AC Circuit

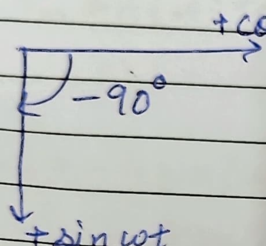
Sinusoids and Phasors

Sinusoidal signal is easy to generate and transmit. Through Fourier analysis, any practical periodic signal can be represented by a sum of sinusoids.

$$v(t) = V_m \sin(\omega t + \phi)$$

ϕ = phase

$\omega = 2\pi f \qquad T = 2\pi/\omega \qquad f = 1/T$



The horizontal axis represents the magnitude of cosine, while the vertical axis (pointing down) denotes the magnitude of sine. Angles are measured +vely counterclockwise from the horizontal.

Adding 2 sinusoids

$$A\cos\omega t + B\sin\omega t = C\cos(\omega t - \theta)$$

$$C = \sqrt{A^2 + B^2} \qquad \theta = \tan^{-1}\frac{B}{A}$$

Q. $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$. Which is leading.

$v_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 50 - 180) = 10\cos(\omega t - 130^\circ)$

$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t - 10 - 90) = 12\cos(\omega t - 100^\circ)$

v_2 leads v_1 by 30°

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Q $i_1 = -4\sin(377t + 55^\circ)$ $\quad$ $I_{max}\cos(\omega t+\theta)$
$i_2 = 5\cos(377t - 65^\circ)$ $\quad$ ~~0, 2π~~
$[-\pi, \pi]$

$I_1(t) = 4\cos(377t + 145)$

$I_2(t) = 5\cos(377t - 65)$

i_1 leads i_2 by $(\theta_1 - \theta_2)$
$= (145 - (-65)) = 210^\circ$

In this que. If $\theta \in [0, 2\pi]$
$\theta_1 = 145^\circ$
$\theta_2 = -65 + 360 = 295^\circ$
$\theta_1 - \theta_2 = -150^\circ$
i_1 lags i_2 by 150°

Inductive Ckt.
V_L leads I_L by 90°

Capacitive Ckt.
I_C leads V_C by 90°

Phasors

Complex no. $z = x + jy$
$j = \sqrt{-1}$

$z = x + jy$ Rectangular form
$z = r\angle\phi$ Polar form
$z = re^{j\phi}$ Exponential form

$r = \sqrt{x^2 + y^2}$ $\quad$ $\phi = \tan^{-1} y/x$
$x = r\cos\phi$ $\quad$ $y = r\sin\phi$

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$z = r\angle\phi = r(\cos\phi + j\sin\phi)$

Reciprocal $\frac{1}{z} = \frac{1}{r}\angle{-\phi}$

~~Root~~ Sq. Root $\sqrt{z} = \sqrt{r}\angle\phi/2$

Conjugation $z^* = r\angle{-\phi} = re^{-j\phi}$

Time domain		Phasor domain	
$V_m\cos(\omega t+\theta)$	$\Longleftrightarrow$	$V_m\angle\theta$	$V_m e^{j\theta}$
$V_m\sin(\omega t+\theta)$	$\Longleftrightarrow$	$V_m\angle\theta-90^\circ$	

$v(t) = -5\sin(\omega t+30^\circ)$

$= 5\cos(\omega t+30+90) = 5\cos(\omega t+120) = 5\angle 120^\circ$

Passive Element

R	$Z=R$
C	$Z = 1/j\omega C$
L	$Z = j\omega L$

For different freq impedance will be different.

