Comprehensive Problem

A telephone wire has a current of 20 A flowing through it. How long does it take for a charge of 15 C to pass through the wire?

$$i = \frac{q}{t}$$

$$20 = \frac{15}{t}$$

$$t = 0.75$$

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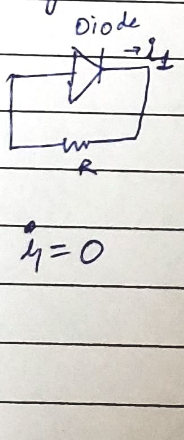
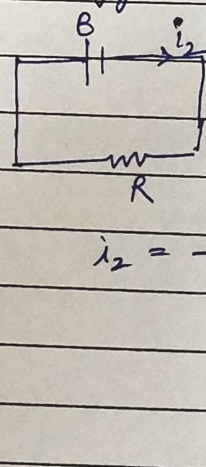
Passive Elements & Active Elements

Active ~~Passive~~ elements in a circuit are those which provides EMF
Eg. Battery, Diode

Passive elements do not generate EMF or energy. Rheostat.





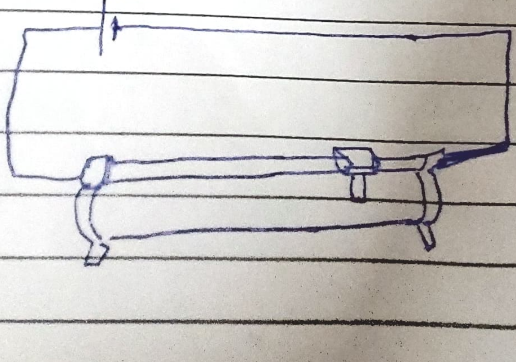
$i_1 = 0$ $\qquad$ $i_2 = -B/R$

Rheostat

specification

Max Current - 2 Amp $\qquad$ Max Resistance - 300 ohm

B volt



Circuit Analysis : Basic Definition

- Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit

- An element is the basic building block of a circuit
- An electric circuit is simply an interconnection of elements

- Active elements is capable of generating energy while a passive element is not

Passive elements
- Resistors, capacitors and inductors

Active elements
- Generators, batteries and operational amplifiers
- voltage or current sources that generally deliver power to the circuit connected to them

Ideal Independent Source
An active element that provides a specified voltage or current that a completely independent of other circuit elements

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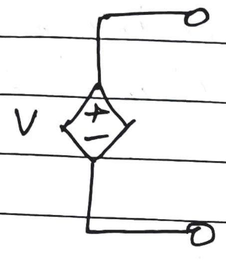
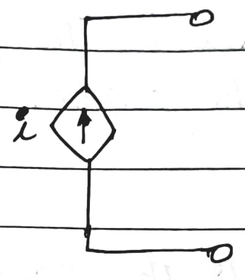
Physical sources such as batteries and generators may be regarded as approximations to ideal voltage sources

Ideal dependent Source

An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current

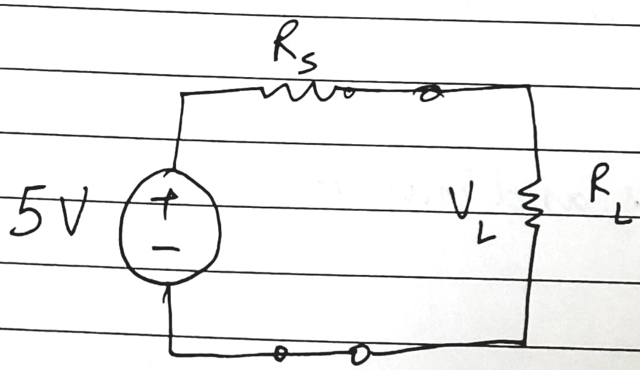


(a)



(b)

