Tutorial 8: Probability and Statistics (MAL403/IC105)

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1. A box contain 5 white balls numbered 0, 1, 2, 3, 4; 4 red balls numbered 0,1,2,3; 3 black balls numbered 0, 1, 2 and 2 blue balls numbered 0,1. A ball was drawn at random from the bag. Let the random variables X and Y defined as follows: X takes values 0, 1, 2, 3 respectively for white, red, black and blue balls. Y denotes the numbered of the ball. Find the joint distribution of X and Y. Also find the marginal distribution of X and Y.

2. Check whether the following functions are joint cdf of some random vector or not.

   (a)
   $$F(x, y) = \begin{cases} 1, & \text{if } x + 2y \geq 1 \\ 0, & \text{if } x + 2y < 1 \end{cases}$$

   (b)
   $$F(x, y) = \begin{cases} 0, & \text{if } x < 0 \text{ or } x + y < 1 \text{ or } y < 0 \\ 1, & \text{otherwise} \end{cases}$$

3. Let (X, Y) be a random vector having joint p.m.f.

   $$p(x, y) = \begin{cases} c(x + 2y), & \text{if } x = 1, 2, \ y = 1, 2 \\ 0, & \text{otherwise}, \end{cases}$$

   where c is a real constant. (a) Find the value of c (b) Find the marginal p.m.f. of X and Y (c) Are X and Y independent (d) Find the conditional p.m.f. of X given $Y = 2$

4. The joint p.m.f. of (X, Y) is given as

   $$p(x, y) = \begin{cases} cxy, & \text{if } x = 1, 2, \ y = 1, 2, \ x \leq y \\ 0, & \text{otherwise}, \end{cases}$$

   where c is a real constant. (a) Find the value of c (b) Find the marginal p.m.f. of X and Y (c) Find the conditional p.m.f. of X given $Y = y$ (d) Find the conditional p.m.f. of Y given $X = x$ (e) Find $P(X > Y), P(X = Y), P\left(X < \frac{2}{3}Y\right), P(X + Y \geq 3)$

5. The joint pmf of a discrete random vector (X, Y) given as

X/Y	-1	0	1
0	0	$1/3$	0
1	$1/3$	0	$1/3$

   (i) Find the marginal distribution of X and Y. (ii) Are X and Y independent?

6. Let (X, Y) be a discrete random vector with p.m.f.

$$p(x, y) = \begin{cases} cy, & \text{if } x \in \{1, 2, 3\}, \ y \in \{1, 2, 3, 4\} \ \& \ x \leq y; \\ 0, & \text{otherwise}; \end{cases}$$

(a) Find value of the constant c; (b) Find the marginal p.m.f.s of X and Y; (c) Find $P(X + Y > 4)$.

7. The joint distribution function of (X, Y) is given as

$$F(x, y) = \begin{cases} 0, & \text{if } x < 0 \text{ or } \ y < 0 \\ \frac{1+xy}{2} & \text{if } 0 \leq x < 1, \ 0 \leq y < 1 \\ \frac{1+x}{2} & \text{if } 0 \leq x < 1, \ y > 1 \\ \frac{1+y}{2} & \text{if } x > 1, \ 0 \leq y < 1 \\ 1 & \text{if } x > 1, \ y > 1 \end{cases}$$

(a) Find the marginal distribution function of X and Y. (b) Find $P(1/2 \leq X \leq 1, \ 1/4 < Y < 1/2)$, $P(X = 1)$ and $P(X \geq 3/2, Y < 1/4)$.