Dependant Sources

Ex: CCCS

$i(t) = 5i_x(t) \longrightarrow 5I_x$

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$$v(t) \Rightarrow V\angle\theta = \vec{V}_{phasor}$$

$$\frac{dv(t)}{dt} \Rightarrow j\omega\vec{V}_{Phasor} = j\omega V\angle\theta = \omega V\angle(\theta+\pi/2)$$

$$\int v\,dt \longrightarrow \frac{\vec{V}_{Phasor}}{j\omega} = \frac{-j}{\omega}V\angle\theta = \frac{V}{\omega}\angle(\theta-\pi/2)$$

Q. Using the phasor approach, determine the current described by the integrodifferential equation.

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t+75^\circ)$$

$$4I + \frac{8I}{j\omega} - 3j\omega I = 50\angle 75^\circ$$

$$\omega = 2$$

$$I(4 - j4 - j6) = 50\angle 75^\circ$$

$$I = \frac{50\angle 75^\circ}{4 - j10} = \frac{50\angle 75}{10.77\angle -68.2^\circ} = 4.642\angle 143.2^\circ \text{ A}$$

$$i(t) = 4.642\cos(2t + 143.2^\circ)\text{ A}$$

Phasor Relationship for Circuit Elements

For Inductor L

$$i = I_m \cos(\omega t + \phi)$$

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$$

$$V = \omega L I_m \cos(\omega t + \phi + 90)$$

$$V = \omega L I_m e^{j(\phi + 90^\circ)} = \omega L I_m \angle \phi + 90^\circ$$

$$V = j\omega L I$$

For capacitor C

$$v = V_m \cos(\omega t + \phi)$$

$$i = C\frac{dv}{dt}$$

$$= -\omega C V_m \sin(\omega t + \phi)$$

$$I = \omega C V_m \cos(\omega t + \phi + 90)$$

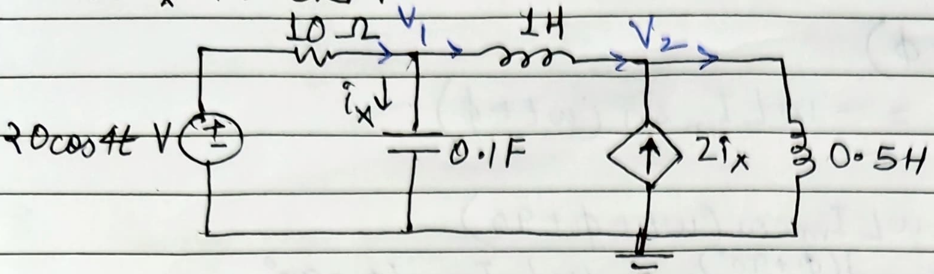
$$I = \omega C V_m e^{j(\phi + 90)} = \omega C V_m \angle \phi + 90^\circ$$

$$I = j\omega C V$$

$$V = \frac{I}{j\omega C}$$

Nodal analysis

Find i_x in ckt.



$20\cos 4t \Rightarrow 20\angle 0^\circ$

$1H = 4j$

$0.5H = 2j$

$0.1F = -2.5j$

KCL at node 1

$$\frac{20 - V_1}{10} = \frac{V_1}{-2.5j} + \frac{V_1 - V_2}{4j}$$

$$(1 + 1.5j)V_1 + 2.5j\, V_2 = 20 \quad - ①$$

KCL at node 2

$$2I_x + \frac{V_1 - V_2}{4j} = \frac{V_2}{2j}$$

$$I_x = V_1 / -2.5j$$

$$\frac{2V_1}{-2.5j} + \frac{V_1 - V_2}{4j} = \frac{V_2}{2j}$$

$$11V_1 + 15V_2 = 0 \quad - ②$$

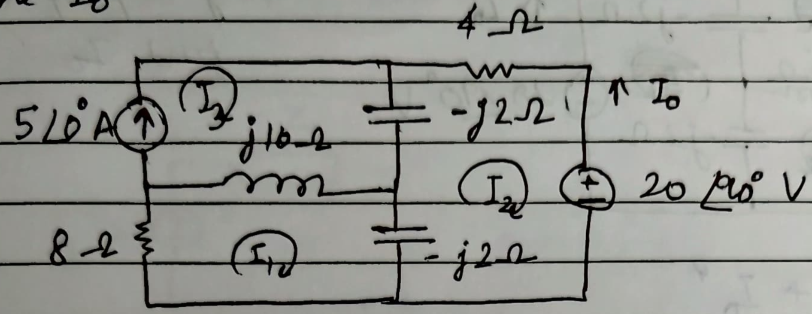
$V_1 = 18.97\angle 18.43^\circ\ V$

$V_2 = 13.91\angle 198.3^\circ\ V$

$$I_x = \frac{V_1}{-2.5j} = \frac{18.97\angle 18.43^\circ}{2.5\angle -90^\circ} = 7.59\angle 108.4^\circ\ A$$

