Tutorial-2 Solution

1) (i) $F\left(\frac{3}{2}\right) = \frac{3}{4} \neq F(3/2+) = 1 \Rightarrow F(x)$ is not right continuous at $\frac{3}{2}$. so $F(x)$ is not a cdf

(ii) $F(1-) = F(1) = F(1+) = 1-1 = 0$.

So $F(x)$ is continuous everywhere hence right continuous.

$F(x)$ is non-decreasing

$\lim_{x\to-\infty} F(x) = 0$, $\lim_{x\to\infty} F(x) = 1$.

$\Rightarrow F(x)$ is a c.d.f.

(iii) $F(0) = 0$, $F(0+) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

So $F(0) \neq F(0+) \Rightarrow$ not right continuous

So F is not a cdf.

(iv) $F(0) = 0$, $F(0+) = 0 \Rightarrow F(0) = F(0+)$

$F(1) = \frac{1+1}{8} = \frac{1}{4} = F(1+)$

$F(2) = \frac{4+1}{8} = 5/8 = F(2+)$

$F(3) = F(3+) = 1$

So F is right continuous at $0, 1, 2, 3$

Also $F(x)$ is continuous on $(-\infty, 1)$, $(1,2)$, $(2,3)$ $(3, \infty)$.

So F is right continuous on $\mathbb{R}$.

F is non-decreasing in $(-\infty, 0)$, $(0,1)$, $(1,2)$, $(2,3)$ & $(3,\infty)$. Also

$$F(0) - F(0-) = 0 \geq 0.$$

$$F(1) - F(1-) = \frac{1}{4} - \frac{1}{8} > 0$$

$$F(2) - F(2-) = \frac{5}{8} - \frac{1}{4} > 0$$

$$F(3) - F(3-) = 1 - 5/8 > 0$$

So $F(x)$ is non-decreasing in $\mathbb{R}$.

$$\lim_{x \to -\infty} F(x) = 0, \quad \lim_{x \to \infty} F(x) = 1.$$

$\Rightarrow F(x)$ is a c.d.f.

(2) $$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-x}, & x \geq 0 \end{cases}$$

$$\lim_{x \to -\infty} F(x) = 0, \quad \lim_{x \to \infty} F(x) = 1$$

Since $F(x)$ is continuous everywhere so right continuous

$F(x)$ is also non-decreasing

$\Rightarrow F(x)$ is a c.d.f.

$P(2 < X \le 3) = F(3) - F(2) = e^{-2} - e^{-3}$

$P(-2 < X \le 3) = 1 - e^{-3}$

$P(1 \le X < 4) = F(4) - F(1) = e^{-1} - e^{-4}$

$P(5 \le X < 8) = F(8) - F(5) = e^{-5} - e^{-8}$

(3) Since F is right continuous so

$$F(20) = F(20+)$$

$$\Rightarrow \quad 16k^2 - 16k + 3 = 0 \quad \Rightarrow k = \frac{1}{4} \text{ or } k = \frac{3}{4} \quad (1)$$

Also F is non-decreasing

$$F(5-) \le F(5) \Rightarrow \frac{2}{3} \le \frac{7}{6} - k$$

$$\Rightarrow k \le \frac{7}{6} - \frac{2}{3} = \frac{1}{2} \quad (2)$$

From (1) & (2) we have $k = \frac{1}{4}$.

$$F(x) = \begin{cases} 0, & x < 2 \\ 2/3 & 2 \le x < 5 \\ \frac{11}{12}, & 5 \le x < 9 \\ \frac{91}{96}, & 9 \le x < 14 \\ 1, & x \ge 14 \end{cases}$$

