MAL100: Mathematics I

Tutorial Sheet 1: Set Theory, Relation and Mapping

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1. If A, B, C be subsets of $\mathbb{R}$, prove that $A \cap (B \triangle C) = (A \cap B) \triangle (A \cap C)$.

2. Let S be a universal set and A be a fixed subset of S.
   (a) If $A \cup B = B$ for all $B \subseteq S$, prove that $A = \phi$.
   (b) If $A \cap B = B$ for all $B \subseteq S$, prove that $A = S$.

3. Let A, B be two subsets of a universal set S. Prove that $A = B$ if and only if $A \triangle B = \phi$.

4. Let S be the set of all positive divisors of 30. Prove that $(S, \leq)$ is a poset where $a \leq b$ means a is a divisor of b, for $a, b \in S$.

5. Let S be the set of all lines in $\mathbb{R}^3$. A relation ρ is defined on S by $L_1 \, \rho \, L_2$ if and only if $L_1 \perp L_2$. Discuss whether ρ is (a) reflexive, (b) symmetric, (c) transitive.

6. (a) A relation ρ is defined on $\mathbb{Z}$ by $a \, \rho \, b$ if and only if $ab > 0$. Discuss whether ρ is (a) reflexive, (b) symmetric, (c) transitive.
   (b) Prove or disprove: Let ρ be a symmetric as well as transitive relation. Then ρ is a equivalence relation.

7. Prove that the functions $f : \mathbb{R} \to \mathbb{R}$ are neither injective nor surjective:
   (a) $f(x) = \dfrac{|x|}{|x| + 1}, \; x \in \mathbb{R}$,
   (b) $f(x) = \lfloor x \rfloor, \; x \in \mathbb{R}$.

8. $f : D \to \mathbb{R}$ and $g : D \to \mathbb{R}$ defined by

$$f(x) = \sin x - \cos x, \; x \in D \text{ and } g(x) = \sqrt{1 - \sin 2x}, \; x \in D$$

   where $D = \{x \in \mathbb{R} : 0 \leq x \leq \frac{\pi}{2}\}$. Are f and g equal? Give reason.

9. Let $f : A \to B, g : B \to C$ and $h : B \to C$ be mappings such that $g \circ f = h \circ g$ and f is surjective. Prove that $g = h$.

10. Let $g : A \to B, h : A \to B$ and $f : B \to C$ be mappings such that $f \circ g = f \circ h$ and f is injective. Prove that $g = h$.