Introduction to Finance

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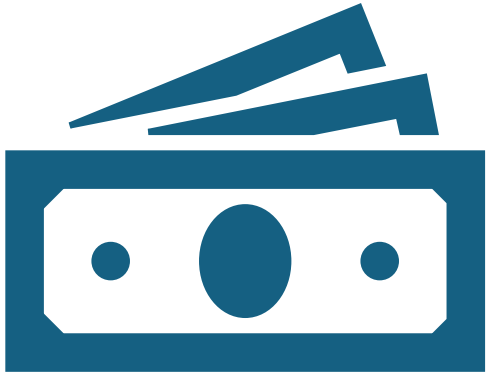

No-Arbitrage, Security Prices and Interest rates

Determining the interest rate:

- Given the risk-free interest rate, the no-arbitrage price of a risk-free bond is determined by
- **Price(Security) = *PV*(All cash flows paid by the security)**
- The reverse is also true: If we know the price of a risk-free bond, we can use the above equation too determine what the risk-free interest rate must be if there are no arbitrage opportunities.

No-Arbitrage, Security Prices and Interest rates

Determining the interest rate: Example

- For example, suppose a risk-free bond that pays $1000 in one year is currently trading with a competitive market price of $929.80 today.
- we know that the bond's price equals the present value of the $1000 cash flow it will pay:
- $929.80 today = ($1000)/ (1 + *r*)
- Then 1+r= 1.0755 and risk-free interest rate r= 7.5%

$$1 + r_f = \frac{\$1000 \text{ in one year}}{\$929.80 \text{ today}} = 1.0755$$

No-Arbitrage and Security Prices

Bond return

- The risk-free interest rate equals the percentage gain that you earn from investing in the bond, which is called the bond's **return**:

$$\text{Return} = \frac{\text{Gain at End of Year}}{\text{Initial Cost}}$$

$$= \frac{1000 - 929.80}{929.80} = \frac{1000}{929.80} - 1 = 7.55\%$$

Bond return

- **If there is no arbitrage, the risk-free interest rate is equal to the return from investing in a risk-free bond.**
- If the bond offered a higher return than the risk-free interest rate, then investors would earn a profit by borrowing at the risk-free interest rate and investing in the bond.
- If the bond had a lower return than the risk-free interest rate, investors would sell the bond and invest the proceeds at the risk-free interest rate.
- No-arbitrage is therefore equivalent to the idea that *all risk-free investments should offer investors the same return*.

No-Arbitrage Price (NAP) and Interest rate computation

- Consider a security that pays its owner Rs.100 today and Rs.100 in one year, without any risk. Suppose the risk-free interest rate is 10%. What is the no-arbitrage price of the security today (before the first Rs.100 is paid)?

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- Compute NAP if interest rate is (a)5% b(12%)

a) Ans: PV= 100 + (100/1.05)=100+95.23=195.23

b) Ans: PV= 100 + (100/1.12)=100+89.28=189.28

(Note: Check the relationship between interest rate and NAP)

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If the security is trading for Rs.195 in the market, what arbitrage opportunity is available?

- Security is trading in premium as market is offering more than its present value (PV=NAP=190.91)

Arbitrage opportunity:

- Sell at 195= gain 95 today (195-100 return from bond today).
- Invest 90.91 at 10% which gives 100 next year

NPV and Firms Decision Making

When securities **trade at no-arbitrage prices**, what can we conclude about the value of trading them?

- The cost and benefit are equal in a normal market and so the NPV of buying a security is zero

$$\begin{aligned} NPV(\text{Buy security}) &= PV(\text{All cash flows paid by the security}) - \text{Price(Security)} \\ &= 0 \end{aligned}$$

- Similarly, if we sell a security, the price we receive is the benefit and the cost is the cashflows we give up. Again, the NPV is zero

$$\begin{aligned} NPV(\text{Sell security}) &= \text{Price(Security)} - PV(\text{All cash flows paid by the security}) \\ &= 0 \end{aligned}$$

NPV and Firms' decision

- The insight that security trading in a normal market is a zero-NPV transaction is a critical building block in our study of corporate finance.
- **Trading securities in a normal market neither creates nor destroys value**: Instead, value is created by the real investment projects in which the firm engages, such as developing new products, opening new stores, or creating more efficient production methods.
- **Financial transactions are not sources of value** but instead serve to adjust the timing and risk of the cash flows to best suit the needs of the firm or its investors.

NPV and firms' decision: Separation Principle

- An important consequence of this result is the idea that we can evaluate a decision by focusing on its real components, rather than its financial ones.
- That is, we can separate the firm's investment decision from its financing choice. We refer to this concept as the **Separation Principle**:
- *Security transactions in a normal market neither create nor destroy value on their own. Therefore, we can evaluate the NPV of an investment decision separately from the decision the firm makes regarding how to finance the investment or any other security transactions the firm is considering.*

NPV and firms decision: Separation Principle

- Your firm is considering a project that will require an up-front investment of Rs.10 million today and will produce Rs.12 million in cash flow for the firm in one year without risk. Rather than pay for the Rs.10 million investment entirely using its own cash, the firm is considering raising additional funds by issuing a security that will pay investors Rs 5.5 million in one year. Suppose the risk-free interest rate is 10%. Is pursuing this project a good decision without issuing the new security? Is it a good decision with the new security?

NPV and firms decision: Separation Principle

Case 1: Without the new security, the cost of the project is Rs.10 million today and the benefit is Rs.12 million in one year. Converting the benefit to the present value at 10% interest rate

- PV= Rs.12 million $/(1.10) =$ Rs. 10.91 million today
- NPV of the project= PV(Cash Flow)-Cost= 10.91-10=0.91 million
- **Case 2:** Now suppose the firm issues the new security. In a normal market, the price of this security will be the present value of its future cash flow:
- Price(Security) $=$ Rs. 5.5 million $/1.10 =$ Rs. 5 million today
- Thus, it raises Rs. 5 million by issuing new security . Then the firm will only need additional 5 million to invest in the project.

NPV and firms decision: Separation Principle

With new security (bond) issue, firm still receives 12 million after 1 year from project investment and needs to pay Rs. 5.5 million after 1 year to the bond investors. Hence firm is left with 6.5 million after 1 year.

PV (CF)=Rs. 6.5 million / (1.10)= Rs. 5.91 million today

NPV (Project)= PV(CF)-Cost= 5.91-5= 0.91 million today

In either case, we get the same result for the NPV. **The separation principle indicates that we will get the same result for any choice of financing for the firm that occurs in a normal market. We can therefore evaluate the project without explicitly considering the different financing possibilities the firm might choose**