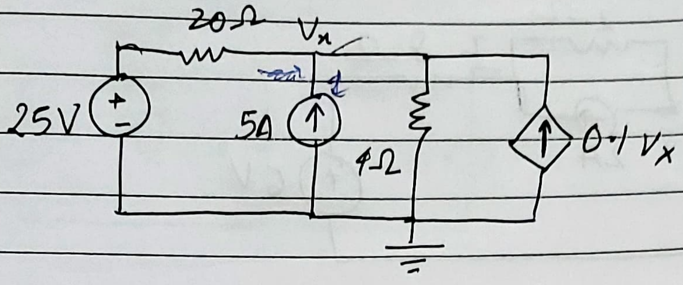
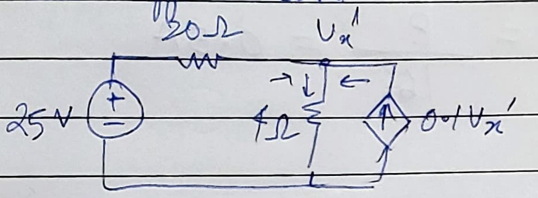
Using superposition theorem, find V_x



Turn off current source



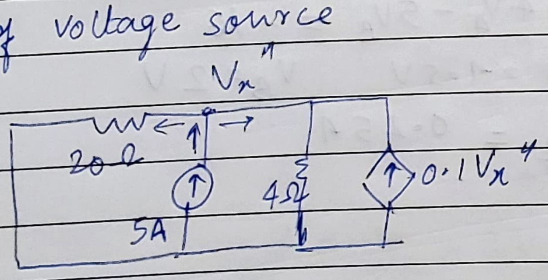
KCL

$$\frac{V_x' - 25}{20} + \frac{-V_x'}{10} + \frac{V_x'}{4} = 0$$

$$25 = 4V_x'$$

$$V_x' = 6.25\ V$$

Turn of voltage source



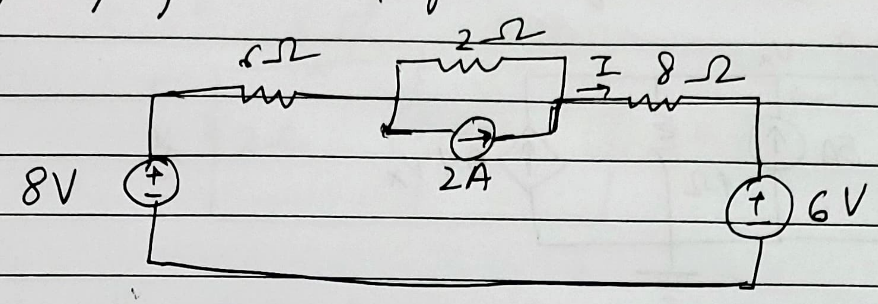
KCL.

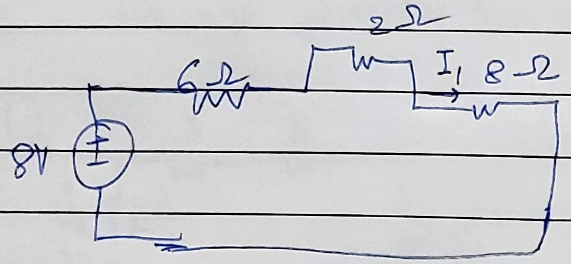
$$\frac{V_x \cancel{20}}{20} + \frac{V_x}{4} = 5 + 0.1 V_x''$$

