Tutorial 3: Probability and Statistics (MAL403/IC105)

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1. Check the following functions are distribution functions are not

(i) $F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{2}, & 0 \leq x \leq \frac{3}{2} \\ 1, & x > \frac{3}{2} \end{cases}$ (ii) $F(x) = \begin{cases} 0, & x \leq 1 \\ 1 - \frac{1}{x^2}, & x > 1 \end{cases}$

(iii) $F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{1}{2} + \frac{e^{-x}}{2}, & x > 0 \end{cases}$ (iv) $F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{8}, & 0 \leq x < 1 \\ \frac{x+1}{8}, & 1 \leq x < 2 \\ \frac{2x+1}{8}, & 2 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$

2. Consider a function F as

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-x}, & x \geq 0 \end{cases}$$

(a) Show that F is a distribution function.

(b) Find the value of $P(2 < X \leq 3),\ P(-2 < X \leq 3),\ P(1 \leq X < 4),\ P(5 \leq X < 8)$.

3. Let X be a random variable with distribution function given as

$$F(x) = \begin{cases} 0, & x < 2 \\ \frac{2}{3}, & 2 \leq x < 5 \\ \frac{7-6k}{6}, & 5 \leq x < 9 \\ \frac{3k^2-6k+7}{6}, & 9 \leq x < 14 \\ \frac{16k^2-16k+19}{16}, & 14 \leq x \leq 20 \\ 1, & x > 20 \end{cases}$$

(a) Find the value of constant k;

(b) Show that the r.v. X is of discrete type and find its support;

(c) Find the p.m.f. of X.

4. Let X be a random variable with distribution function given as

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{4}, & 0 \leq x < 1 \\ \frac{x}{3}, & 1 \leq x < 2 \\ \frac{3x}{8}, & 2 \leq x < 5/2 \\ 1, & x \geq 5/2 \end{cases}$$

(a) Prove that X is neither continuous nor discrete random variable.

(b) Find the value of $P\left(1 < X \leq \frac{5}{2}\right)$, $P\left(1 < X < \frac{5}{2}\right)$, $P\left(1 \leq X < \frac{5}{2}\right)$, $P(-2 \leq X < 1)$, $P(X \geq 2)$, $P(X > 2)$.

5. A burnt out bulb was mistakenly placed in a box containing 3 good bulbs. In order to locate the bad bulb, the bulbs are randomly tested one by one, without replacement. Let X denote the number of bulbs tested to determine the bad bulb.

6. An guinea pig either dies (D) or survives (S) in the course of a surgical experiment. The experiment is to be performed first with two guinea pigs. If both survive, no further trials are to be made. If exactly one guinea pig survives, one more guinea pig is to undergo the experiment. If both guinea pigs die, two additional guinea pigs are to be tried. Assume that the trials are independent and that the probability of survival in each trial is $\frac{2}{3}$. Let X of the number of survivals and Y detonate the number of deaths. Find the p.m.f. of X and Y.

7. Let $F : \mathbb{R} \to \mathbb{R}$ defined by

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x+4}{16}, & 0 \leq x < 2 \\ \frac{x^2+4b}{32}, & 2 \leq x < 4 \\ \frac{x+c}{8}, & 4 \leq x \leq 6 \\ 1, & x > 6 \end{cases}$$

where b and c are real constants.

(a) Find the value of b and c so that F is a distribution function of some random variable.

(b) Show that the random variable is neither discrete nor continuous.