The set of discontinuity points $D = \{2, 5, 9, 14\}$

Moreover

$P(X=2) = F(2) - F(2-) = 2/3$

$P(X=5) = F(5) - F(5-) = \frac{1}{4}$

$P(X=9) = F(9) - F(9-) = \frac{1}{32}$

$P(X=14) = F(14) - F(14-) = 5/96$

$P(X \in D) = P(X=2) = P(X=5) + P(X=9) + P(X=14)$

$= 1$

so X is a discrete r.v. with support

$S = \{2, 5, 9, 14\}$

p.m.f of X given as

$$f_X(x) = P(X=x) = \begin{cases} 2/3, & x=2 \\ 1/4, & x=5 \\ 1/32, & x=9 \\ \frac{5}{96}, & x=14 \\ 0, & o/w. \end{cases}$$

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(4) The set of discontinuity points of $F(x)$ are

$D = \{1, 2, 5/2\}$. Since $D \neq \phi$ so the r.v. is not of continuous type.

$P(X \in D) = P(X=1) + P(X=2) + P(X=5/2)$

$= F(1) - F(1-) + F(2) - F(2) + F(5/2) - F(5/2-)$

$= \frac{11}{48} < 1 \Rightarrow X$ is not a discrete r.v.

(b)

$P(1 < X \le 5/2) = F(5/2) - F(1) = 1 - \frac{1}{3} = 2/3$

$P(1 < X < 5/2) = F(5/2-) - F(1) = \frac{15}{16} - \frac{1}{3} = \frac{29}{48}$

$P(1 \le X < 5/2) = F(5/2-) - F(1-) = \frac{15}{16} - \frac{1}{4} = \frac{11}{16}$

$P(-2 \le X < 1) = F(1-) - F(-2-) = \frac{1}{4} - 0 = \frac{1}{4}$

$P(X \geq 2) = 1 - F(2-) = 1 - \frac{1}{3} = \frac{1}{3}$

$P(X > 2) = 1 - F(2) = 1 - \frac{3}{4} = \frac{1}{4}$.

(5) Let X denote the number of bulbs to be tested

$X \to 1, 2, 3$

$P(X=1) = \frac{1}{4}$, $P(X=2) = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$

$P(X=3) = \frac{1}{2}$.

$$F_X(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{4}, & 1 \le x < 2 \\ \frac{1}{2}, & 2 \le x < 3 \\ 1, & x \geq 3 \end{cases}$$

(6) Let S denote a survival and D denote a death of a guinea pig during the trial. Then the sample space is

$$\Omega = \{(S,S), (D,S,S), (D,S,D), (S,D,S), (S,D,D), (D,D,S,S), (D,D,D,S), (D,D,S,D), (D,D,D,D)\}$$

Let X = the number of survivors

Y = the number of deaths

$$P(X=2) = P(\{(S,S), (S,D,S), (D,S,S), (D,D,S,S)\})$$

$$= \frac{64}{81}$$

$$P(X=1) = 16/81$$

$$P(X=0) = 1/81$$

So p.m.f. of X is $p_X(x) = \begin{cases} \frac{1}{81}, & x=0 \\ \frac{16}{81}, & x=1 \\ \frac{64}{81}, & x=2 \\ 0, & o/w \end{cases}$

$$P(Y=0) = 4/8, \quad P(X=1) = \frac{8}{27}$$

$$P(Y=2) = \frac{16}{81}, \quad P(Y=3) = \frac{4}{81}$$

$$P(Y=4) = \frac{1}{81}$$

so p.m.f Y is

$$p_Y(y) = \begin{cases} 4/8, & x=0 \\ 8/27, & x=1 \\ 16/81, & x=2 \\ 4/81, & x=3 \\ 1/81, & x=4 \\ 0, & o/w. \end{cases}$$

7 a $F(6) = F(6^+) = \frac{6+c}{8} = 1 \Rightarrow c = 2$

$F(2^-) \leq F(2) \Rightarrow \frac{6}{16} \leq \frac{1+b}{8} \Rightarrow b \geq 2$

$F(4-) \leq F(4) \Rightarrow \frac{4+b}{8} \leq \frac{4+c}{8} \Rightarrow b \leq 2$

$\Rightarrow b = 2.$

(b) Do yourself.