$$V_x'' = 25$$

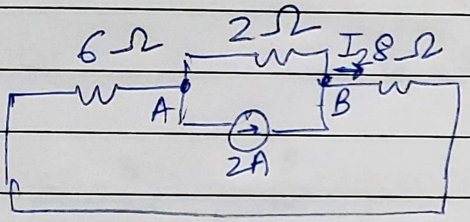
$$V_x = V_x' + V_x'' = 31.25\ V$$

Using superposition, find I





$$I_1 = \frac{8}{16} = 0.5A$$



KCL at A

$$-\frac{V_A}{6} = \frac{V_A - V_B}{2} + 2$$

$$-4V_A + 3V_B = 12$$

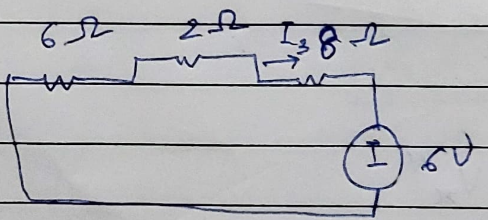
KCL at B

$$\frac{V_A - V_B}{2} + 2 = \frac{V_B}{8}$$

$$4V_A - 5V_B = -16$$

$V_A = -1.5\,V$ $\quad V_B = 2\,V$

$$I_2 = 0.25\,A$$

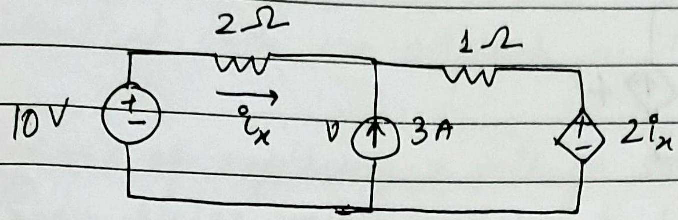


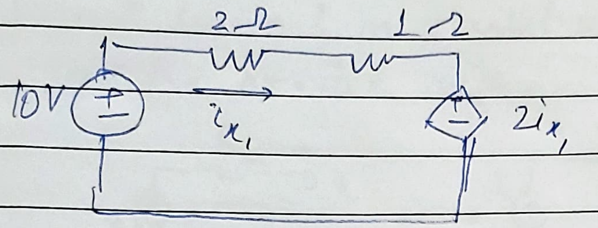
$$I_3 = -\frac{6}{16} = -0.375$$

$$I = I_1 + I_2 + I_3$$

$$= 0.375\,A$$

Using superposition, find i_x

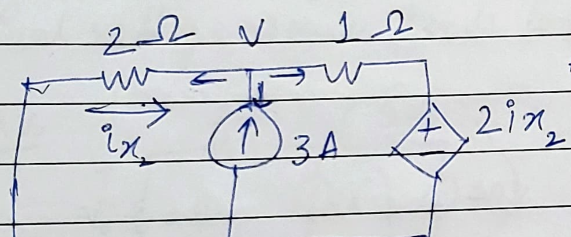




Applying KVL

$$10 - 2i_{x_1} - i_{x_1} - 2i_{x_1} = 0$$

$$i_{x_1} = 2A$$



Applying KCL

$$3 = \frac{V-0}{2} + \frac{V-2i_{x_2}}{1}$$

$$i_{x_2} = \frac{0-V}{2} = \frac{-V}{2}$$

$$2i_{x_2} = -V$$

$$3 = \frac{V}{2} + 2V$$

$$V = 6/5$$

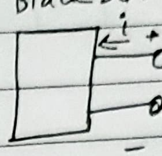
$$i_{x_2} = -0.6A$$

$$i_x = i_{x_1} + i_{x_2} = 2 - 0.6 = 1.4A$$

Source Tranformation

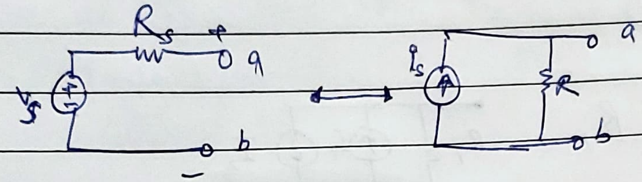
An equivalent circuit is one whose V-I characteristics are identical with the original circuit.

Black box



Process of replacing a voltage source V_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.

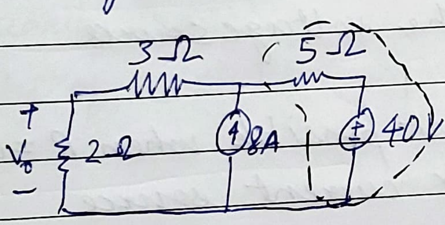
Current entering +ve terminal
passive sign convention
vi = power absorbed.

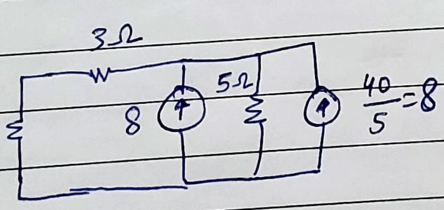


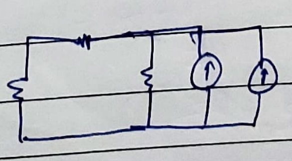
$V_s = i_s R$ or $i_s = \frac{V_s}{R}$

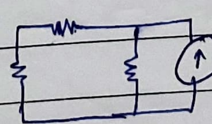
As R_s tends to zero
Real voltage source tends to ideal voltage source

Find V_o.

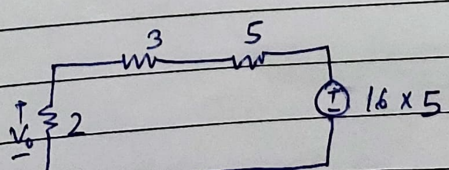






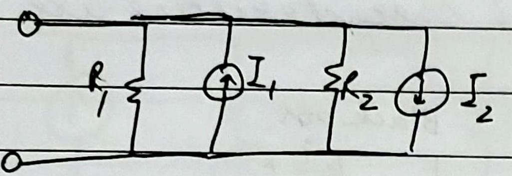


$(I_1 + I_2) = 8+8 = 16$



Voltage divider $= V_o = \frac{2}{2+3+5} \times 86$

Example.



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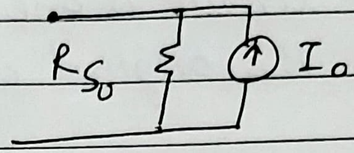
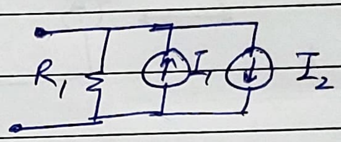


$$I_o = I_1 - I_2$$

$$R_{S0} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\lim_{R_2 \to 0} \frac{R_1 R_2}{R_1 + R_2} = 0$$

$$\lim_{R_2 \to \infty} \frac{R_1 R_2}{R_1 + R_2} = R_1$$



- The arrow of the current source is directed toward the positive terminal of the voltage source

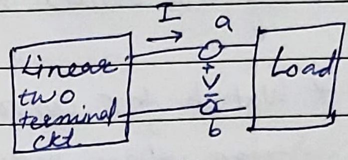
- The source transformation is not possible when $R = 0$ for voltage source and $R = \infty$ for current source.

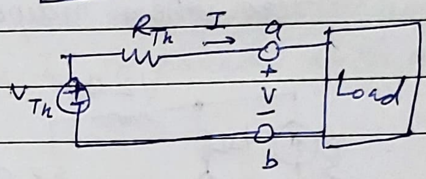
R_s

Thevenin's Theorem

- A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}'

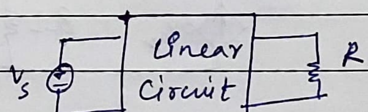
  - V_{Th} : open-circuit voltage at the terminals.
  - R_{Th} : input or equivalent resistance at the terminals when the independent sources are turned off





Linearity Property

- Property of an element describing a linear relationship between cause and effect



- Homogeneity (scaling) property.

  $v = f(i) \longrightarrow k^*v = f(k^*i)$

  $v - iR \longrightarrow kv = kiR$

- Superposition (additivity) property

  $*\ v_1 = f(i_1)$ and $v_2 = f(i_2) \longrightarrow v = f(i_1 + i_2) = v_1 + v_2$

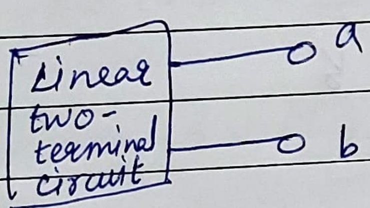
  $v_1 = i_1 R$ and $v_2 = i_2 R \longrightarrow v = (i_1 + i_2) R = v_1 + v_2$

- A circuit element (or device) is said to possess the property of linearity when it follows the principle of Homogeneity (scaling) and superposition (additivity)
- A linear circuit is one whose output is linearly related (or directly proportional) to its input

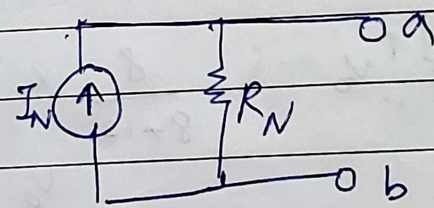
Norton's Theorem

- A linear two-terminal circuit can be replaced by an equivalent circuit of a current source I_N in parallel with a resistor R_N.