Symbols for (a) dependent voltage source
(b) dependent current source



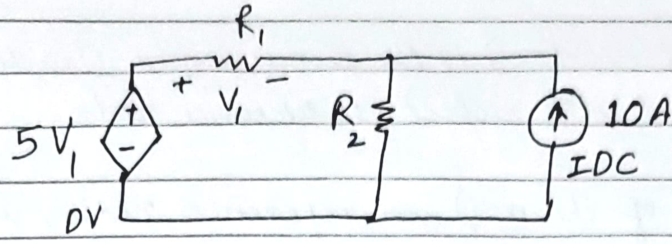
$$i = 5/(R_L + R_s)$$

$$V_L = i R_L$$

$$= 5 \frac{R_L}{R_L + R_s}$$

$$= \frac{5}{1 + \frac{R_s}{R_L}}$$

$\frac{R_s}{R_L} << 1$ For Ideal



~~Voltage Induced Voltage~~

VCVS: Voltage Controlled Voltage Source

CCVS: Current Controlled Voltage Sources

VCCS: Voltage Controlled Current Source

CCCS: Current Controlled Current Source

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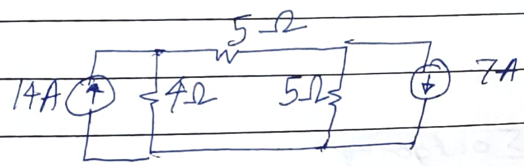
Nodal Analysis

Ch 3 Section 3.2



No. of Nodes in the given circuit = 3

No. of circuit elements = 5 (3 resistors, 2 current source)

Using KCL, KVL and Ohm's law we solve circuits

Node - Voltage Method

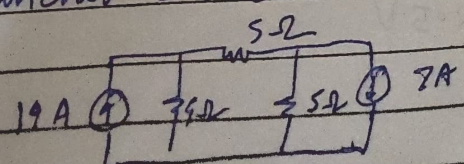
Systematic application of KCL

General Procedure

→ First Identify no. of Nodes

→ Select a node as the reference node or datum node (commonly called ground)

* Choose node which has maximum no. of branches connected to it

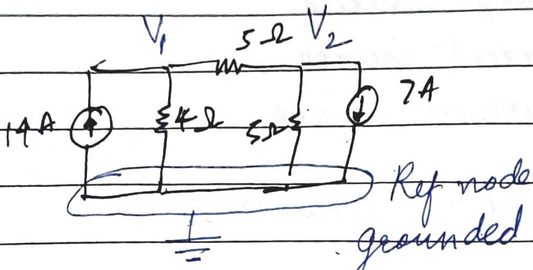


In this ckt. choose the bottom node as ref node

→ Assign voltages $V_1, V_2, \ldots V(n-1)$ to the remaining $(n-1)$ nodes. The voltages are refered with respect to reference node

→ Apply KCL to each of the $(n-1)$ non reference nodes, using Ohm's law to express the branch currents in terms of node voltages

→ Solve the resulting simultaneous eq^n to obtain the unknown node voltages



Apply KCL

KCL at node 1

Σ incoming i = Σ outgoing i

$$14 = \frac{V_1}{4} + \frac{V_1 - V_2}{5}$$

$$14 \times 20 = 5V_1 + 4V_1 - 4V_2$$

$$\boxed{9V_1 - 4V_2 = 280} \quad \text{—(1)}$$

KCL at node 2

Algebraic sum of all incoming current $=0$

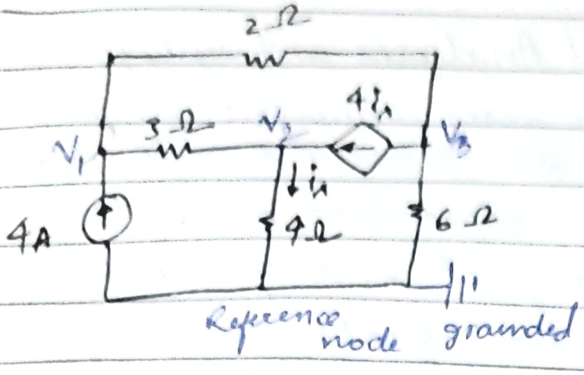
$$\frac{V_1 - V_2}{5} + \left(\frac{0 - V_2}{5}\right) + (-7) = 0$$

$$V_1 - V_2 - V_2 - 35 = 0$$

$$\boxed{V_1 - 2V_2 = 35} \quad \text{—(2)}$$

$V_1 = 30V$

$V_2 = -2.5V$



Node 1

Σ Incoming current = Σ outgoing current

$$4 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{2}$$

$$5V_1 - 2V_2 - 3V_3 = 24 \quad -① $$

Node 2

$$\frac{V_1 - V_2}{3} + V_2 = \frac{V_2}{4}$$

$$4V_1 + 5V_2 = 0 \quad -②$$

Node 3

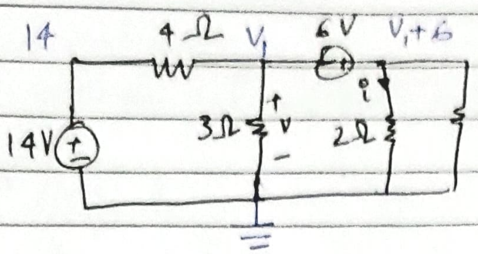
$$\frac{V_1 - V_3}{2} = \frac{V_3}{6} + V_2$$