Mesh Analysis

Determine: I_o



Mesh 1

$$\cancel{8+j10} - 8I_1 - 10j(I_1 - I_3) - (-2j)(I_1 - I_2) = 0$$

Mesh 2

$$-4I_2 - 20\angle 90^\circ - (-2j)(I_2 - I_1) - (-2j)(I_2 - I_3) = 0$$

Mesh 3

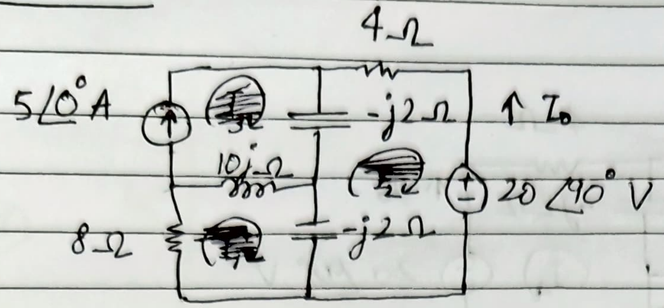
$$I_3 = 5$$

Solving

$$I_2 = 6.12 \angle -35.22^\circ \text{ A}$$

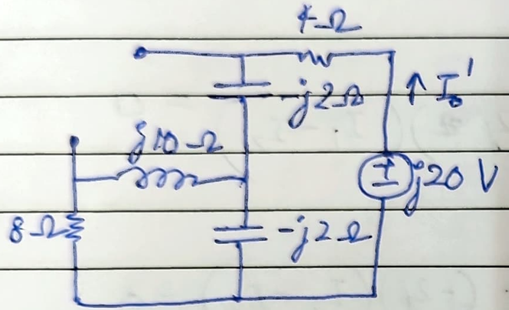
$$I_o = -2 = 6.12 \angle 144.78^\circ \text{ A}$$

