Introduction to Finance

L12

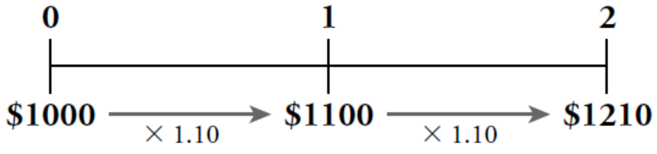
Rules of Time Value-1) compare and combining values

- 1. It is only possible to compare or combine values at the same point in time
  - A rupee today and rupee in future are not equivalent (inflation effect)
  - Having money now is more valuable than having money in the future (interest income and inflation)
  - Only cash flows in the same unit/same point of time can be compared or combined
  - Hence to compare cash flows at different points of time-convert them to same unit or move them to same point in time

Rules of Time Value-2) Moving cash flows forward in time

2) To move cash flow forward in time, we have to **compound** it.

- If the market interest rate for the year is *r*, then we multiply by the interest rate factor, (1 + *r*), to move the cash flow from the beginning to the end of the year. This process of moving a value or cash flow forward in time is known as **compounding**.
- Note that the value grows as we move the cash flow further in the future. Note also that the equivalent value (i.e. $1000) grows by $100 the first year, but by $110 the second year. In the second year we earn interest on our original $1000, plus we earn interest on the $100 interest we received in the first year. This effect of earning "interest on interest" is known as **compound interest**
- The difference in value between money today and money in the future represents the **time value of money**, and it reflects the fact that by having money sooner, you can invest it and have more money later as a result



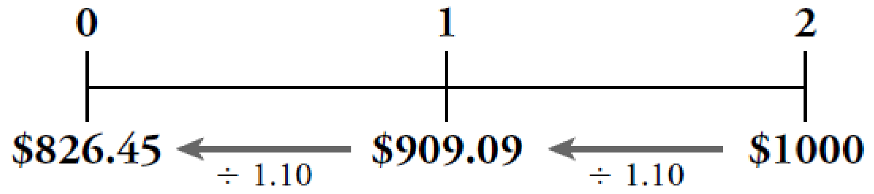
Future Value of a Cash Flow

$$FV_n = C \times \underbrace{(1+r) \times (1+r) \times \cdots \times (1+r)}_{} = C \times (1+r)^n$$

Rules of Time Value-3) Moving cash flows Back in time

3) To move cash flow back in time we must **discount** it

- Thie process of moving a value or cash flow backward in time—finding the equivalent value today of a future cash flow—is known as **discounting**.
- They represent the same value in different units(different points in time).
- In general, to move a cash flow *C* backward *n* periods, we must discount it by the *n* intervening interest rate factors.

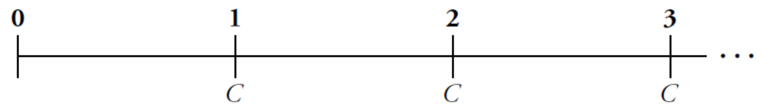


Present Value of a Cash Flow

$$PV = C \div (1 + r)^n = \frac{C}{(1 + r)^n}$$

Perpetuities

- A **perpetuity** is a stream of equal cash flows that occur at regular intervals and last forever.
- Example: British government bond called a **consol.** Consol bonds promise the owner a fixed cash flow every year, forever.
- Note from the timeline that the first cash flow does not occur immediately; *it arrives at the end of the first period*. This timing is sometimes referred to as payment *in arrears*
- Suppose we invest an amount *P* in the bank. Every year we can withdraw the interest we have earned, $C = r * P$, leaving the principal, *P*, in the bank. The present value of receiving *C* in perpetuity is therefore the upfront cost $P = C/r$.



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots = \sum_{n=1}^{\infty} \frac{C}{(1+r)^n}$$

Present Value of a Perpetuity

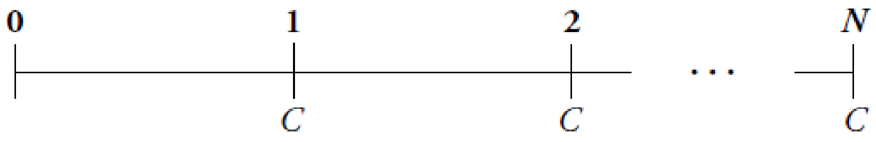
$$PV(C \text{ in perpetuity}) = \frac{C}{r}$$

Example:

- You want to endow an annual MBA graduation party at your alma mater. You budget \$30,000 per year forever for the party. If the university earns 8% per year on its investments, and if the first party is in one year's time, how much will you need to donate to endow the party?

Annuities

- An **annuity** is a stream of *N* equal cash flows paid at regular intervals.
- The difference between an annuity and a perpetuity is that an annuity ends after some fixed number of payments. Most car loans, mortgages, and some bonds are annuities.
- We represent the cash flows of an annuity on a timeline as follows.



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{C}{(1+r)^N} = \sum_{n=1}^{N} \frac{C}{(1+r)^n}$$

$$PV(\text{annuity of } C \text{ for } N \text{ periods with interest rate } r) = C \times \frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)$$

Annuities

- You are the winner of the $30 million state lottery. You can take your prize money either as
- (a) 30 payments of $1 million per year (starting today), or
- (b) $15 million paid today.
- If the interest rate is 8%, which option should you take?

