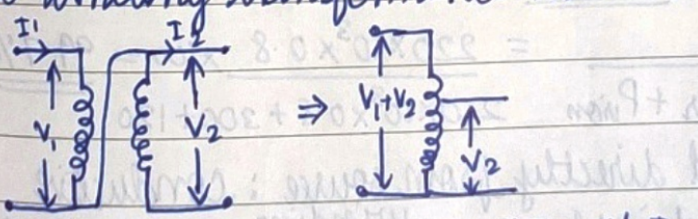
$\boxed{\text{load(VA)} \leq \text{transformer (VA)}}$

Required ratings are reduced, so a given 2 winding transformer rated for S can be operated to load higher than S if connected as an auto transformer.

Disadvantage: No isolation between input and output

Advantages: can connect a higher load, saves copper

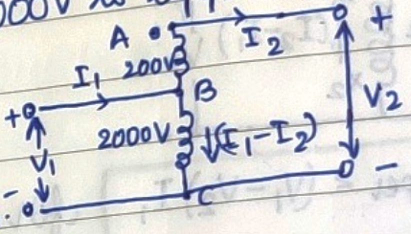
## Two winding transformer connected as auto transformer




losses: $V_1 I_1$

$$(V_1+V_2)\,I_1 = V_2 I_2$$

Q: ~~A~~ 2000/200-V, 20-KVA is connected as a ~~set~~ step-up autotransformer as in figure below in which AB is 200V winding and BC is 2000-V winding. Full load copper loss = 300W. Iron loss when 2000V is applied to HV winding = 120W. Calculate:




1) The LV and HV side voltage ratings of the autotransformer.
2) KVA rating of autotransformer.
3) efficiency at full load 0.8 pf
4) KVA transferred inductively and conductively

ans: 1) LV voltage = $\underline{2000\,V}$
HV voltage = $\underline{2200\,V}$

2) KVA = $V_2 I_2 = 2200 \times I_2$

$I_2 = \frac{20{,}000}{200} = 100$ A (step up rated voltage)

$\Rightarrow$ KVA = $2200 \times 100 = \underline{220\text{ KVA}}$

3) $\eta = \frac{P_{(load)}}{P_{(load)} + P_{copper} + P_{iron}} = \frac{220 \times 10^3 \times 0.8}{220 \times 10^3 \times 0.8 + 300 + 120} \times 100 = \underline{99.7\%}$

4) The power transferred directly from source: conductive
power transferred directly to ~~finding~~ winding: ~~con~~ inductive

$P^{KVA}_{conduct} = 2000 \times I_1 = 2000 \times 10 = 20$ KVA (common)

$P^{KVA}_{induct} = 200 \times I_2 = 200 \times 100 = 20$ KVA (not common)

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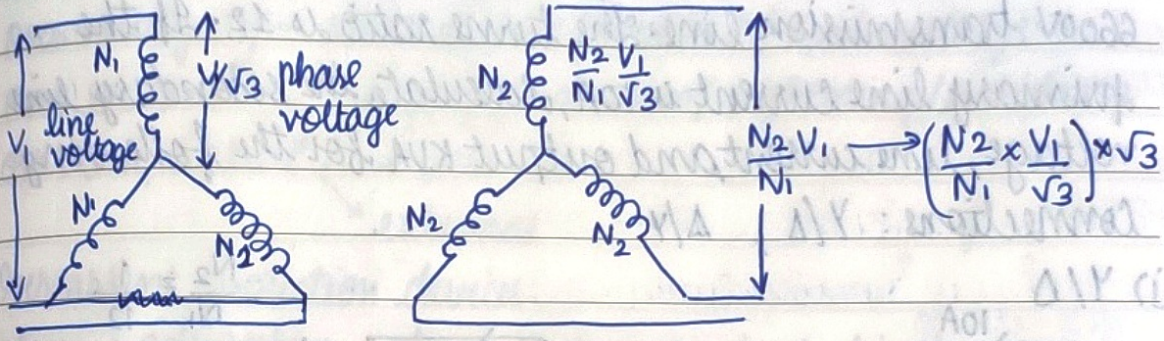
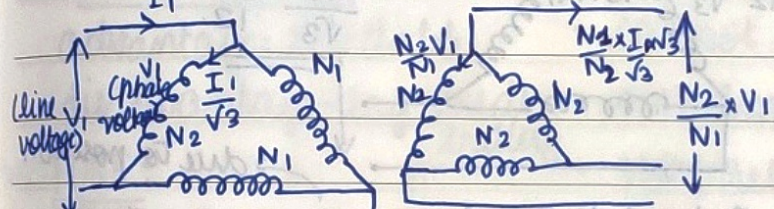
## Three phase transformer :