- I_N : short circuit current through the terminals
- R_N : input or equivalent resistance at the terminals when the independent sources are turned off



(a)

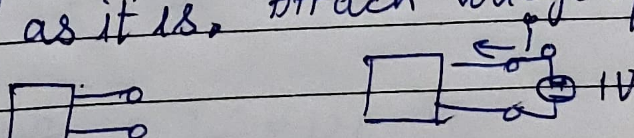


(b)

The thevenin's and Norton equivalent are related by source transformation

- finding R_N & R_{Th} with dependant sources

Turn off independent source and leave dependant source as it is. Attach voltage of 1V and find i_o in that branch



$$R_N / R_{Th} = \frac{1}{i_o}$$

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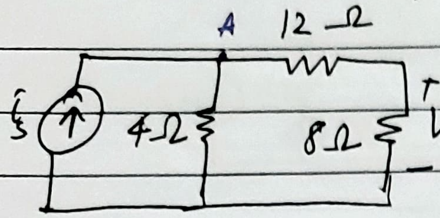
For this circuit, find V_o when $i_s = 30$ and $i_s = 45A$



KCL at A

$$I_s = \frac{V_A}{4} + \frac{V_A}{20}$$

$$V_A = \frac{20 \times I_s}{6}$$

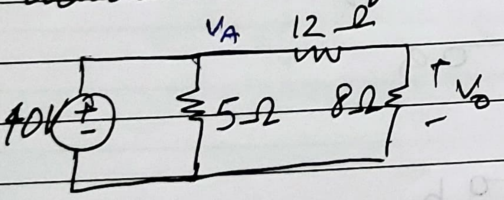
$$V_o = \frac{8}{8+12} \times V_A$$

$$= \frac{8}{20} \times \frac{20\, i_s}{6}$$

$$V_o = \frac{4}{3} \times I_s$$

$i_s = 30 \quad V_o = 40 \qquad i_s = 45, \; V_o = 60$

Assume that $V_o = 1V$ and use linearity to calculate the actual value of V_o in the circuit



$$\frac{8}{8+12} \times V_A = V_o$$

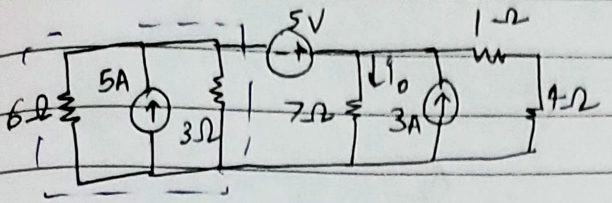
$$V_o = 1$$

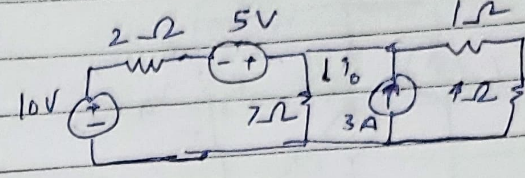
$$V_A = 20/8 = 2.5\ V$$

But in reality $V_A = 40$

$$V_o = \frac{1 \times 40}{2.5} = 16\ V$$

Find i_o in the circuit shown using source transformation

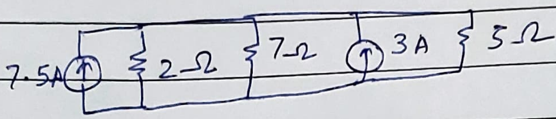




$6 \| 3 \qquad \frac{6\times 3}{6+3} = 2\,\Omega$

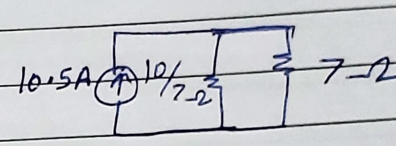
$V = IR = 10\ V$

$I = \frac{V}{R} = 15/2 = 7.5$

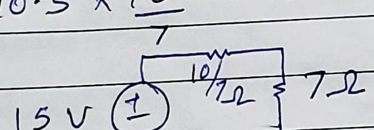


$I = 7.5 + 3$

$R = \frac{2\times 5}{2+7} = \frac{10}{7}\,\Omega$

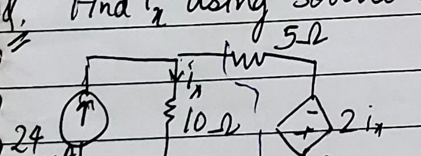


$V = 10.5 \times \frac{10}{7} = 15\ V$

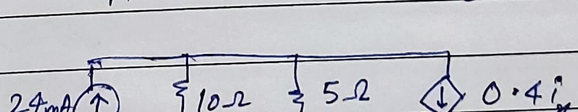


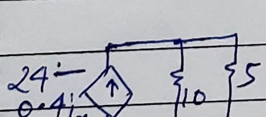
$i_o = \frac{15}{\frac{10}{7}+7} = 1.78\ A$

Q. Find i_x using source transformation



$R = 5\,\Omega$, $i = \frac{2i_x}{5} = 0.4\,i_x$

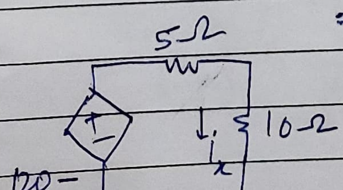




$V = i \times R$

$= (24 - 0.4 i_x)5$

$= 120 - 2 i_x$



$i_x = \frac{120 - 2i_x}{15}$

$i_x = \frac{120\,mA}{17}$

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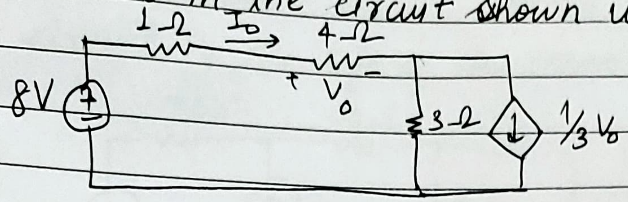
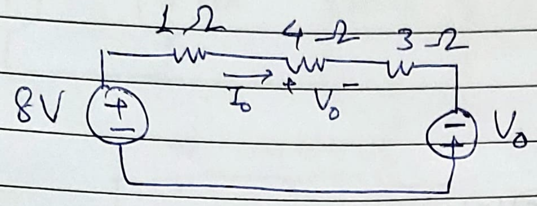
Find I_o in the circuit shown using source transformation





