Tutorial 12: Probability and Statistics (MAL403/IC105)

Indian Institute of Technology Bhilai

---

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Pareto population with density $f_X(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, x > \alpha, \alpha > 0, \beta > 2$. Find method of moment estimator of α and β.

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $U(-\theta, \theta)$ population. Find the MME of θ.

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from an $Exp(\mu, \sigma)$ population. Find the method of moment estimators (MMEs) of μ and σ.

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$.

   (a) Find MLE of μ when σ is known.

   (b) Find MLE of μ when σ is known with $\mu \geq \mu_0$.

   (c) Find MLE of μ when σ is known with $\mu \leq \mu_0$.

   (d) Find MLE of μ when σ is known with $a \leq \mu \leq b$.

5. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Find MLE of σ^2 when μ is known.

6. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Find MLE and MME of μ and σ^2

7. Let $X_1, X_2, \ldots, X_n$ be a random sample from $Bin(m, p)$ population with m known, $0 < p < 1$. Find the MLE of $p(1-p)$.

8. Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with following uniform distributions (a) $U(0, \theta)$ (b) $U(0, 2\theta)$ (c) $U(\theta - 1, \theta + 1)$ (d) $U(\theta, \theta + 1)$. Find the MLE of θ for each case.

9. Let $X_1, X_2, ..., X_n$ be a random sample from a uniform $U(-\theta, 2\theta)$ population. Find an unbiased estimator of θ.

10. Let $X_1, X_2, ..., X_n$ be a random sample from a normal $N(\mu, \sigma^2)$ population. Find an unbiased estimator of $\frac{\mu}{\sigma}$ and $\mu + b\sigma$, where b is a real constant.

11. Find a 90% confidence interval for the mean of a normal distribution with $\sigma = 3$, given the sample $(3.3, -0.3, -0.6, -0.9)$. What would be the confidence interval if σ were unknown?

12. In pouring glass for use in automobile windshields uniformity of thickness is desirable to prevent distortion. If a random sample of 100 wind shields yields a sample standard deviation of 0.01 inch, construct a 95% confidence interval on the standard deviation in thickness.

13. Independent random samples of sizes 36 and 49 are taken from two normal populations have sample mean 5 and 3.5 respectively. The population standard deviations are 1.2 and 1.8 respectively. Based on these samples find a 90% confidence interval for the difference in the means of two populations.