Y - Y ( Star - star)



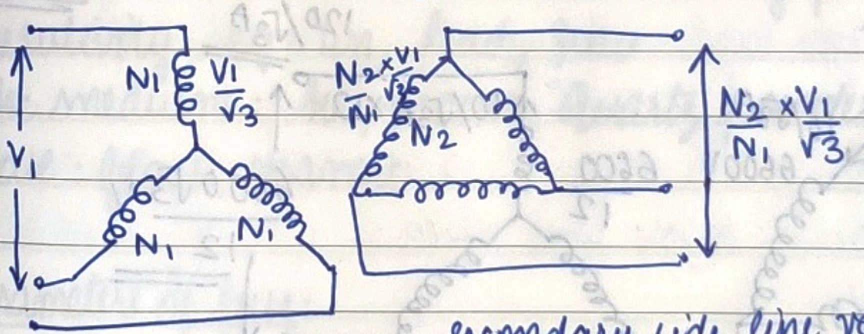

$\Delta - \Delta$ ( Delta-Delta)




* Star connection - voltage rating is reduced

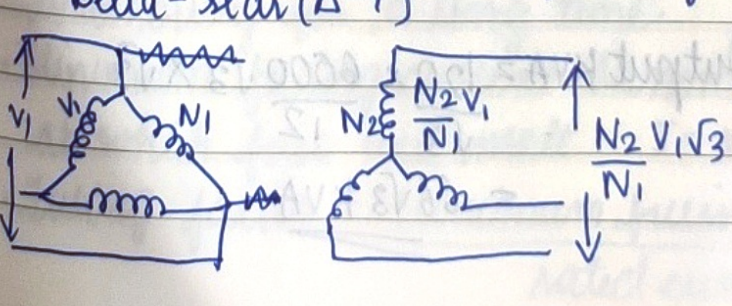
* Delta connection - current rating is reduced.

Star-Delta (Y-Δ)




secondary side line voltage lags primary side line voltage by 30°

Delta-Star (Δ-Y)

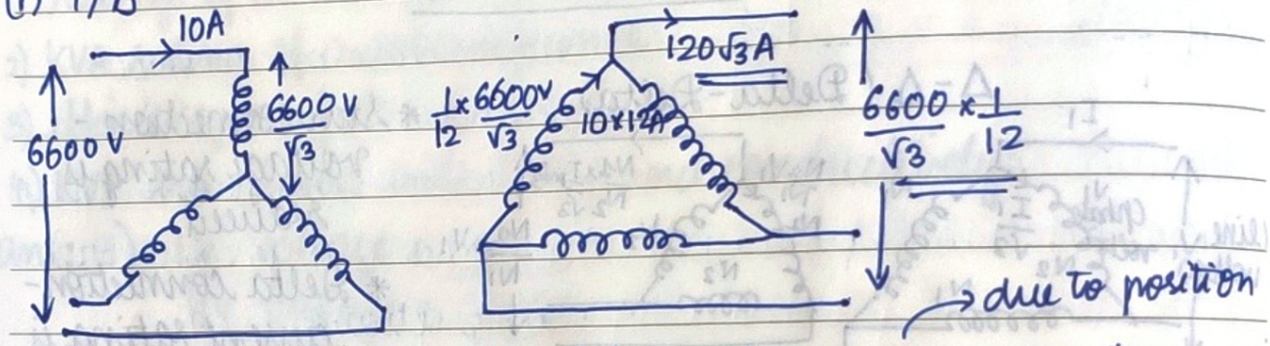



secondary side line voltage leads primary side line voltage by 30°

Q: A 3-phase transformer bank consisting of three 1-phase transformers is used to stepdown the voltage of a 3-phase 6600V transmission line. The turns ratio is 12. If the primary line current is 10A, calculate the secondary line voltage, line current and output KVA for the following connections: Y/Δ, Δ/Y