$$3V_1 - 6V_2 - 4V_3 = 0 \quad -③$$

$V_1 = 32V$

$V_2 = -25.6V$

$V_3 = 62.4V$

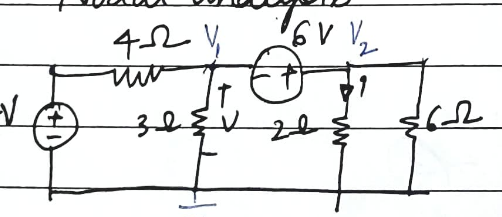


Nodal Analysis with Voltage Source

- Voltage source connected between ~~simply set~~ reference node and a non-reference node
  → simply set voltage at the non reference node equal to the voltage of the voltage source.
- Voltage source connected between two non reference nodes
  → These two non reference nodes form a generalized node or super node
  → We apply both KCL and KVL to determine the node voltages

- A supernode is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes & any elements connected in parallel with it.

Q. Find the node voltages and v and i in the circuit using Nodal analysis



There is supernode between V_1 & V_2

Applying supernode analysis

$$\frac{V_1-14}{4}+\frac{V_1}{3}+\frac{V_2}{2}+\frac{V_2}{6}=0$$

$$3V_1-42+4V_1+6V_2+2V_2=0$$

$$7V_1+8V_2=42 \quad -①$$

Applying KVL between V_1 & V_2

$$V_1+6=V_2 \quad -②$$

from ① & ② $15V_1=-6$

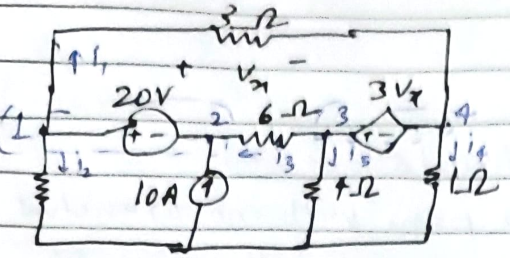
$$V_1=-400\,mV$$

$$V_1=v \Rightarrow v=-400\,mV$$

$$i=V_2/2$$

$$V_2=-6/15+6=84/15\,V$$

$$i=42/15=2.8\,A$$



1 & 2 form super node

$$\frac{V_3 - V_2}{6} + 10 = \frac{V_1 - V_4}{3} + \frac{V_1}{2}$$

$$5V_1 + V_2 - V_3 - 2V_4 = 60 \quad \text{--- (1)}$$

3 & 4 form a super node

$$\frac{V_3 - V_2}{6} + \frac{V_4}{1} + \frac{V_3}{4} = \frac{V_1 - V_4}{3}$$

$$4V_1 + 2V_2 - 5V_3 - 16V_4 = 0 \quad \text{--- (2)}$$

Applying KVL, $V_1 - V_2 = 20$

$$-V_3 + 3V_x + V_4 = 0$$

$$V_x = V_1 - V_4$$

$$3V_1 - V_3 - 2V_4 = 0 \quad \text{--- (3)}$$

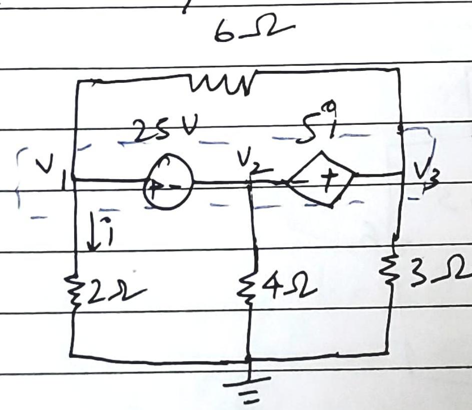
$$V_x - 3V_x + 6i_3 - 20 = 0$$

