Department of Mathematics
Indian Institute of Technology Bhilai
IC104: Linear Algebra-I
Quiz-1 Answers

**Note: Text in blue color is the answer of the question. The marking scheme for descriptive questions is mentioned in red color against each question. All the questions except 2 and 7 are of 2 marks each. Questions 2 and 7 are of 4 marks each.**

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1. Please, take selfie for attendance. This is mandatory and may lead to non evaluation of your answers if you fail to upload.

2. Describe all the row reduced echelon forms of a $3 \times 2$ matrix.
   Ans: We have the following 4 types of RREs possible for a $3 \times 2$ matrix
   $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & \star \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$
   where $\star$ denotes any arbitrary element of the field.
   Distribution of marks:
   - Full marks only if all above types are mentioned
   - No mark if incorrect types of RRE forms or less RRE forms except zero matrix are mentioned
   - 1 mark for each of the cases, either you missed zero matrix or you have particular RRE, will be deducted.

3. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be an invertible matrix. Then choose the correct options.
   (a) If $a = 0$, then RRE of $A$ has a zero row.
   (b) Rank of $A$ is 2
   (c) RRE of $A$ is an identity matrix
   (d) The system $Ax = 0$, has a nontrivial solution

4. Let $ax + y = 1,\ bx + y = c$ be a system of two equations in two variables, where $a, b, c$ are non zero real numbers. Then which of the following options are correct
   (a) System is always consistent
   (b) System has unique solution if $a \neq b$

(c) $(0, 0)$ is not a solution of the system

(d) If $a = b = c$, then system will have infinitely many solutions

5. Let $A = \begin{bmatrix} 1 & -4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then which of the following statements are correct

(a) $A$ is invertible

(b) Inverse of $A$ is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $A$ is an elementary matrix

(d) $A$ is in RRE form

6. Let $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = A \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$, then the matrix $A$ will be equal to

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

7. Let $V = \{(a, b, c) \in \mathbb{R}^3 : a + b + c = 0\}$ be a subspace of $\mathbb{R}^3$. Find the dimension and basis of $V$.

Ans: Let $(x, y, z) \in V$, then $x + y + z = 0$ implies $x = -y - z$ and hence a general element of $V$ can be written as $(x, y, z) = (-y - z, y, z) = y(-1, 1, 0) + z(-1, 0, 1)$. Therefore, $V$ is spanned by $(-1, 1, 0), (-1, 0, 1)$ which is a linearly independent subset of $V$. Hence basis of $V$ is $\{(-1, 1, 0), (-1, 0, 1)\}$ and dimension is 2.

Distribution of marks:

- If basis is correct then 3 marks
- If dimension is mentioned correct then 1 mark
- Full marks iff both basis and dimension are correct
- No mark if both dimension and basis are wrong

8. Let $V$ be the vector space of all $2\times 2$ matrices over $\mathbb{R}$ . Let $W_1 = \left\{ A \in V : A = \begin{bmatrix} x & y \\ z & 0 \end{bmatrix} \right\}$ and $W_2 = \left\{ A \in V : A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \right\}$ , where $x, y, z, a, b$ are arbitrary scalars in $\mathbb{R}$ . Then select the correct statements from the following options

   (a) $W_1$ and $W_2$ are sub spaces of $V$

   (b) $V = W_1 + W2$

   (c) $V$ is a direct sum of $W_1$ and $W_2$

   (d) $W_1 \cap W_2$ is a one dimensional subspace of $V$.

9. Which of the following statements are correct

   (a) The only sub spaces of $\mathbb{R}$ are $\mathbb{R}$ and zero subspace

   (b) The only subspaces of $\mathbb{R}^2$ are zero subspace, $\mathbb{R}$ and $\mathbb{R}^2$

   (c) A spanning set of a $n-$dimensional vector space can have more than $n$ elements

   (d) Let $V$ be a $n$dimensional vector space , then a linearly independent subset of $V$ having $n$ elements is a basis of $V$