

Tutorial 10: Probability and Statistics (MAL403/IC105)

Indian Institute of Technology Bhilai

1. X_1, X_2, X_3, X_4, X_5 independent random variables with $X_1 \sim N(200, 8)$, $X_2 \sim N(104, 8)$, $X_3 \sim N(108, 15)$, $X_4 \sim N(120, 15)$ and $X_5 \sim N(210, 15)$. Let $U = \frac{X_1+X_2}{2}$ and $V = \frac{X_3+X_4+X_5}{3}$. Then find the value of $P(U > V)$.
2. (a) Six fair dice are thrown independently. Let S denote the number of dice showing even numbers on their upper faces. Then find the mean and the variance of S .
(b) Let X_1 and X_2 be independent random variables with respective moment generating function $M_1(t) = \left(\frac{3}{4} + \frac{1}{4}e^t\right)^3$ and $M_2(t) = e^{2(e^t-1)}$. Find the value of $P(X_1 + X_2 = 1)$.
3. Let X and Y be jointly distributed random variables with $E(X) = E(Y) = 0$, $E(X^2) = E(Y^2) = 2$ and $\text{Corr}(X, Y) = 1/3$. Find $\text{Corr}\left(\frac{X}{3} + \frac{2Y}{3}, \frac{2X}{Y} + \frac{Y}{3}\right)$.
4. Let X_1, \dots, X_n be random variable with $E(X_i) = \mu_i$ and $\text{Var}(X_i) = \sigma_i^2$ and $\rho_{i,j} = \text{Corr}(X_i, X_j)$, $i, j = 1, \dots, n$, $i \neq j$. For real numbers a_i, b_i , $i = 1, 2, \dots, n$ define $Y = \sum_{i=1}^n a_i X_i$ and $Z = \sum_{i=1}^n b_i X_i$. Find $\text{Cov}(Y, Z)$.
5. Let $(x_i, y_i) \in \mathbb{R}^2$, $i = 1, 2, \dots, n$ be such that $\sum x_i = \sum y_i = 0$. Using statistical argument show that $(\sum x_i y_i)^2 \leq (\sum x_i^2)(\sum y_i^2)$.
6. Let (X_1, X_2) have the joint p.m.f.

$$p_{X_1, X_2}(x_1, x_2) = \begin{cases} \left(\frac{2}{3}\right)^{x_1+x_2} \left(\frac{1}{3}\right)^{2-x_1-x_2}, & (x_1, x_2) = (0, 0), (0, 1), (1, 1) \\ 0, & \text{Otherwise} \end{cases}$$

- (a) Find the joint p.m.f. of $Y_1 = X_1 - X_2$ and $Y_2 = X_1 + X_2$; (b) Find the marginal p.m.f.s of Y_1 and Y_2 ; (c) Find $\text{Var}(Y_2)$ and $\text{Cov}(Y_1, Y_2)$; (d) Are Y_1 and Y_2 independent?
7. Let (X_1, X_2, X_3) be a random vector having joint p.d.f.

$$f_{X_1, X_2, X_3}(x_1, x_2, x_3) = \begin{cases} 2e^{-(x_2+2x_3)}, & 0 < x_1 < 1, x_2 > 0, x_3 > 0 \\ 0, & \text{Otherwise} \end{cases}$$

- (a) Find the marginal p.d.f.s of X_1 , X_2 and X_3 ; (b) Are X_1, X_2, X_3 independent?; (c) Are $X_1 + X_2$ and X_3 independent.? (d) Find the conditional p.d.f. of X_1 given $X_2 = 2$.
8. Let (X, Y) have bivariate normal distribution with density function

$$f_{X,Y}(x, y) = \frac{1}{\pi\sqrt{3}} \exp\left[-\frac{2}{3}(x^2 - xy + y^2)\right], -\infty < x, y < \infty$$

- (a) Find $E(X)$, $E(Y)$, $\text{Var}(X)$, $\text{Var}(Y)$ and $\rho_{X,Y}$. (b) Find $P(-1 < X < 1 | Y = 1)$, $V(2X + 3Y)$ and $P(-5 < 2X + 3Y < 8)$.