$$6i_3 = V_3 - V_2 \quad \& \quad V_x = V_1 - V_4$$

$$-2V_1 - V_2 + V_3 + 2V_4 = 20 \quad \text{--- (4)}$$

$V_1 = 26.67V$ $V_2 = 6.667V$ $V_3 = 173.33V$ $V_4 = -46.67V$

Q Find V_1, V_2 and V_3 in the ckt using nodal analysis



$$\frac{V_1}{2} + \frac{V_2}{4} + \frac{V_3}{3} = 0$$

$$6V_1 + 3V_2 + 4V_3 = 0 \quad \text{--- (1)}$$

$$V_1 - V_2 = 25 \quad \text{--- (2)}$$

$$V_3 - V_2 = 5i$$

$$i = V_1/2$$

$$V_3 - V_2 = 5V_1/2$$

$$5V_1 + 2V_2 - 2V_3 = 0$$

$V_1 = 7.608\,V$

$V_2 = -17.39V$

$V_3 = 1.6305\,V$

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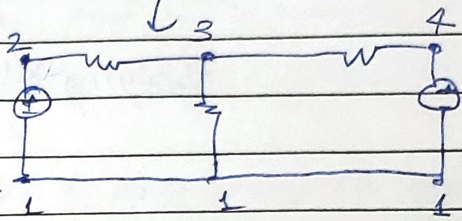
Mesh Analysis

Systematic application of KVL

→ It is only applicable to planar circuits

A circuit that can be drawn in a plane with no branches crossing one another is planar circuit, otherwise it is non planar.

planar ckt

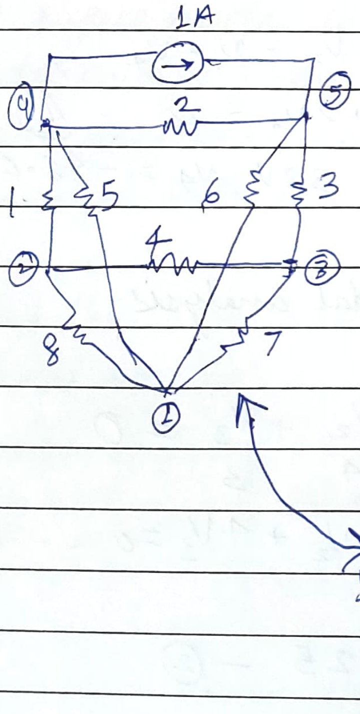


No. of nodes = 4

(3 unknown in nodal analysis)

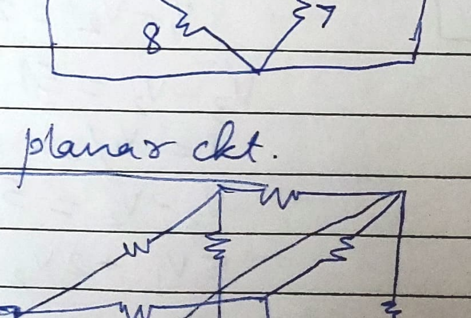
If we use nodal analysis we require 3x3 matrix inversion

m=2 : Mesh analysis requires 2x2 matrix inversion

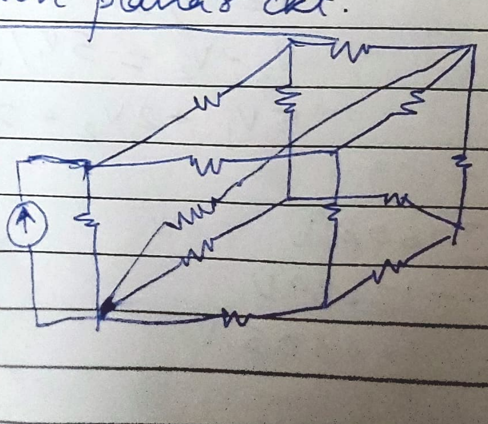


Is the ckt planar?

Yes it is planar as we can redraw the circuit



* Example of Non planar ckt.



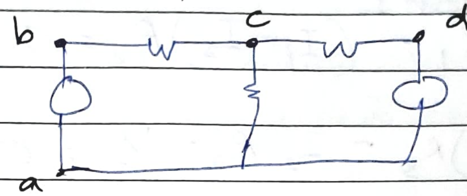
Nodes, Branches, Loops and Meshes

Branch: a single element such as a voltage source or a resistor

Node: point of connection between two or more branches

Loop: any closed path in a circuit. abcda

Mesh: A loop that does not contain any other loops within it abca,



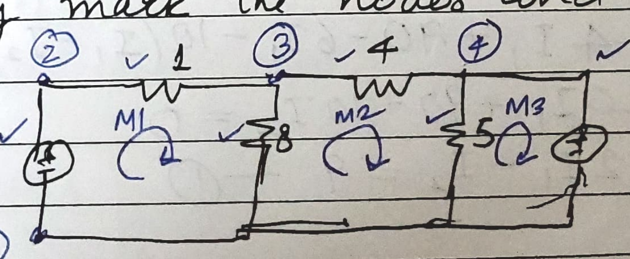
A network with b branches, n nodes and m meshes will satisfy the fundamental theorem of network topology.