$$\frac{N_2}{N_1} = \frac{1}{12}$$

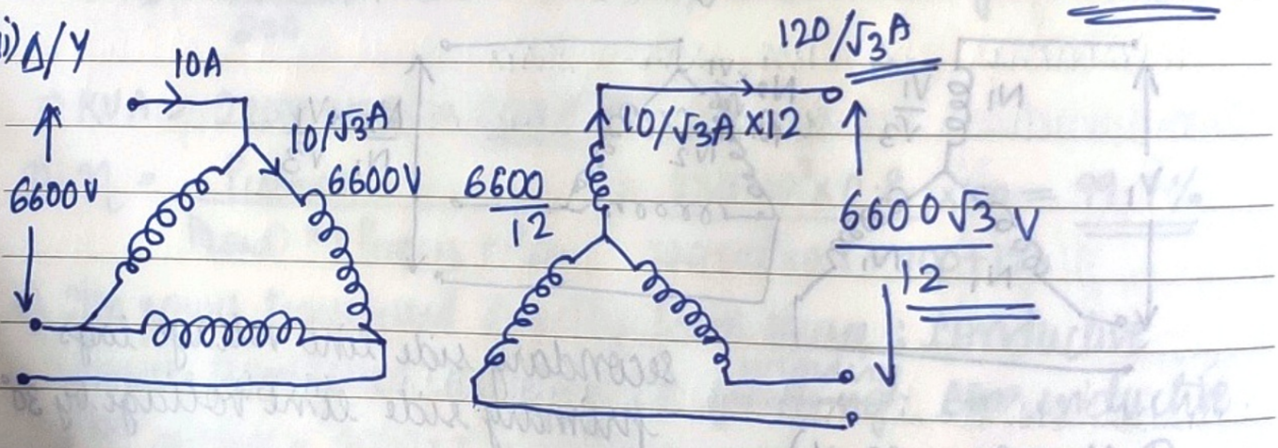
(i) Y/Δ




Output KVA $= \frac{6600}{\sqrt{3}} \times \frac{\sqrt{3}}{12} \times 120\sqrt{3} = \cancel{3}66\,KVA$

$= 66\sqrt{3}\,KVA$

(ii) Δ/Y




Output KVA $= \frac{120}{\sqrt{3}} \times \frac{6600\sqrt{3}}{12} \times \sqrt{3}$

$= 66\sqrt{3}\,KVA$

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Circuit protection devices

Circuit Abnormal conditions:

- over voltage → short circuit / overload / undervoltage
- over current → supply fluctuation / lightning / switching
- over temperature → internal
  → external

Overcurrent protection devices:

- fuse : advantage - cheap, automatic, fast, disadvantage:
  - threshold current is not precise
  - cannot be synchronized (non controlable)
  - not smart (no indication)
  - non resettable
- circuit breaker :
  advantage: resettable
  disadvantage: lag, needs battery

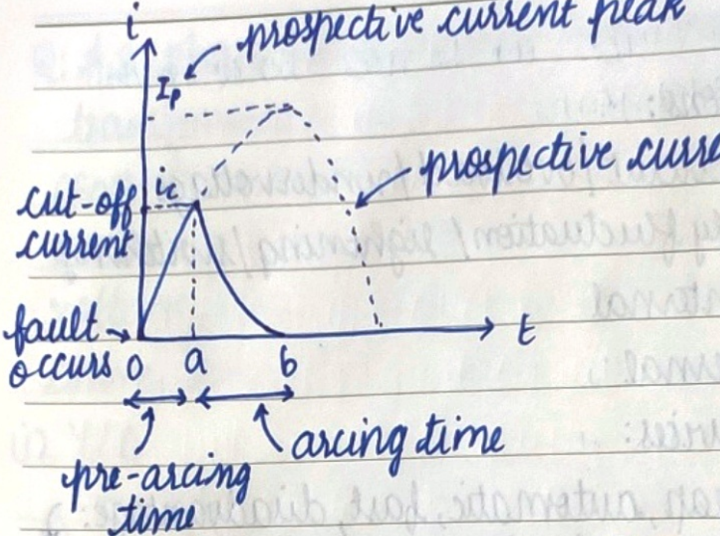
FUSE :

