

MAL100: Mathematics I

Tutorial Sheet 10

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1. Find the limits of the following functions:

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{e^y \sin x}{x}.$

(b) $\lim_{(x,y) \rightarrow (\pi,0)} \frac{1 - \sin y}{y + \cos x}.$

(c) $\lim_{\substack{(x,y) \rightarrow (1,1) \\ x \neq y}} \frac{x^2 - y^2}{x + y}.$

(d) $\lim_{\substack{(x,y) \rightarrow (0,0) \\ x \neq y}} \frac{\sqrt{x} + \sqrt{y}}{x + 2\sqrt{x} - y + 2\sqrt{y}}.$

(e) $\lim_{\substack{(x,y) \rightarrow (4,3) \\ x \neq y+1}} \frac{\sqrt{x} - \sqrt{y+1}}{x - y - 1}.$

(f) $\lim_{(x,y,z) \rightarrow (\frac{-1}{4}, \frac{\pi}{2}, 2)} \tan^{-1}(xyz).$

(g) $\lim_{(x,y,z) \rightarrow (\pi, 0, 3)} ze^{-2y} \cos 2x.$

2. At what points (x, y) in the plane are the following functions continuous:

(a) $f(x, y) = \ln(x^2 + y^2).$

(b) $f(x, y) = \frac{x^2 + y^2}{x^2 - y^2}.$

(c) $f(x, y) = \frac{x^2 y}{x^2 + y^2}.$

3. At what points (x, y, z) in the space are the following functions continuous:

(a) $h(x, y, z) = xy \cos \frac{1}{z}.$

(b) $h(x, y, z) = \frac{1}{|y| + |z|}.$

(c) $h(x, y, z) = \frac{1}{|xy| + |z|}.$