$$\boxed{b = m + n - 1}$$

$$m = (b+1) - n$$

Example

How many branches, nodes and meshes are there?
Clearly mark the nodes and meshes



6 Branches
4 nodes
3 meshes

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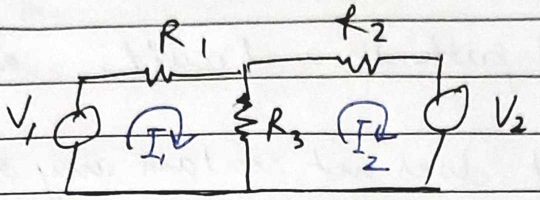
- Assign mesh currents i1, i2, ... to the m meshes
- Apply KVL to each of the m meshes, using Ohm's law to express the voltages in terms of the mesh current
- Solve the resulting m simultaneous equations to get the mesh currents

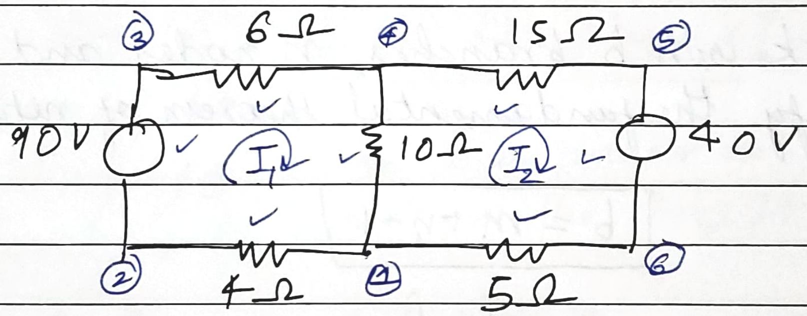


KVL in mesh 1

$$V_1 - I_1 R_1 - (I_1 - I_2) R_3 = 0$$

mesh 2

$$(I_1 - I_2) R_3 - I_2 R_2 - V_2 = 0$$



$$n = 6 \qquad b = 7$$

$$m = (7+1) - 6 = 2$$

KVL in mesh 1

$$-4I_1 + 90 - 6I_1 - 10(I_1 - I_2) = 0$$

$$-20I_1 + 90 + 10I_2 = 0$$

$$\Rightarrow \quad 2I_1 - I_2 = 9 \quad \text{—} \; 1$$

KVL in mesh 2,

$$10(I_1 - I_2) - 15I_2 - 40 - 5I_2 = 0$$

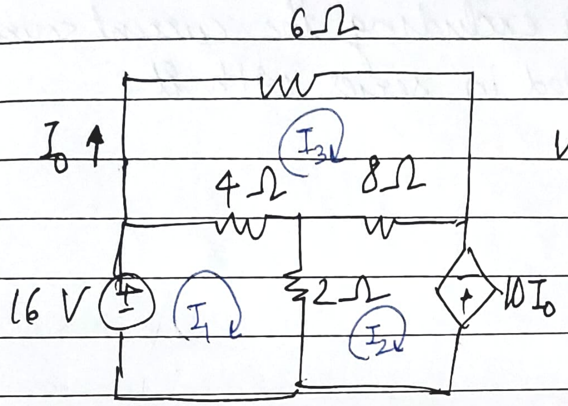
$$I_1 - 3I_2 = 4 \quad \text{—} \; 2$$

$$\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$$

$I_1 = 4.6A$, $I_2 = 0.2A$

Find power dissipated in 10 Ω resistance

$$= (I_1 - I_2)^2 R$$
$$= (4.6 - 0.2)^2 \times 10$$



Using mesh analysis, find I_0 in the circuit.

$m = 3$

KVL in mesh 1

$$16 - 4(I_1 - I_3) - 2(I_1 - I_2) = 0$$
$$16 - 4I_1 + 4I_3 - 2I_1 + 2I_2 = 0$$
$$16 + 4I_3 - 6I_1 + 2I_2 = 0$$
$$6I_1 - 2I_2 - 4I_3 = 16 \quad -(1)$$

KVL in mesh 2

$$10I_0 - 2(I_2 - I_1) - 8(I_2 - I_3) = 0$$
$$-10I_2 + 8I_3 + 2I_1 + 10I_0 = 0 \quad -(2)$$

KVL in mesh 3

$$-6I_3 - 8(I_3 - I_2) - 4(I_3 - I_1) = 0$$
$$-6I_3 - 8I_3 + 8I_2 - 4I_3 + 4I_1 = 0$$
$$4I_1 + 8I_2 - 18I_3 = 0 \quad -(3)$$

We can see in ckt $I_0 = I_3$ $-(4)$

$$\boxed{I_3 = I_0 = -4A}$$

From (2) & (4)

$$-10I_2 + 8I_3 + 2I_1 + 10I_3 = 0$$
$$2I_1 - 10I_2 + 18I_3 = 0$$

Super Mesh

Mesh analysis with current source

- Circuit with current source existing only in one mesh.
  - This current source becomes the mesh current.