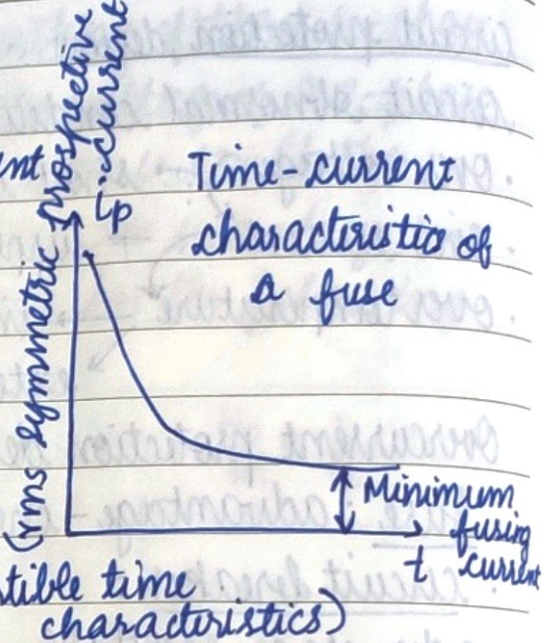
Components of fuse:

- fuse wire : material with low melting point & high resistivity . eg : Tin, lead, zinc
- Arc medium : Air, vaccum, Quartz powder
- Case : glass, ceramic

Parameters of fuse:

- Rated current - current it can carry (maximum) continously for a long time.
- Minimum fusing current: Minimum rms value current at which fuse will melt. $I = Kd^{1.5}$
- Fusing factor = $\dfrac{\text{Minimum fusing current}}{\text{rated current}}$




Types of fuse:

- Rewirable (open or semi enclosed)
- Enclosed or cartridge fuse - possible to explode at high current
- high rupturing capacity fuse - industrial application or distribution network for high current
- Expulsion type high voltage fuse - poles for distribution network.

Fuse rating

- current rating - before fuse melts
- $I^2t$ rating
- voltage rating - after fuse melts, depends on arc medium & case
- Temperature
- voltage drop

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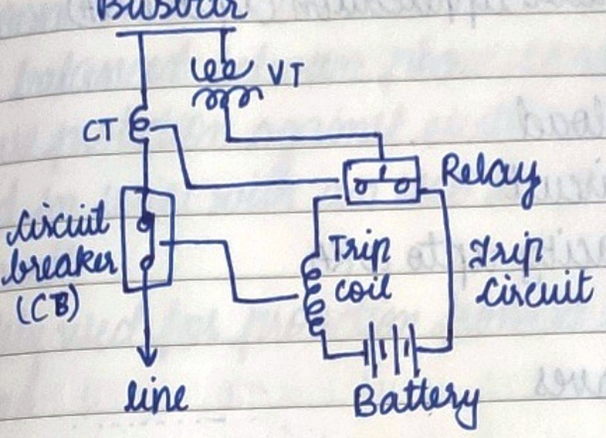
Circuit breaker : Switch which can be used to open or close circuit in power network




Events during fault clearing

1) fault initiation
2) Trip coil energization
3) Opening of contacts
4) Arc extinction

} circuit breaker is slower than fuse

Parameters of circuit breaker:

- Rated voltage ($U_n$)
- Rated current ($I_n$)
- Short circuit trip current setting ($I_m$) → over load → Threshold → short circuit
- Short circuit breaking current capacity ($I_{cu}/I_{cn}$)

## Types of Circuit Breaker in Low voltage network:

- Miniature Circuit breaker (MCB)
  - * for lower current/domestic application (upto 125A)
  - * Reset very fast
  - * Bimetallic strip – over load
  - * Magnetic coil – short circuit
  - * Interrupt current capacity upto 10KA
  - * Non adjustable
  - * Characterized by i-t curves
  - * MCB trip curves:
    - → Z : Trips at 2 to 3 times rated current Suitable for highly sensitive applications eg: semiconductor devices
    - → B : Trips 3 to 5 times rated current
    - → C : Trips at 5 to 10 times rated current Suitable for medium inrush current eg: motors, transformers
    - → K : Trips at 10 to 14 times rated current suitable for loads with high inrush current
    - → D : Trips at 10 to 20 times rated current - Suitable for high starting currents.