Superposition



using superposition find I_o

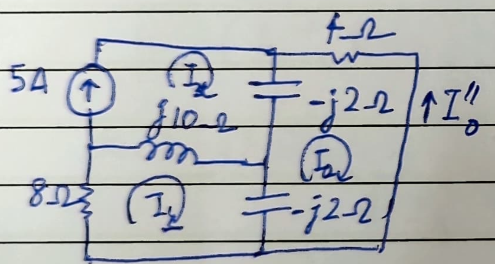
$I_o = I_o' + I_o''$



$$Z = \frac{-j2(8+j10)}{-2j+8+10j} = 0.25 - j2.25$$

$$I_o' = \frac{j20}{4-j2+Z} = \frac{j20}{4.25-j4.25}$$

$$I_o' = -2.353 + j2.353$$



Mesh 1

$(8+j8)I_1 - j10 I_3 + j2 I_2 = 0$

Mesh 2

$(4-j4)I_2 + j2 I_1 + j2 I_3 = 0$

Mesh 3

$I_3 = 5$

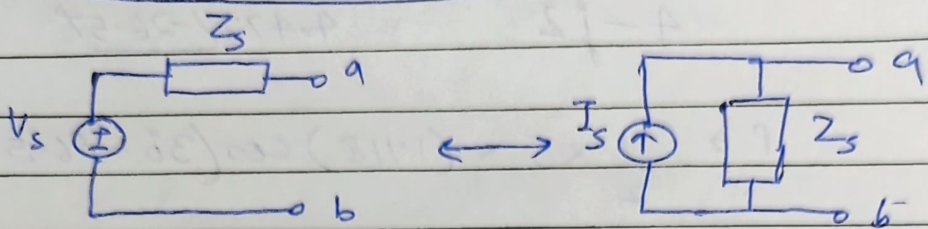
$I_2 = 2.647 - j1.176$

$I_o'' = -I_2 = -2.647 + j1.176$

$I_o = I_o' + I_o'' = -5 + j3.529$

Source Transformation

$$\boxed{V_s = Z_s I_s \iff I_s = \frac{V_s}{Z_s}}$$



$V_s = Z_s I_s$ $\qquad$ $I_s = \dfrac{V_s}{Z_s}$

Thevenin and Norton Equivalent Circuits

$$\boxed{V_{Th} = Z_N I_N, \quad Z_{Th} = Z_N}$$

AC Power analysis

$$p(t) = v(t)\,i(t)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

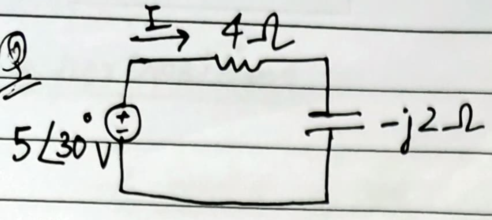
$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Average power $\quad P = \dfrac{1}{T}\displaystyle\int_0^T p(T)\,dt$

$$P = \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\,dt$$
$$+ \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

Q



Find Avg. power supplied by source and avg power absorbed by resistor

$$I = \frac{5\angle 30^\circ}{4-j2} = \frac{5\angle 30^\circ}{4.472\angle -26.57^\circ} = 1.118\angle 56.57^\circ$$

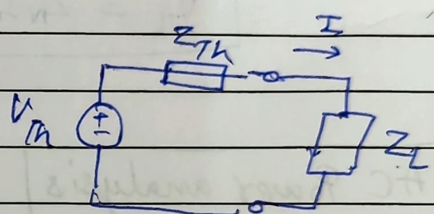
$$P = \frac{1}{2} \times 5 \times (1.118)\cos(30^\circ - 56.57^\circ) = 2.5\,W$$

$$I_R = I = 1.118\angle 56.57^\circ \text{ A}$$

$$V_R = 4 I_R = 4.472\angle 56.57^\circ \text{ V}$$

$$P = \frac{1}{2}(4.472)(1.118) = 2.5\,W$$

Maximum Average Power Transfer



$$Z_{Th} = R_{Th} + jX_{Th}$$

$$Z_L = R_L + jX_L$$

$$I = \frac{V_{Th}}{Z_{Th} + Z_L} = \frac{V_{Th}}{(R_{Th} + jX_{Th}) + (R_L + jX_L)}$$

$$P = \frac{1}{2}|I|^2 R_L = \frac{|V_{Th}|^2 R_L/2}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}$$

$$\frac{\partial P}{\partial X_L} = \frac{-|V_{Th}|^2 R_L (X_{Th}+X_L)}{[(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2]^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{|V_{Th}|^2[(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2 - 2R_L(R_{Th}+R_L)]}{2[(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2]^2}$$

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$$\frac{\partial P}{\partial X_L} = 0$$

$$X_L = -X_{Th}$$

$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_L)^2}$$

$$Z_L = R_L + jX_L = R_{Th} - jX_{Th}$$

$$P_{max} = \frac{|V_{Th}|^2}{8R_{Th}}$$

RMS

Root mean square

$$X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2\,dt}$$

① square
② mean
③ root

Power Factor

cos of the phase difference between voltage and current

Complex Power

$$S = \frac{1}{2} V I^* \quad (I^* = \text{complex conjugate of current})$$

$$= V_{rms} I_{rms}^*$$

$$S = I_{rms}^* Z$$

$$\boxed{\begin{aligned} S &= V_{rms} I_{rms} \angle \theta_v - \theta_i \\ &= V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i) \end{aligned}}$$

$$Z = R + jX$$

$$S = I_{rms}^2 (R + jX) = P + jQ$$

$$P = \mathrm{Re}(S) = I_{rms}^2 R$$

$$Q = \mathrm{Im}(S) = I_{rms}^2 X$$

P is average or real power — depends on load's resistance R

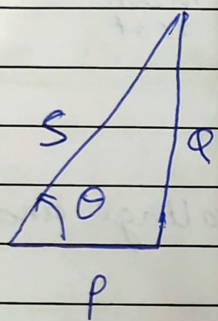
Q is reactive power — depends on load's reactance X