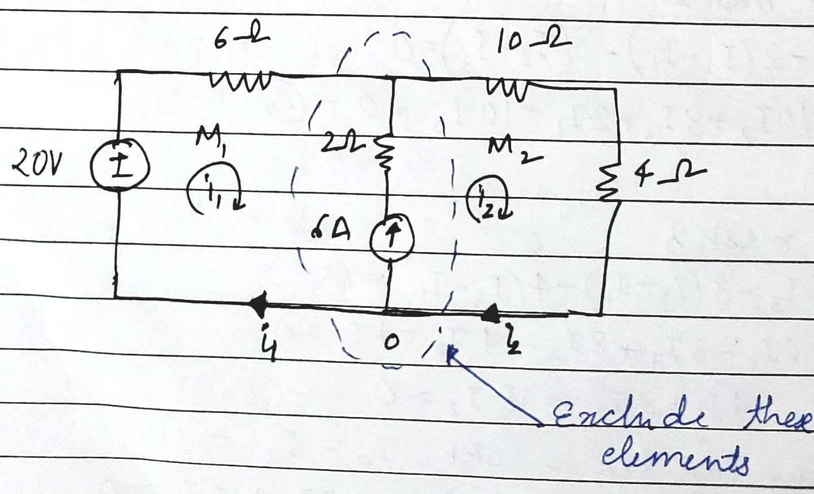
- Circuit where two meshes have a (dependent or independent) current source in common
  - Create a super mesh by excluding the current source and any elements connected in series with it.

Properties of super-mesh

- The current source in the super mesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents

- A super mesh has no current of its own

- A super mesh requires the application of both KVL and KCL



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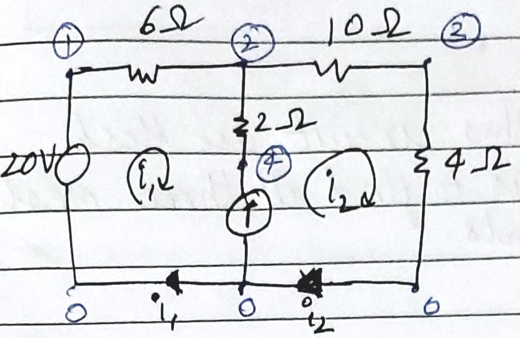


Using mesh analysis, find i_1 & i_2

KVL in mesh 1

$$20 - 6i_1 - 2(i_1 - i_2) - V_{40} = 0 \quad -(1)$$

KVL in mesh 2

$$V_{40} - 2(i_2 - i_1) - 10i_2 - 4i_2 = 0 \quad -(1')$$

$$i_2 - i_1 = 6 \quad -(2)$$

Add (1) & (1')

V_{40} eliminated

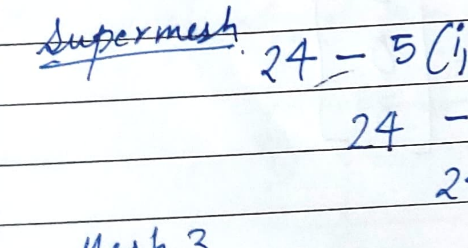
$$6i_1 + 14i_2 = 20 \quad -(3)$$

From (2) & (3)

$i_1 = -3.2\,A$,

$i_2 = 2.8\,A$

Use mesh analysis to determine i_1, i_2, i_3



mesh 1 & mesh 2 makes a supermesh

KVL for supermesh $-(1)$

KVL for mesh 3 $-(2)$

$$i_1 - i_2 = 4 \quad -(3)$$

Supermesh

$$24 - 5(i_1 - i_3) - 10(i_2 - i_3) - 20i_2 = 0 \quad -(1)$$

$$24 - 5i_1 + 5i_3 - 10i_2 + 10i_3 - 20i_2 = 0$$

$$24 = 5i_1 + 30i_2 - 15i_3 \quad -(1)$$

Mesh 3

$$-5(i_3 - i_1) - 5i_3 - 10(i_3 - i_2) = 0$$

$$-5i_3 + 5i_1 - 5i_3 - 10i_3 + 10i_2 = 0$$

$$5i_1 - 20i_3 + 10i_2 = 0$$

$$i_1 + 2i_2 - 4i_3 = 0 \quad -(2)$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 5 & 30 & -15 \\ 1 & 2 & -4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 24 \\ 0 \end{bmatrix}$$

