Solⁿ of Tutorial - 13

① composite, simple, simple, composite, composite.

② $X \sim P(\lambda)$, $H_0 : \lambda = 1$

$H_1 : \lambda = 2$

The test is $\phi(x) = \begin{cases} 1, & x > 3 \\ 0, & x \leq 3 \end{cases}$

i.e. reject H_0 $x > 3$, not reject H_0 if $x \leq 3$.

$$\alpha = P(\text{Type - I error})$$

$$= P_{\lambda=1}(X > 3) = 1 - P_{\lambda=1}(X \leq 3)$$

$$= 1 - \left[\frac{e^{-1}}{0!} + \frac{e^{-1}}{1!} + \frac{e^{-1}}{2!} + \frac{e^{-1}}{3!}\right]$$

$$= 1 - \frac{11}{6} e^{-1}$$

$\beta = P(\text{Type-II error})$

$$= P_{\lambda=2}(X \leq 3) = \frac{25}{6} e^{-2}$$

Power of the test is $(1-\beta)$

(3) Let X denote the number heads

$$X \sim Bin(4, p)$$

$$H_0: p = 0.5 \quad \text{vs} \quad H_1: p = 0.75$$

We reject H_0 if $X \geq 3$

Prob of type-I error

$$\alpha = P_{p=0.5}(X \geq 3) = \binom{4}{3}(0.5)^3(0.5)$$

$$+ \binom{4}{4}(0.5)^4 \cdot 1.$$

$$= 0.3125$$

$$\beta = P(\text{type-II error}) = P_{p=0.75}(X < 3)$$

$$= 1 - P_{p=0.75}(X \geq 3)$$

$$= 1 - \left[\binom{4}{3} (0.75)^3 (0.25) + \binom{4}{4} (0.75)^4 \right]$$

④ Let $X_1, \ldots, X_{10} \sim N(\mu, 1)$

$H_0 : \mu = 4$

$H_1 : \mu = 7$

Do yourself. Similar problem we have done in the class.

⑤ $n = 20$, $\bar{X} = 42$. $\sigma = 6$

$H_0 : \mu \leq 44$

$H_1 : \mu > 44$

$$Z = \frac{\sqrt{20}\,(\bar{X} - 44)}{6}, \qquad \alpha = 0.05$$

Reject H_0 if $Z > z_\alpha$.

$$Z = \frac{\sqrt{20}\,(42-44)}{6} = -1.49$$

$$Z_{0.05} = 1.645.$$

So we will not reject H_0.

(6) $n = 30$, $\sigma = 40$.

$H_0: \mu = 800$ vs $H_1: \mu > 800$, $\alpha = 0.05$

$$Z = \frac{\sqrt{30}\,(\bar{x} - 800)}{40}$$

Reject H_0 if $Z > z_\alpha$

$$\frac{\sqrt{30}\,(\bar{x} - 800)}{40} > 1.64$$

$$\bar{x} > (40 \times 1.64)/\sqrt{30} + 800$$

Reject H_0 $\quad \bar{x} > \frac{40 \times 1.64}{\sqrt{30}} + 800$

⑦ $n = 16$, $\quad s^2 = 3$

$H_0: \sigma^2 = 2$ vs $H_1: \sigma^2 > 2$, $\alpha = 0.05$

Let $\quad W = \frac{(n-1)s^2}{\sigma^2}$.

We know reject H_0 if $W > \chi_{15, 0.05}$

$$\chi_{15, 0.05} = 24.996$$

$$W = \frac{15 \times 3}{2} = 22.5$$

So we do not reject H_0.

(8) $H_0: \theta = 3$ vs $H_1: \theta = 4$

$f_0 = 3x^2, \quad 0 < x < 1$

$f_1 = 4x^3, \quad 0 < x < 1$

Reject H_0 if $\dfrac{f_1(x)}{f_0(x)} > K$

$\Rightarrow \quad x > K_1$

Now K_1 will be determined by

$P(\text{type-I error}) = 0.05$

$P_{\theta=3}(X > K_1) = 0.05$

$\Rightarrow \displaystyle\int_{K_1}^{1} 3x^2 \, dx = 0.05$

$\Rightarrow \left[x^3\right]_{K_1}^{1} = 0.05$

$\Rightarrow K_1^3 = 1 - 0.05$

$\Rightarrow K_1 = (0.95)^{1/3}$

So MP test at level $\alpha = 0.05$ is

Reject H_0 if $X > (0.95)^{1/3}$.