$V = IR$

$= \frac{1}{3} V_o \times 3 = V_o$

$8 + V_o = I_o \times 8$

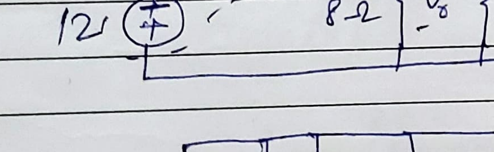
$V_o = I_o \times 4$

$8 + 4I_o = 8I_o$

$8 = 4I_o$

$I_o = 2\,A$

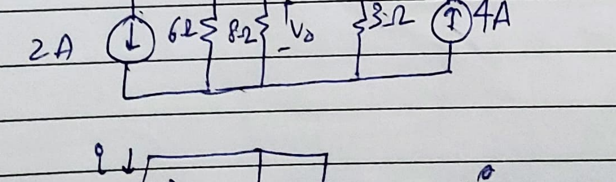
Find V_o using source transformation



$V = 4 \times 3$

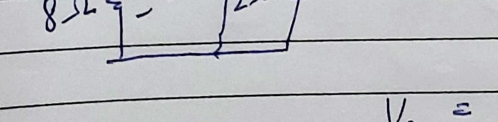
$= 12$

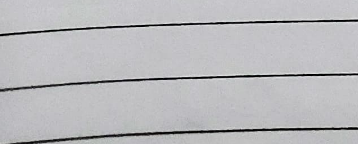
$i = \frac{12}{3} = 4$



$12 = i \times 6$

$i = 2$

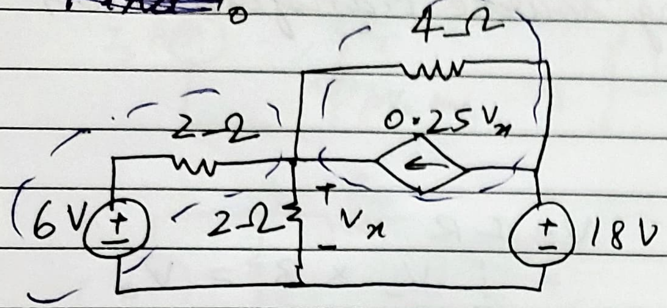


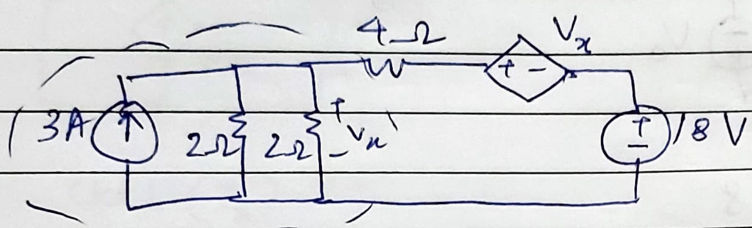


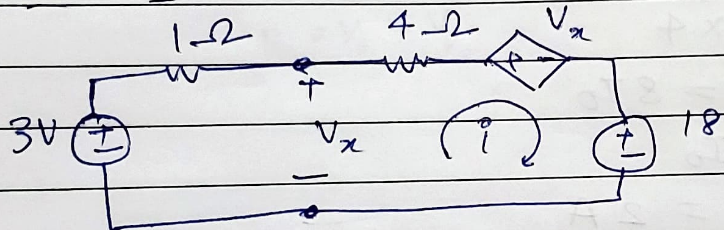
$i = \frac{2}{10} \times 2 = 0.4\,A$

$V_o = 3.2\,V$

~~Find~~ i_o







$$3 - 5i - V_x - 18 = 0 \quad \text{— ①}$$

$$-3 + i + V_x = 0$$

$$V_x = 3 - i \quad \text{— ②}$$

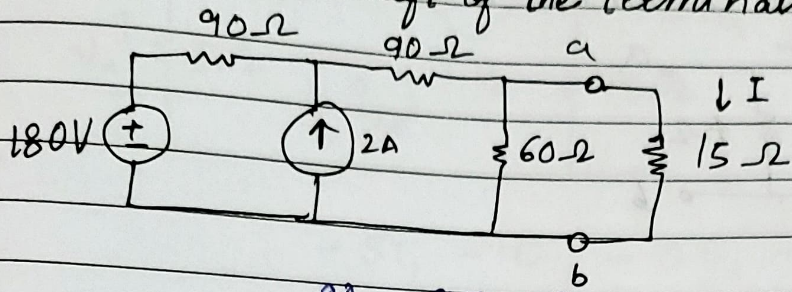
① & ②

$$15 + 5i + 3 - i = 0$$

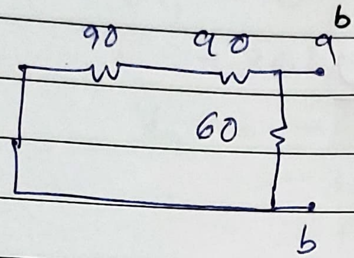
$$i = -4.5A$$

$$V_x = 3 - i = 7.5\,V$$

For the circuit shown, obtain the Thevenin equivalent of the circuit to the left of the terminals a and b. Then find I.



R_{Th}

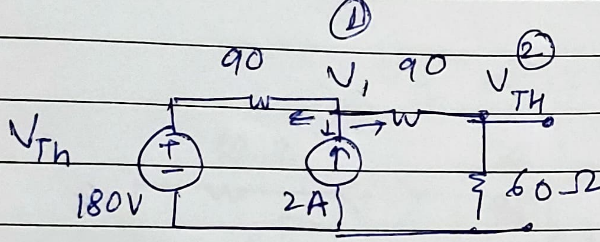


$$= (90+90) \parallel 60$$

$$= 180 \parallel 60$$

$$R_{Th} = \frac{180 \times 60}{240} = 45\Omega$$

V_{Th}



KCL at Node 1

$$\frac{V_1 - 180}{90} - 2 + \frac{V_1 - V_{Th}}{90} = 0$$

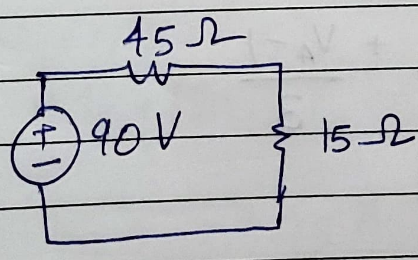
$$2V_1 - V_{Th} - 180 = 0 \quad -(1)$$

KCL at Node (2)

$$\frac{V_{Th} - V_1}{90} + \frac{V_{Th} - 0}{60} = 0$$

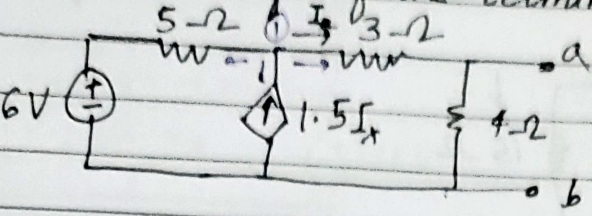
$$5V_{Th} - 2V_1 = 0 \quad -(2)$$

From (1) & (2) $\quad V_{Th} = 90V$



$$I = \frac{90}{45+15} = \frac{90}{60} = 1.5A$$

For the circuit shown, obtain the Thevenin equivalent of the circuit to the left of the terminals a and b