$i_1 = 4.8\,A$

$i_2 = 0.8\,A$

$i_3 = 1.6\,A$

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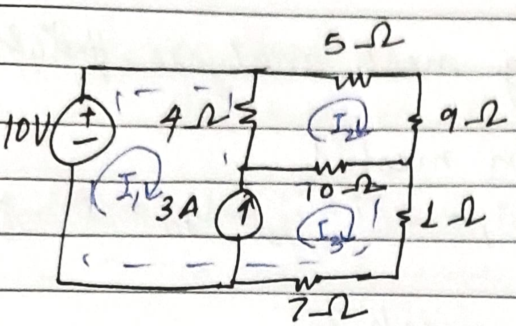


For this circuit use Mesh analysis to find all three mesh currents.

Supermesh.

$$10-4(I_1-I_2)-10(I_3-I_2)-I_3-7I_3=0$$

$$10-4I_1+4I_2-10I_3+10I_2-I_3-7I_3=0$$

$$10-4I_1+14I_2-18I_3=0$$

$$4I_1-14I_2+18I_3=10$$

$$2I_1-7I_2+9I_3=5 \quad -1$$

Mesh 2

$$-4(I_2-I_1)-5I_2-9I_2-10(I_2-I_3)=0$$

$$-4I_2+4I_1-5I_2-9I_2-10I_2+10I_3=0$$

$$4I_1-28I_2+10I_3=0 \quad -2$$

$$I_1-I_3=3 \quad -3$$

$$\begin{bmatrix} 2 & -7 & 9 \\ 4 & -28 & 10 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}=\begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}$$

$I_1 = 3.26$ A

$I_2 = 0.56$ A

$I_3 = 0.26$ A

Nodal vs Mesh Analysis

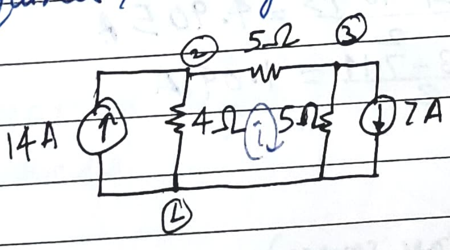
- To select the method that results in the smaller number of equations.

- For example
  - Choose nodal analysis for circuit with fewer nodes than meshes.
  - Choose mesh analysis for circuit with fewer meshes than nodes

- Networks with parallel-connected elements, current sources, or supernodes
  - More suitable for nodal analysis.

- Networks that contain many series connected elements, voltage sources, or super ~~node~~ meshes
  - More suitable for mesh analysis

- If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis



In this circuit.

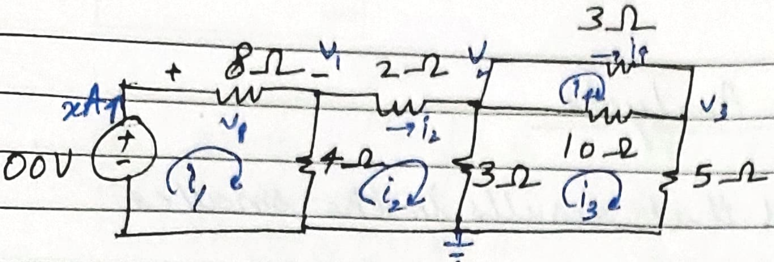
No. of nodes = 3

Nodal eq^n = 2.

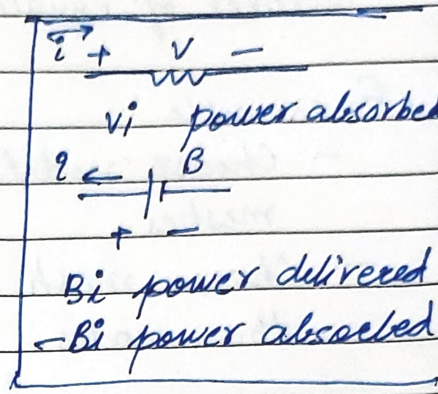
No. of mesh = 3

But the no. of unknown current is only one i as denoted as we know the current from the current source in other two meshes.

So it is better to use mesh analysis.



