Tutorial 13: Probability and Statistics (MAL403/IC105)

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1. Consider a normal population $N(\mu, \sigma^2)$. Which of the following is a simple hypothesis and which are composite hypotheses:

   (a) $H_0 : \mu > 2, \sigma^2 = 25$

   (b) $H_0 : \mu = 2$

   (c) $H_0 : \mu = 4, \sigma^2 = 9$

   (d) $H_0 : \mu = 2, \sigma^2 \neq 9$

   (3) $H_0 : \sigma^2 = 25$

2. A sample of size 1 is taken from a population distribution Poisson $P(\lambda)$. To test $H_0 : \lambda = 1$ against $H_1 : \lambda = 2$ consider the test

$$\phi(x) = \begin{cases} 1 & \text{if } x > 3 \\ 0 & \text{if } x \leq 3 \end{cases}$$

   Find the probabilities of type I and type II errors and the power power of the test.

3. The probability that a given coin will show up heads when tossed is p . In order to test $H_0 : p = 0.5$ against $H_1 : p = 0.75$, the coin was tossed 4 times independently. Let a test be : Reject H_0, if the number of heads is at least 3, and accept H_1 otherwise. Find the Probabilities of Type I and Type II errors.

4. Suppose $X_1, X_2, \ldots, X_{10}$ be a random sample form $N(\mu, 1)$ population. For testing $H_0 : \mu = 4$ vs. $H_1 : \mu = 7$ at level α, find the most powerful test.

5. A random sample of size 20 from a normal population gives a sample mean of 42 and sample standard deviation of 6. At 5% level of significance test the hypothesis that the population mean is more than 44. State clearly the null and alternative hypotheses.

6. An electrical firm manufactures batteries which have lifetimes normally distributed with mean μ and standard deviation 40 hours. What is the rejection region for $H_0 : \mu = 800$ vs $\mu > 800$ at 5% level based on a sample of size $n = 30$.

7. The life (in years) of a certain battery is normally distributed. A random sample of 16 batteries produced a sample variance $S^2 = 3$. Test the hypothesis $H_0 : \sigma^2 = 2$ vs $H_1 : \sigma^2 > 2$ at 5% level of significance.

8. Suppose X is random sample with p.d.f. $f(x|\theta) = \theta x^{\theta-1}$ for $0 < x < 1$ find the most powerful test with significance level $\alpha = 0.05$ for testing the null hypothesis $H_0 : \theta = 3$ against $H_1 : \theta = 4$.