- Molded case Circuit breaker (MCCB)
  - * for current higher than MCB (upto 1600A)
  - * Adjustable Over load setting
  - * Interrupt capacity upto 85KA
  - * Industrial application

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• Residual current circuit Breaker (RCCB)
* Detects and trips against electrical leakage currents
* Disconnects the circuit whenever there is a current which is not balanced between phase conductor
* Ensure protection against electric shock caused by direct contacts
* used in series with an MCB to provide overcurrent protection as well.
* Widely used for protection from a leakage current of 30, 100 & 300mA.

• Earth Leakage Circuit Breaker (ELCB) → replaced by RCCB&SFU
* Voltage operated earth leakage device
* Measure the voltage on the earth conductor; if this voltage is not zero this indicates a current leakage to earth.

Switch fuse unit:
• Switch plus fuse enclosed in a metal unit
• Can mange only one load at a time
• has ON/OFF option
• overload protection
• available as 2 pole, 3 pole, 4 pole

Wire & cables:

- wire is one electrical conductor while cable is multiple conductors or a group of wires encased in sheathing.
- current rating → thickness, insulation, temperature, envirionment
- Voltage rating
- temperature rating
- resistance per unit length
- inductance & capacitance per unit length (at HF)
- Wires → single strand, multi strand, enamel wire

# Electrical Machines

Electric machines
- AC Electrical machines
  - Induction machines
  - Synchronous machines
- DC Electrical Machines
  - Series Motor
  - Shunt DC Motor
  - separately excited

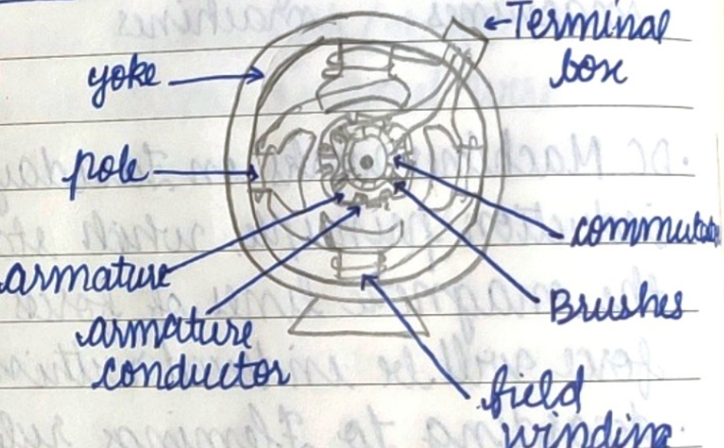
• DC Machines works on Faraday's law of electromagnetic induction principle, which states that a conductor cut the magnetic lines of forces then an electromagnetic force will be induced within the conductors.

• According to Flemings rule, the direction of induced current changes whenever the direction of motion of the conductor changes. Let's consider an armature rotating clockwise and a conductor at the left is moving upward. When the armature ~~rotating~~ completes a half rotation, the direction of motion of that particular conductor will be reversed to downward. Hence, the direction of motion of that current in every armature conductor will be alternating. If you look you will know how the direction of the induced current is alternating in an armature conductor.

- But with a split ring commutator, connections of the armature conductors also gets reversed when the current reversal occurs. And therefore, we get unidirectional current at the terminals.

<u>Construction of DC Machines</u>

Basically, there is no constructional difference between a DC motor and a DC generator. The same DC machine can be run as a generator or motor.

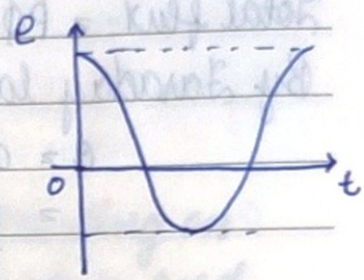



DC machines: field winding & armature winding
The rotor has been classified into two parts: lap winding
↳ armature winding & wave winding