Find all the element currents, voltages and power absorbed



V	I	VI
100 V	x	−100x
V_8	x	$x V_8$

KCL at node 1

$$\frac{V_1-100}{8}+\frac{V_1}{4}+\frac{V_1-V_2}{2}=0$$

$$7V_1-4V_2=100 \quad -(1)$$

KCL at node 2

$$\frac{V_2-V_1}{2}+\frac{V_2}{3}+\frac{V_2-V_3}{3}+\frac{V_2-V_3}{10}=0$$

$$15V_1-38V_2+13V_3=0 \quad -(2)$$

KCL at node 3

$$\frac{V_3-V_2}{3}+\frac{V_3-V_2}{10}+\frac{V_3}{5}=0$$

$$13V_2-19V_3=0$$

Solving (1), (2) & (3) $V_1 = 20.24V$, $V_2 = 10.43V$, $V_3 = 7.14V$.

$i_1 = \frac{100-20.24}{8} = 9.97A$, $i_2 = \frac{20.24-10.43}{2} = 4.905A$

$i_3 = \frac{7.14}{5} = 1.43A$, $i_4 = \frac{10.43-7.14}{3} = 1.09A$

Power absorbed

(1) $-100\times 9.97A = -997W$ (By voltage source)

(2) $\cancel{9.97\times 8 = 79.76W}$ (By 8Ω) $9.97\times(100-20.24) = 795.20W$

(3) $\cancel{4\left(\frac{20.24}{4}\right) = 20.24W}$ (By 4Ω) $\frac{20.24}{4}\times 20.24 = 102.4W$

(4) $\cancel{2(4.90) = 9.8W}$ (By 2Ω) $\frac{(20.24-10.43)^2}{2} = 48.11W$

(5) $\cancel{3\left(\frac{10.43}{3}\right) = 10.43W}$ (By 3Ω) $\frac{10.43}{3}\times 10.43 = 36.26W$

(6) (By 10Ω) $(10.43-7.14)^2/10 = 1.08W$

(7) (By 5Ω) $(7.14)^2/5 = \cancel{50.97W}$ $10.19W$ (8) (By 3Ω) $\frac{(10.43-7.14)^2}{3} = 3.60$

Circuit theorems

Superposition Theorem

- It states that the voltage across (or current ~~throug~~ through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to EACH independent source acting alone.

- The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.

1. Turn of all independent sources except one source.
   a. Independent voltage sources are replaced by 0 V (short ckt)
   b. Independent current sources are replaced by 0A (open ckt)
   c. Dependent sources are left intact because they are controlled by circuit variables

2. Find the output (voltage or current) due to that active source using nodal or mesh analysis

3. Repeat ~~st~~ steps 1 and 2 for each of the other independent sources.

4. Find the total contribution by adding algebraically all the contributions due to the independent sources

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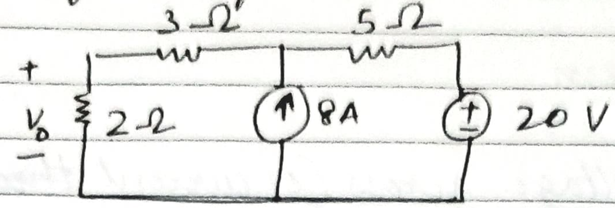
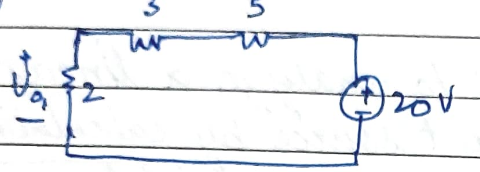
Determining voltage across 2Ω resistor in given ckt.

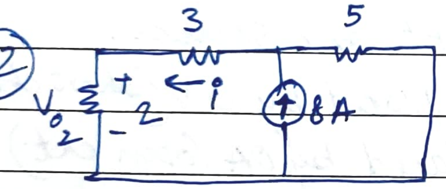


We will redraw the circuit into 2 circuits and use superposition principle

① 

We have short circuited the current source ~~Now~~ Now we will find the voltage across 2Ω resistor in this ckt.

$$V_{o_1} = \frac{2}{2+3+5} \times 20 = \frac{2 \times 20}{10} = 4V$$

② 

We have short circuited the voltage source. Now we will find the voltage across 2Ω resistor in this ckt.

$$V_{o_2} = i \times 2$$

$i = 4A$ (using current division)

$$V_{o_2} = 8V$$

$$\boxed{V_o = V_{o_1} + V_{o_2}}$$

$$= 4 + 8 = 12V$$