KCL at 1

$$\frac{V_1-6}{5} - 1.5\, I_x + \frac{V_1}{7} = 0$$

$$I_x = \frac{V_1}{7}$$

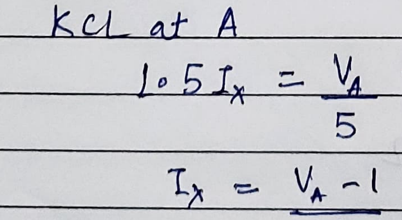
$$\frac{V_1-6}{5} - \frac{1.5\, V_1}{7} + \frac{V_1}{7}$$

$$84 = 9V_1$$

$$V_1 = 84/9$$

$$V_{Th} = \frac{\cancel{84}^{12}}{9} \times \frac{4}{\cancel{7}} = \frac{48}{9} = 5.333V$$

R_{Th}



$$R_{Th} = \frac{1}{i_o}$$

KCL at A

$$1.5\, I_x = \frac{V_A}{5} + \frac{V_A - 1}{3}$$

$$I_x = \frac{V_A - 1}{3}$$

$$1.5 \times \frac{V_A - 1}{3} = \frac{V_A}{5} + \frac{V_A - 1}{3}$$

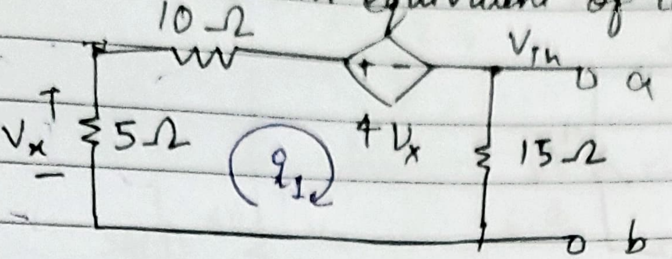
$$V_A = -5V$$

$$i_o + I_x = I_1$$

$$i_o = I_1 - I_x$$

$$= \frac{1}{4} - \frac{-5-1}{3} = 2.25A$$

Obtain Thevenin equivalent of the circuit.



$$-5i_1 - 10i_1 - 4V_x - 15i_1 = 0$$

$$-30i_1 = 4V_x$$

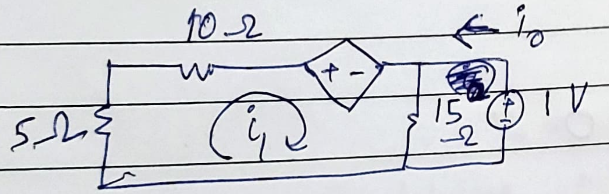
$$V_x = -i_1 \times 5$$

$$-30i_1 = -20i_1$$

$$10i_1 = 0$$

$$i_1 = 0$$

$$V_{Th} = 15i_1 = 0$$



$$R_{Th} = \frac{1}{i_0}$$

Mesh 1 $\quad V_x - 10i_1 - 4V_x - 15(i_1 + i_0) = 0$

$$3V_x + 25i_1 + 15i_0 = 0$$

$$-15i_1 + 25i_1 + 15i_0 = 0$$

$$15i_0 + 10i_1 = 0 \quad -(1)$$

Mesh 2

$$-15(i_0 + i_1) + 1 = 0$$

$$15i_0 + 15i_1 = 1 \quad -(2)$$

$$i_0 = -2/15 \text{ A}$$

$$R = \frac{1}{-2/15} = -7.2\ \Omega$$

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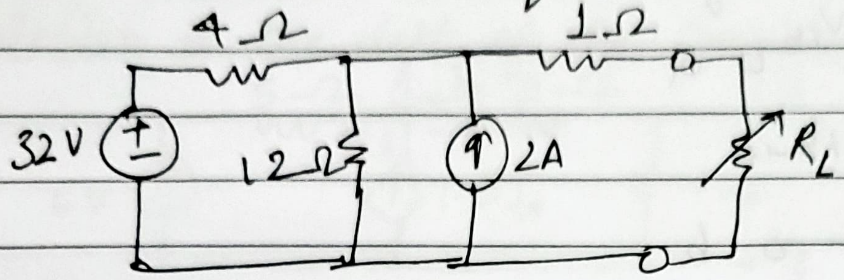
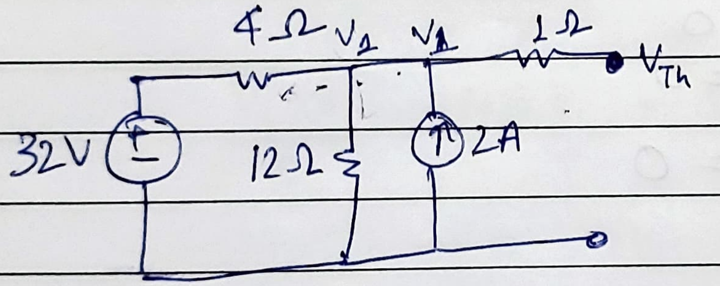
Q. Obtain Thevenin equivalent.



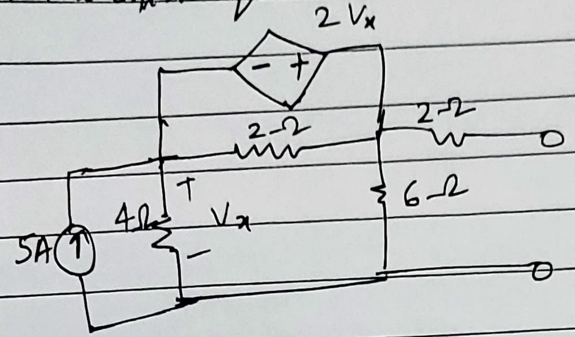
$$R_{Th} = (4 \| 12) + 1$$
$$= 3 + 1$$
$$= 4\ \Omega$$

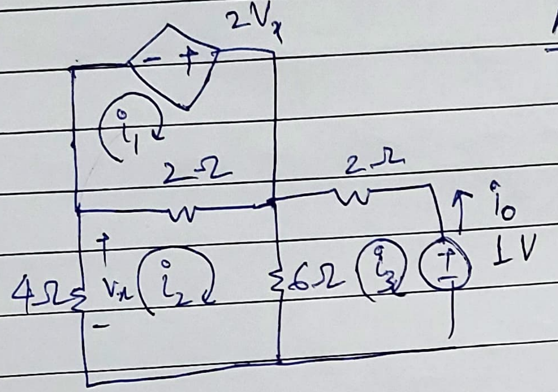


$V_{Th} = V_1$

$$\frac{V_1 - 32}{4} + \frac{V_1}{12} - 2 = 0$$
$$3V_1 - 96 + V_1 - 24 = 0$$
$$4V_1 = 120$$
$$V_1 = 30\ V$$
$$V_{Th} = 30\ V$$