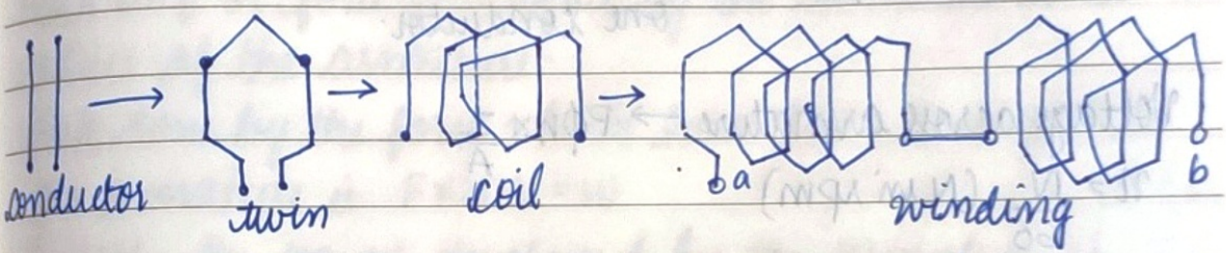
- Current rating of the machine depends on the number of parallel paths

Armature can be connected as wave winding, in this case.

Armature winding - torque
field winding - flux

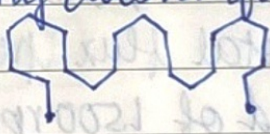



Armature winding resistance → 0-30Ω
Field winding resistance ∽ 500Ω




• lap connection - winding which folds back
The value of parallel path (a) = No. of poles

• Wave connection - winding doesn't fold back, it goes on continously



No. of parallel paths in case of wave connection (a) = 2

EMF equation of DC Machine
Flux ($\phi$)(flux per pole)
No. of conductors (Z)(No. of conductors in the machine)
A - no. of parallel paths
Speed of armature (n is rps)
No. of conductors in each parallel path = Z/A

No. of poles - P

Total flux = $P\phi$

By Faraday law of EMI

n revolution in 1 sec

1 revolution 1/n sec

$$e = d\phi/dt$$

$$= \frac{P\phi}{1/n} = P\phi n$$ voltage induced in one conductor.

Voltage across armature → $P\phi n \times \frac{Z}{A}$

$n = \frac{N}{60}$ (N in rpm)

$$\boxed{E_g/E_b = \frac{NP\phi Z}{60A}}$$

Q: A 4 pole DC machine has 60 slots and 8 conductors per slots. The total flux per pole is 20 mWb. For a relative speed of 1500 rpm between field flux and armature winding, calculate the generated armature voltage if the armature winding is connected as a) lap connected b) wave connected.

$P = 4$, $N = 1500$ rpm, $\phi = 20$ mWb, $Z = 60 \times 8 = 480$

$$E = \frac{NP\phi Z}{60A}$$

(a) lap connected : $A = P$

$$\Rightarrow E = \frac{NP\phi Z}{60P} = \frac{150\cancel{0} \times 2\cancel{0} \times 10^{-3} \times \cancel{480}\,8}{\cancel{60}} = \underline{240V}$$

(b) Wave connected, A = 2

$\Rightarrow E = \frac{N\phi Z}{60A} = 2 \times 240 = \underline{\underline{480\,V}}$

## Determination of Torque of a machine

Torque produced by the armature is depending on back emf or force developed by the armature and radius of the armature.

Work done by the force which have been developed in the armature is $F \times 2\pi r = W$

Finally the power developed by the armature is

$P = \frac{\text{Workdone}}{\text{time}} = W/t = F \cdot 2\pi r / 1/n = nF \times 2\pi r$

time $\longrightarrow$ time taken by armature to complete one revolution.

$T = F \times r$ $\qquad$ $P = T 2\pi n = T \times 2\pi N/60$

$\boxed{T = P/2\pi n}$ $\qquad$ $\boxed{P = \omega T}$

Q: A 6 pole DC machine has 300 conductors and each conductor is capable of carrying 80A without excessive temperature rise. The total flux per pole is 15mWb and the machine is driven at speed of 1800 rpm. Calculate the total current, emf, power developed in the armature and electromagnetic torque, if the armature winding is a) wave connected b) lap connected.

(b) Wave connected, A = 2

$\Rightarrow E = \frac{N\phi Z}{60A} = 2 \times 240 = \underline{480\,V}$

## Determination of Torque of a machine

Torque produced by the armature is depending on back emf or force developed by the armature and radius of the armature.

Work done by the force which have been developed in the armature is $F \times 2\pi r = W$

Finally the power developed by the armature is

$\frac{\text{Workdone}}{\text{time}}$ $P = W/t = F \cdot 2\pi r / 1/n = nF \times 2\pi r$

time ⟶ time taken by armature to complete one revolution.