$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$, $Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

Real power is the actual power dissipated ~~by~~ by the load.
Reactive power Q is a measure of energy exchange between source and reactive part of load. Unit of Q is volt-ampere reactive

$Q = 0$ for resistive loads (unity Power Factor)
$Q < 0$ for capacitive loads (leading pf)
$Q > 0$ for inductive loads (lagging pf)

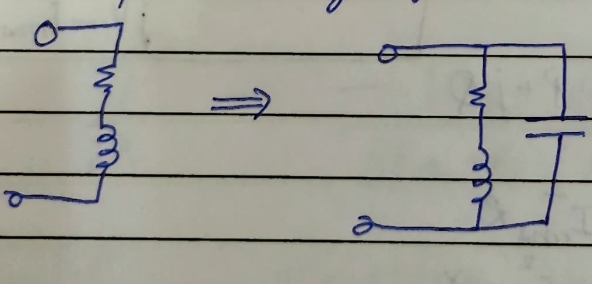
Power Triangle (EE lale)

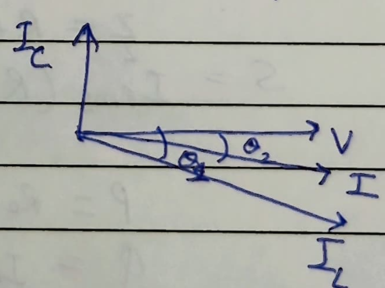


S – Complex power
Q – Reactive power
P – Real Power
θ – Power Factor angle

Power Factor Correction

Most ~~inductive~~ domestic & industrial loads are inductive and operate at a low lagging power factor
PF improved by ~~by~~ installing a capacitor in parallel





Sir's questions

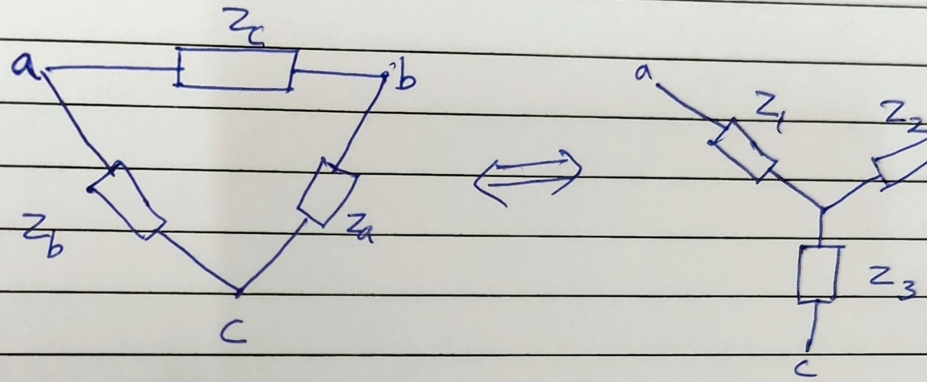
1. Appliances which have leading/ lagging power factor

Lagging PF – Transformers, induction motors
Leading PF – radio ckts

2. Example of 3 phase load in steel industry

Electric Arc Furnace

Star-Delta.



Star to Delta

$$Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1} \qquad Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

Delta to Star

$$Z_1 = \frac{Z_bZ_c}{Z_a+Z_b+Z_c} \qquad Z_2 = \frac{Z_cZ_a}{Z_a+Z_b+Z_c}$$

$$Z_3 = \frac{Z_aZ_b}{Z_a+Z_b+Z_c}$$