Tutorial 4: Probability and Statistics (MAL403/IC105)

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1. Let X be a continuous type random variable with p.d.f.

$$f(x) = \begin{cases} k - |x|, & |x| < \frac{1}{2} \\ 0, & \text{Otherwise,} \end{cases}$$

where $k \in \mathbb{R}$.

   (a) Find the value of constant k;

   (b) Evaluate $P(X < 0),\ P(X \leq 0),\ P(0 < X \leq 1/4)\ P(-1/8 \leq X \leq 1/4)$

   (c) Find the distribution function of X.

2. Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1 - x^2), & -1 < x < 1 \\ 0, & \text{Otherwise.} \end{cases}$$

   (a) What is the value of c ?

   (b) Find the cumulative distribution function of X.

3. Consider a function $f(x)$ defined as

$$f(x) = \begin{cases} \frac{1}{\beta}\left(1 - \frac{|x-\alpha|}{\beta}\right), & \alpha - \beta < x < \alpha + \beta \\ 0, & \text{Otherwise,} \end{cases}$$

where $\alpha \in \mathbb{R},\ \beta > 0$.

   (a) Show that $f(x)$ is a probability density function.

   (b) Find the distribution function.

4. Let X be a random variable with distribution function F. Then find the distribution function fo $|X|,\ aX + b$, where $a \neq 0, b \in \mathbb{R},\ \max\{X, 0\}$ and $\min\{X, 0\}$.

5. Let X be a discrete random variable with p.m.f. $P(X = -2) = \frac{1}{5},\ P(X = -1) = \frac{1}{6},\ P(X = 0) = \frac{1}{5},\ P(X = 1) = \frac{1}{15}$ and $P(X = 2) = \frac{11}{30}$. Find the p.m.f. and d.f. of $Y = X^2$.

6. Let X be a random variable with pdf

$$f_X(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1 \\ 0, & \text{Otherwise.} \end{cases}$$

Find the distribution function of $Y = \max\{X, 0\}$.

7. The random variable X has pdf $f_X(x) = \frac{1}{2}e^{-|x|},\ -\infty < x < \infty$. Find the distribution of $Y = X^2$.

8. Suppose X have the density function

$$f_X(x) = \begin{cases} c(x+1), & -1 < x < 2 \\ 0, & \text{Otherwise.} \end{cases}$$

   Find the value of c. Hence calculate the pdf and cdf of $Y = X^2$.

9. Suppose X have the density function

$$f_X(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{Otherwise.} \end{cases}$$

   Find the density function of $Y = 40(1 - X)$.

10. Among the 15 applicants for a job, 9 are women and 6 are men. 5 applicants are randomly selected from the applicant pool for final interviews. Let X is the number of female applicants among the final 5. (i) Give the probability mass function for X . (ii) Define Y, the number of male applicants among the final 5, as a function of X . Find the probability mass function for Y.