Obtain Thevenin equivalent





Mesh 1

$2V_x - 2(i_1 - i_2) = 0$

$V_x = i_1 - i_2$ — (1)

Mesh 2

$-4i_2 - 2(i_2 - i_1) - 6(i_2 - i_3) = 0$ — (2)

Mesh 3

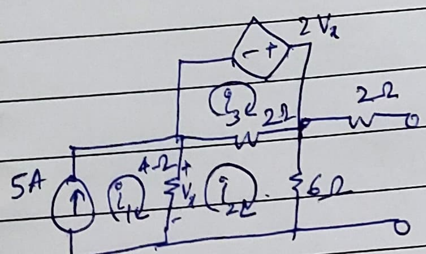
$-6(i_3 - i_2) - 2i_3 - 1 = 0$ — (3)

$V_x = -4i_2 = i_1 - i_2$ — (4)

solving 2, 3 & 4

$i_3 = -1/6\ A$ $i_o = -i_3$

$R_{Th} = \frac{1}{-i_3} = 6\ \Omega$



$i_1 = 5$

$2V_x - 2(i_3 - i_2) = 0$

$V_x = i_3 - i_2$

$-4(i_2 - i_1) - 2(i_2 - i_3) - 6i_2 = 0$

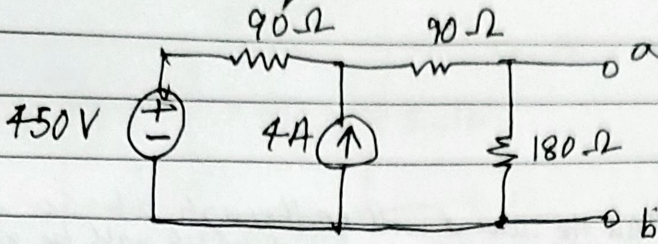
$12i_2 - 4i_1 - 2i_3 = 0$

$V_x = -4(i_2 - i_1)$

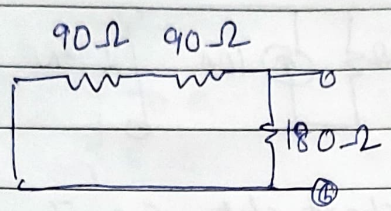
$i_2 = 10/3$

$V_{Th} = 6i_2 = 20V$

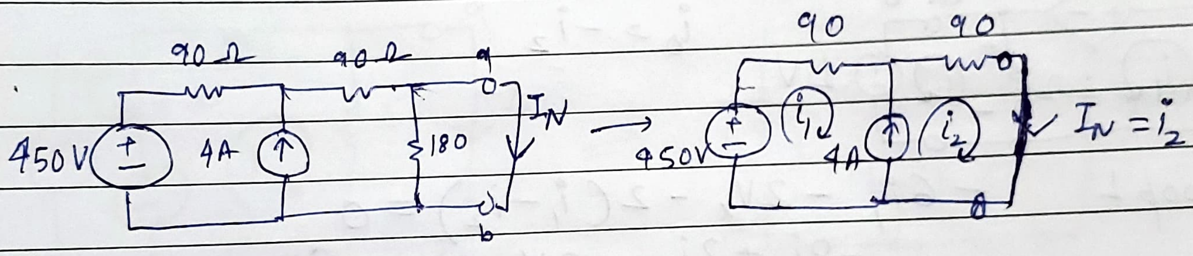
Obtain Norton equivalent



R_N



$$R_N = (90+90) \parallel 180$$
$$= 90\ \Omega$$



KVL in Supermesh.

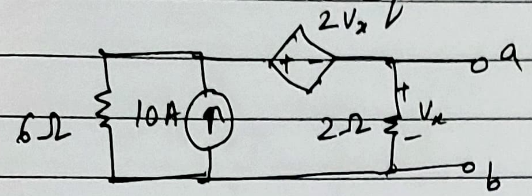
$$450 - 90i_1 - 90i_2 = 0$$
$$i_1 + i_2 = 5 \quad -(1)$$

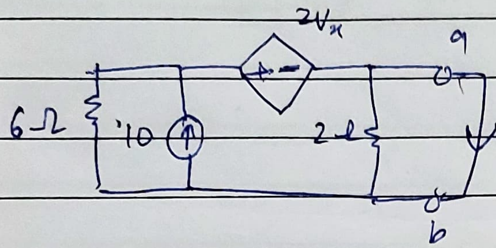
$$i_2 - i_1 = 4 \quad -(2)$$

$$2i_2 = 9$$
$$i_2 = 4.5A$$

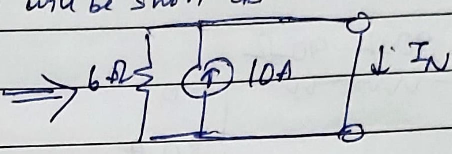
$$I_N = 4.5A$$

Obtain Norton equivalent



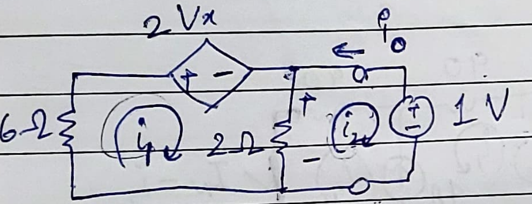


(~~All the current will go through~~ ab so 2Ω will be short ckt. and VCVS will be removed)



Similarly 6Ω will be short ckt. So I_N will be 10A

$I_N = 10A$



$i_0 = -i_2$

Loop 1 $\quad -6i_1 - 2V_x - 2(i_1 - i_2) = 0$

$-8i_1 + 2i_2 = 2V_x$ —(1)

Loop 2 $\quad -2(i_2 - i_1) = 1$ —(2)

Also $\quad -2(i_2 - i_1) = V_x$ —(3)

$V_x = 1.$

$-8i_1 + 2i_2 = 2$

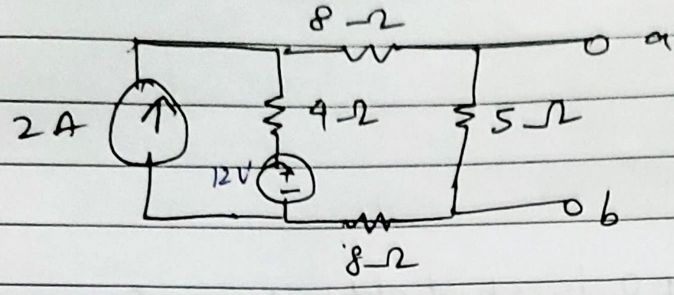
$2i_1 - 2i_2 = 1$

$i_1 = \frac{-1}{2}$ A

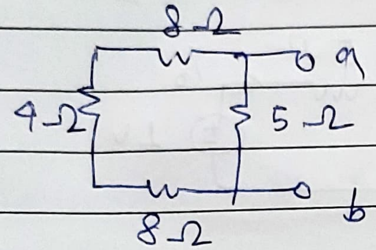
$i_2 = -1$ A

$i_0 = 1$A

Obtain Thevenin equivalent.