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Solution:

a) N=29 (as first payment is today)

$$PV(\text{29 yr annuity of \$1 million/yr}) = \$1 \text{ million} \times \frac{1}{.08}\left(1 - \frac{1}{1.08^{29}}\right)$$

$$= \$11.16 \text{ million today}$$

Adding the $1 million we receive upfront, this option has a present value of $12.16 million

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Solution:

- b) if you have the $15 million today, you can use $1 million immediately and invest the remaining $14 million at an 8% interest rate. This strategy will give you $14 million * 8% = $1.12 million per year in perpetuity
- Or Alternatively, you can spend $15 million - $11.16 million = $3.84 million today, and invest the remaining $11.16 million, which will still allow you to withdraw $1 million each year for the next 29 years.
- Hence option b is better.

Annuities: Future value

- Future value of annuity is useful if we want to know how a savings account will grow over time.

Future Value of an Annuity

$$FV(\text{annuity}) = PV \times (1+r)^N$$

$$= \frac{C}{r}\left(1 - \frac{1}{(1+r)^N}\right) \times (1+r)^N$$

$$= C \times \frac{1}{r}\left((1+r)^N - 1\right)$$

Annuities: Future value

- Paul is 35 years old, and he has decided to invest in retirement plan. At the end of each year until he is 65, he will save $10,000 in a retirement account. If the account earns 10% per year, how much will Paul have saved at age 65?

Future Value of an Annuity

$$
\begin{aligned}
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$$

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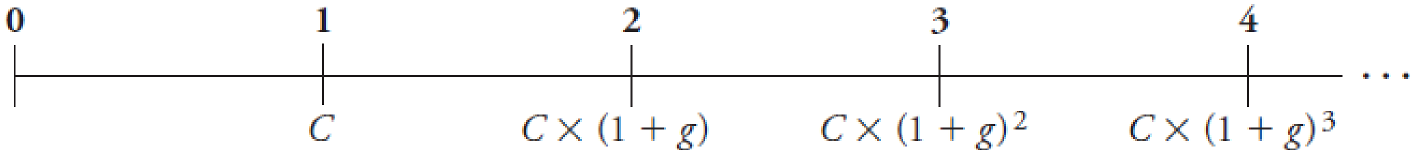
$$FV = \$10{,}000 \times \frac{1}{0.10}(1.10^{30} - 1)$$

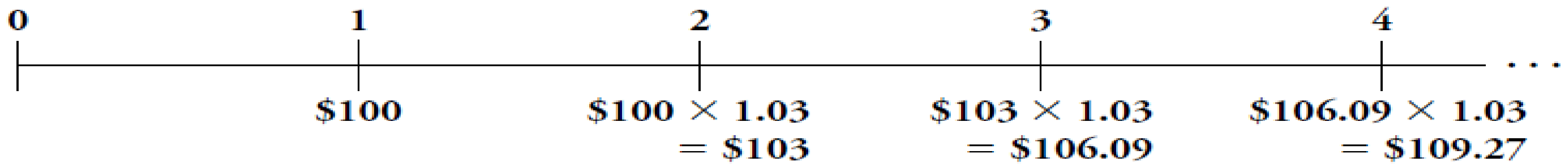
$$= \$10{,}000 \times 164.49$$

$$= \$1.645 \text{ million at age } 65$$

Growing perpetuity

- A **growing perpetuity** is a stream of cash flows that occur at regular intervals and grow at a constant rate forever
- Condition g<r





Present Value of a Growing Perpetuity

$$PV(\text{growing perpetuity}) = \frac{C}{r - g}$$

Growing Annuity

Present Value of a Growing Annuity

$$PV = C \times \frac{1}{r-g}\left(1 - \left(\frac{1+g}{1+r}\right)^N\right)$$

- A **growing annuity** is a stream of *N* growing cash flows, paid at regular intervals. It is a growing perpetuity that eventually comes to an end.
- The present value of an *N*-period growing annuity with initial cash flow *C*, growth rate *g*, and interest rate *r*
- (1) The first cash flow arrives at the end of the first period, and (2) the first cash flow does not grow. The last cash flow therefore reflects only *N* - 1 periods of growth.

Example

- In the previous example of retirement plan Paul considered saving \$10000 per year of his retirement. Although \$10,000 is the most he can save in the first year, he expects his salary to increase each year so that he will be able to increase his savings by 5% per year. With this plan, if he earns 10% per year on his savings, how much will Paul have saved at age 65?

Annuity payment: loan payment

- In general, when solving for a loan payment, think of the amount borrowed (the loan principal) as the present value of the payments when evaluated at the loan rate.
- If the payments of the loan are an annuity, we can solve for the payment of the loan by inverting the annuity formula.

Loan or Annuity Payment

$$C = \frac{P}{\frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)}$$

Example

- A biotech firm plans to buy a new machinery for $500,000. The firm pay 20% of the purchase price as a down payment and finance the remainder by taking a 48-month loan with equal monthly payments and an interest rate of 0.5% per month. What is the monthly loan payment?

- C= $9394

Internal rate of return (IRR)

- IRR is the interest rate that sets the net present value (NPV) of the cash flow equal to zero
- For bond, IRR=yield