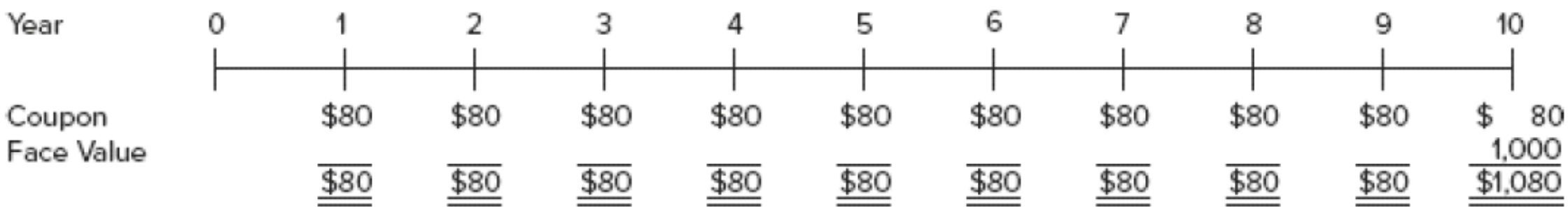
Introduction to Finance

L11

Bond Valuation

- Suppose the X Co. were to issue a bond with 10 years to maturity. The X bond has an annual coupon of $80. Similar bonds have a yield to maturity of 8 percent. Based on our preceding discussion, the X bond will pay $80 per year for the next 10 years in coupon interest. In 10 years, X bond will pay $1,000 to the owner of the bond. What would this bond sell for at present?

Cash flows


Year
0
1
2
3
4
5
6
7
8
9
10
Coupon
$80
$80
$80
$80
$80
$80
$80
$80
$80
$ 80
Face Value
1,000
$80
$80
$80
$80
$80
$80
$80
$80
$80
$1,080

$$\text{Present value} = \$1{,}000/1.08^{10} = \$1{,}000/2.1589 = \$463.19$$

$$\begin{aligned}\text{Annuity present value} &= \$80 \times (1 - 1/1.08^{10})/.08 \\ &= \$80 \times (1 - 1/2.1589)/.08 \\ &= \$80 \times 6.7101 \\ &= \$536.81\end{aligned}$$

$$\text{Total bond value} = \$463.19 + 536.81 = \$1{,}000$$

- **This bond sells for exactly its face value, a**s the going interest rate in the market is 8 percent.
- Considered as an interest-only loan, what interest rate does this bond have? With an $80 coupon, this bond pays exactly 8 percent interest only when itsells for $1,000.

Bond price at discount

- To illustrate what happens as interest rates change, suppose that a year has gone by. The X bond now has nine years to maturity.
- If the interest rate in the market has risen to 10 percent, what will the bond be worth?

Bond price at discount

- we repeat the present value calculations with 9 years instead of 10, and a 10 percent yield instead of an 8 percent yield.
- Therefore, the bond should sell for about $885.
- We say that this bond, with its 8 percent coupon, is priced to yield 10 percent at $885.

$$\text{Present value} = \$1{,}000/1.10^{9} = \$1{,}000/2.3579 = \$424.10$$

$$\begin{aligned}\text{Annuity present value} &= \$80 \times (1 - 1/1.10^{9})/.10 \\ &= \$80 \times (1 - 1/2.3579)/.10 \\ &= \$80 \times 5.7590 \\ &= \$460.72\end{aligned}$$

$$\text{Total bond value} = \$424.10 + 460.72 = \$884.82$$

Bond value/price at discount

- The X Co. bond now sells for less than its $1,000 face value. Why?
- The market interest rate is 10 percent. Considered as an interest-only loan of $1,000, this bond only pays 8 percent, its coupon rate.
- Because this bond pays less than the going rate, investors are only willing to lend something less than the $1,000 promised repayment.
- Because the bond sells for less than face value, it is said to be a *discount bond*
- The only way to get the interest rate up to 10 percent is to lower the price to less than $1,000 so that the purchaser, in effect, has a built-in gain.
- For the X bond, the price of $885 is $115 less than the face value, so an investor who purchased and kept the bond would get $80 per year and would have a $115 gain at maturity as well. This gain compensates the lender for the below-market coupon rate.

Bond value/price at discount

- Another way to see why the bond is discounted by $115 is to note that the $80 coupon is $20 below the coupon on a newly issued par value bond, based on current market conditions.
- The bond would be worth $1,000 only if it had a coupon of $100 per year.
- In this sense, an investor who buys and keeps the bond gives up $20 per year for nine years.
- At 10 percent, this annuity stream is worth $115 and this is the amount of discount

$$\begin{aligned}\text{Annuity present value} &= \$20 \times (1 - 1/1.10^{9})/.10 \\ &= \$20 \times 5.7590 \\ &= \$115.18\end{aligned}$$

Bond value at Premium

- What would the X bond sell for if interest rates had dropped by 2 percent instead of rising by 2 percent?
- The bond would sell for more than $1,000. Such a bond is said to sell at a *premium* and is called a *premium bond.*

Bond value at Premium

- This case is the opposite of that of a discount bond. The X bond has a coupon rate of 8 percent when the market rate is now only 6 percent. Investors are willing to pay a premium to get this extra coupon amount.
- In this case, the relevant YTM is 6 percent, and there are nine years remaining.
- The present value of the bond or bond price is:

Bond value at Premium

- This case is the opposite of that of a discount bond. The X bond has a coupon rate of 8 percent when the market rate is now only 6 percent. Investors are willing to pay a premium to get this extra coupon amount.
- In this case, the relevant YTM is 6 percent, and there are nine years remaining.
- The present value of the bond or bond price is $1136.03

$$\text{Present value} = \$1{,}000/1.06^{9} = \$1{,}000/1.6895 = \$591.90$$

$$\begin{aligned}\text{Annuity present value} &= \$80 \times (1 - 1/1.06^{9})/.06 \\ &= \$80 \times (1 - 1/1.6895)/.06 \\ &= \$80 \times 6.8017 \\ &= \$544.14\end{aligned}$$

$$\text{Total bond value} = \$591.90 + 544.14 = \$1{,}136.03$$

Bond value at Premium

- The total bond value is therefore about $136 in excess of par value.
- We can verify this amount by noting that the coupon is now $20 high, based on current market conditions.
- The present value of $20 per year for nine years at 6 percent is $136.03

$$\begin{aligned}\text{Annuity present value} &= \$20 \times (1 - 1/1.06^{9})/.06 \\ &= \$20 \times 6.8017 \\ &= \$136.03\end{aligned}$$