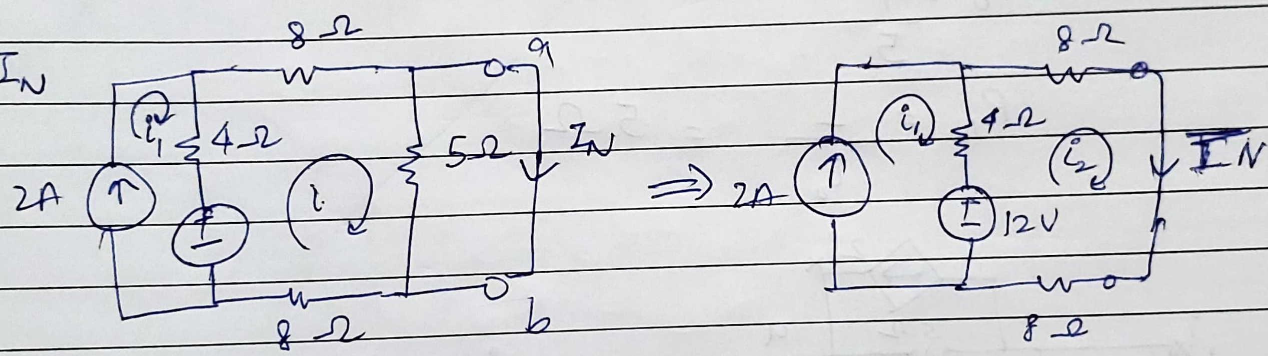
R_N



$$R_N = (8+8+4) \,||\, 5$$

$$= \frac{20 \times 5}{25} = 4\,\Omega$$

I_N



$$i_1 = 2A$$

KVL in loop 2

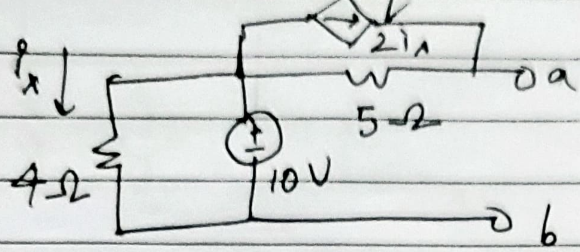
$$12 - 4(i_2 - 2) - 8i_2 - 8i_2 = 0$$

$$20 i_2 = 20$$

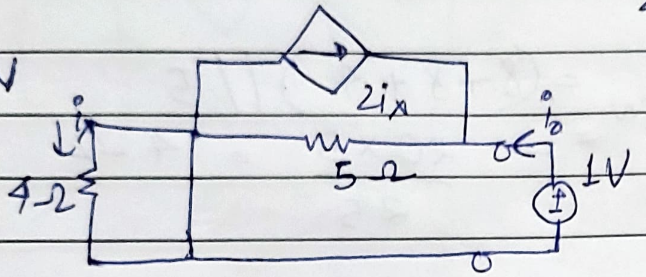
$$i_2 = 1A$$

$$I_N = 1A$$

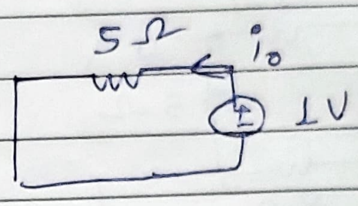
Q. Obtain Norton equivalent.



R_N



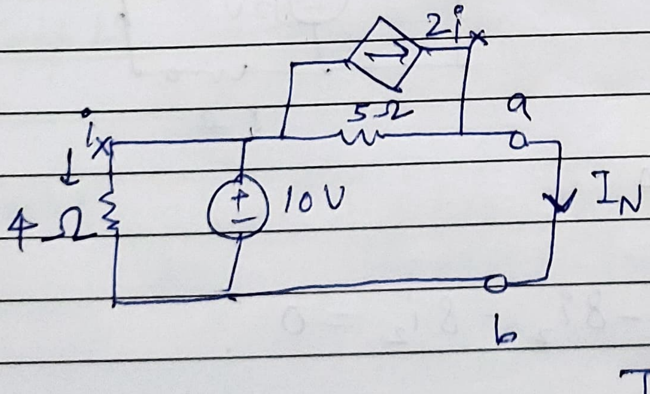
4Ω is short ckt'd. so $i_x = 0$



$$i_o = \frac{1}{5}$$

$$R_N = \frac{1}{i_o} = 5\Omega$$

I_N



$$i_x = \frac{10}{4} = 2.5A$$

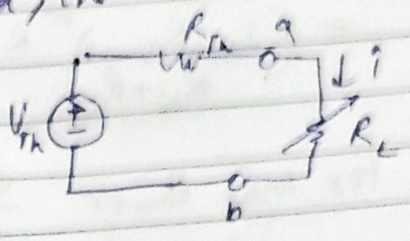
$$I_N = \frac{10}{5} + 2i_x$$

$$= \frac{10}{5} + 2 \times 2.5$$

$$= 7A$$

Maximum Power transfer

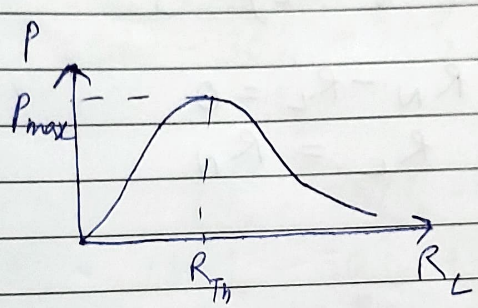
- If the entire circuit is replaced by its Thevenin equivalent except for the load, the power delivered to the load is:



$$P = i^2 R_L = \left[\frac{V_{Th}}{R_{Th} + R_L}\right]^2 R_L$$

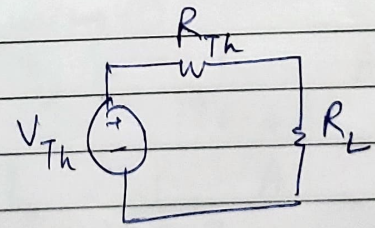
- For maximum power dissipated in R_L, P_{max} (for a given R_{Th} and V_{Th})

$$R_L = R_{Th} \implies P_{max} = \frac{V_{Th}^2}{4R_L}$$



__Proof of MPT__

Thevenin ckt.