$T = F \times r$ $P = T\,2\pi n = T \times 2\pi N/60$

$\boxed{T = P/2\pi n}$ $\boxed{P = \omega T}$

Q: A 6 pole DC machine has 300 conductors and each conductor is capable of carrying 80 A without excessive temperature rise. The total flux per pole is 15 mWb and the machine is driven at speed of 1800 rpm. Calculate the total current, emf, power developed in the armature and electromagnetic torque, if the armature winding is a) wave connected b) lap connected.

a) Wave connection:, $A=2$

$I_a$, Total current $= 2\times 80 = \underline{160A}$

$E_g = \dfrac{NP\phi Z}{60A} = \dfrac{\cancel{1800}\times \cancel{6}\times 15\times 10^{-3}\times 3\cancel{00}}{\cancel{60}\times 2} = \underline{\underline{405V}}$

$P = E_g\times I_a = 405\times 160 = 64.8\,KW$

$T = \dfrac{E_g I_a}{2\pi N/60} = \underline{\underline{343.77\,Nm}}$

b) Lap connected, $A=6$

$I_a = 6\times 80 = \underline{480\,A}$

$E_g = \dfrac{NP\phi Z}{60A} = \dfrac{\cancel{1800}\times \cancel{6}\times 15\times 10^{-3}\times 3\cancel{00}}{\cancel{60}\times \cancel{6}} = \underline{\underline{135V}}$

$P = E_g\times I_a = 135\times 480 = \underline{64.8kW}$

$T = \dfrac{E_g I_a}{2\pi N/60} = \underline{\underline{343.77\,Nm}}$

<u>Types of DC machines</u>

These are categorized into two parts:

(i) Self excited DC machines

(ii) separately excited DC machines

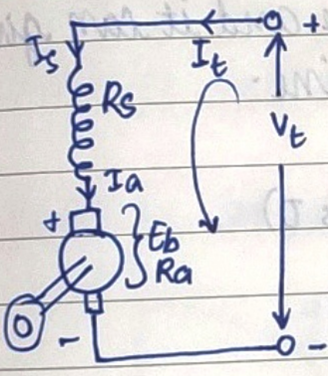
If the armature & field windings both are connected with the same source - self excited DC machines

If these windings are connected with separate source - separately excited DC machines.

## DC machines

- self excited
  - Series DC motor
  - Shunt DC motor
  - Compound DC motor
    - long shunt
    - short shunt
- separately excited

## Series DC Motor




$R_a <<< R_s$

$R_a$: $1-5\Omega$ ; $R_s$: $100-500\Omega$

$V_t - I_s R_s - I_a R_a - E_b = 0$

$\Rightarrow \boxed{E_b = V_t - I_s R_s - I_a R_a}$

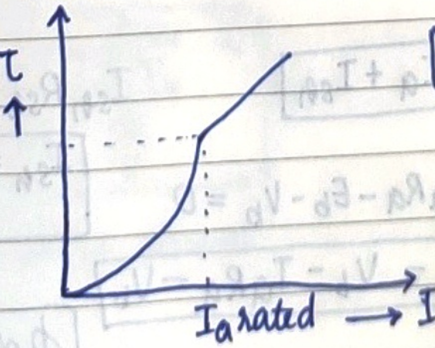
$\boxed{I_t = I_s = I_a}$

$\boxed{E_b = V_t - I_t(R_s + R_a) - V_b}$

$V_b$ is voltage drop on brush

## Characteristics of DC Series motor:

1) Torque Vs Armature current characteristics ($\tau$ vs $I_a$)




$\boxed{\tau \propto I_a^2}$

until saturation, after $\phi = K$

$\varepsilon_b = \frac{NP\phi Z}{60A}$, $P_d = \omega\tau$

$\varepsilon_b I_a = \frac{2\pi N}{60} \times \tau$

$\Rightarrow \frac{NP\phi Z}{60A} I_a = \frac{2\pi N}{60}\tau$

$\Rightarrow \tau \propto \phi I_a$

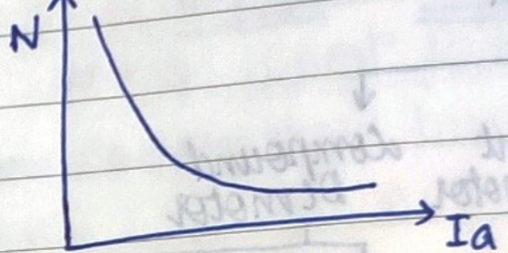
As flux is proportional to $I_a$ because, both armature & field windings are connected in series

2) Speed Vs Armature current (N Vs Ia)

$E_b = \frac{NP\phi Z}{60A}$, In case of series DC motor, $\phi \propto I_a$

$\Rightarrow E_b = \frac{NPI_aZ}{60A}$

$\boxed{N \propto E_b/I_a}$



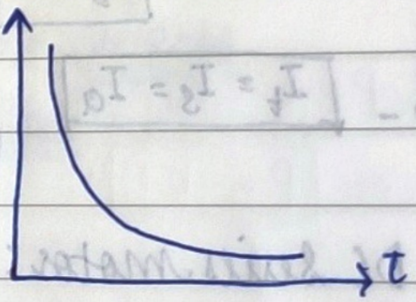

under light load or no load condition, the magnitude of armature current is very small which results in high speed operation of the machine and it can give mechanical damage on the machine.

3) Speed Vs Torque characteristics (N Vs $\tau$)

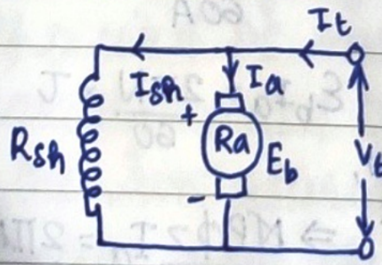
$P = \omega \tau$

$\frac{2\pi N \tau}{60} = P$

$\Rightarrow \boxed{N \propto P/\tau}$




DC shunt motor




$\boxed{I_t = I_a + I_{sh}}$

$I_{sh}R_{sh} = V_t$

$\boxed{I_{sh} = \frac{V_t}{R_{sh}}}$

$V_t - I_aR_a - E_b - V_b = 0$

$\Rightarrow \boxed{E_b = V_t - I_aR_a - V_b}$

$\boxed{\phi \propto I_{sh}}$

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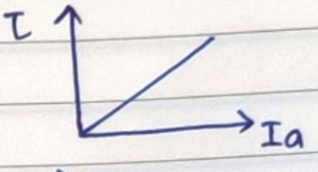
Characteristics of DC shunt motor:

1) Torque Vs Armature current ($T$ Vs $I_a$)

In case of DC shunt motor, the magnitude of field current is very small, so magnitude of flux produced will be constant.

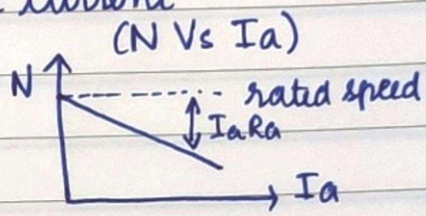



2) Speed Vs Armature current

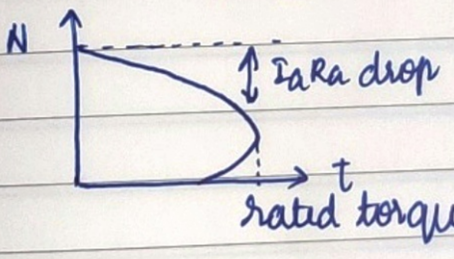
($N$ Vs $I_a$)

$N \propto \frac{E_b}{\Phi}$ & K by $I_{sh}$

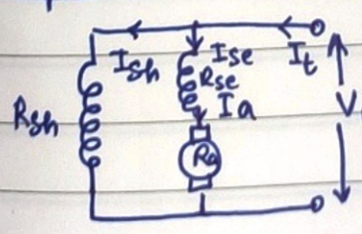
$\Rightarrow \boxed{N \propto V_t - I_a R_a}$



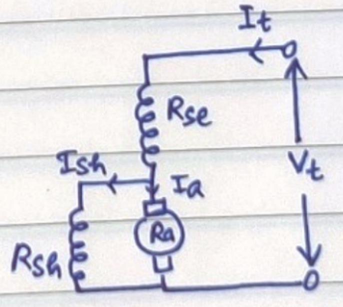

3) Speed Vs Torque ($N$ Vs $T$)




Compound DC motor:




long shunt




short shunt