$$P = i^2 R_L$$

$$= \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

For Max Power $\frac{dP}{dR_L} = 0$

$$\frac{dP}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right]$$

$$= V_{Th}^2 \left[\frac{R_{Th} - R_L}{(R_{Th} + R_L)^3}\right] = 0$$

$$\implies R_{Th} = R_L$$

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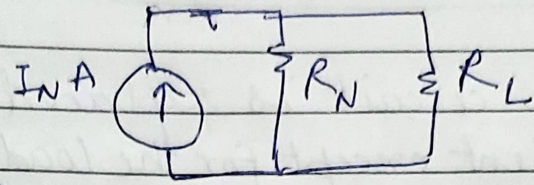
Norton Ckt.



$$P = i^2 R_L$$

$$= \left(\frac{I_N R_N}{R_N + R_L}\right)^2 R_L$$

for Max Power $\frac{dP}{dR_L} = 0$

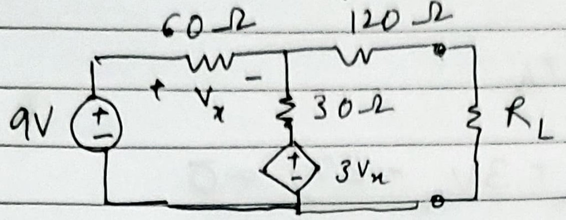
$$\frac{dP}{dR_L} = I_N^2 R_N^2 \left[\frac{(R_N+R_L)^2 - 2R_L(R_N+R_L)}{(R_N+R_L)^4}\right]$$

$$= I_N^2 R_N^2 \left[\frac{(R_N+R_L) - 2R_L}{(R_N+R_L)^3}\right] = 0$$

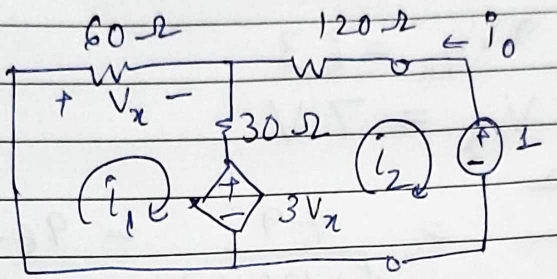
$$R_N - R_L = 0$$

$$R_L = R_N$$

Find the value of R_L that will draw maximum power from the rest of the ckt. Find max power



R_{Th}



$i_0 = -i_2$

Loop 1

$$-60i_1 - 30(i_1 - i_2) - 3V_x = 0$$

$$V_x = 60i_1$$

$$-60i_1 - 30i_1 + 30i_2 - 180i_1 = 0$$

$$i_2 = 9i_1 \quad \text{— (1)}$$

Loop 2

$$3V_x - 30(i_2 - i_1) - 120i_2 - 1 = 0$$

$$180i_1 - 30i_2 + 30i_1 - 120i_2 = 1$$

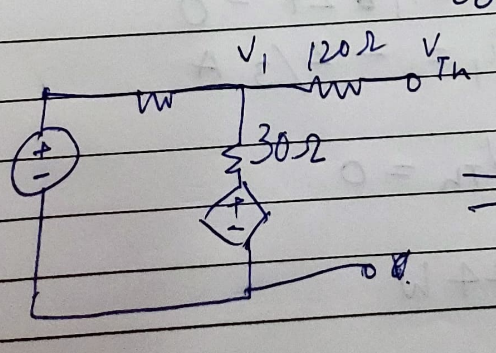
$$210i_1 - 150i_2 = 1$$

using (1)

$$i_2 = -\frac{1}{126.67} \text{ A}$$

$$i_0 = \frac{1}{126.67} \text{ A}$$

$$R_{Th} = \frac{1}{i_0} = 126.67\,\Omega$$



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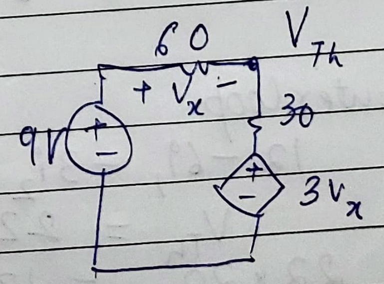


$$\frac{V_{Th}-9}{60} + \frac{V_{Th}-3V_x}{30} = 0$$

$$V_x = 9 - V_{Th}$$

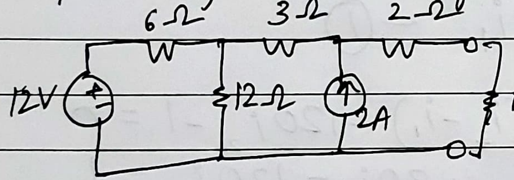
$$V_{Th} - 9 + 2(V_{Th} + 3V_{Th} - 27) = 0$$

$$9V_{Th} = 63$$

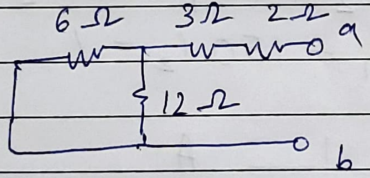
$$V_{Th} = 7V$$

$$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{49}{4\times 126.67} = 96.71\, mW$$

Q. Find the value of R_L that will draw maximum power from rest of the circuit. Find max power



R_{Th}

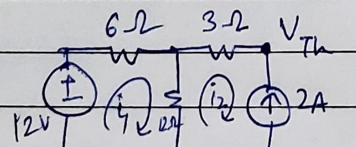


$$(6\|12) + 3 + 2 = R_{Th}$$

$$= \frac{6\times 12}{18} + 3 + 2$$

$$= 4 + 3 + 2 = 9\Omega$$

V_{Th}



$$i_2 = -2A$$

Loop 1
$$12 - 6i_1 - 12(i_1 - i_2) = 0$$

$$12 - 6i_1 - 12i_1 - 24 = 0$$

$$i_1 = -2/3\ A$$

KVL in outer loop

$$12 - 6i_1 - 3i_2 - V_{Th} = 0$$

$$V_{Th} = 22V$$

$$P_{max} = \frac{22\times 22}{4\times 9} = 13.44\, W$$