

Quiz-1: Probability and Statistics (MAL403/IC105)

Indian Institute of Technology Bhilai

Name:

Roll:

Marks: 10, Time: 1hr

Multiple choice question may have more than one answer. To get mark you have to tick all the correct options.

1. Let E and F be two events with $P(E) = 0.7, P(F) = 0.4$ and $P(E \cap F^c) = 0.4$ Then $P(F|E \cup F^c)$ is equal to _____ $\frac{1}{3}$
2. Let E and F be two mutually disjoint events. Further, let E and F be independent of G . If $P(E) + P(F) = p$ and $P(G) = q$. Then $P(E \cup F \cup G) =$ _____ $p + q - pq$
3. The distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{4} + \frac{1}{6}(4x - x^2), & 0 \leq x < 1 \\ 1, & x \geq 1 \end{cases}$$

Then $P(X = 0|0 \leq X < 1) =$ _____ $\frac{1}{3}$

4. Let X be a random variable having distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{4}{9}, & 0 \leq x < 1 \\ \frac{8}{9}, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

Then which of the following statement is (are) correct?

- (a) Then random variable takes positive probability at two points.
 - (b) $E(X) = \frac{2}{3}$
 - (c) $P(1 \leq X \leq 2) = \frac{5}{9}$
 - (d) $P(0 < X < 1) = \frac{4}{9}$
5. Let X be a discrete random variable having pmf $P(X = -2) = \frac{1}{6}, P(X = -1) = \frac{1}{3}, P(X = 0) = P(X = 1) = P(X = 2) = \frac{1}{6}$. Then which of the following is (are) correct
 - (a) $E(X^2) = \frac{11}{6}$
 - (b) The m.g.f of X is $\frac{e^t}{6} + \frac{e^{2t}}{2} + \frac{e^{4t}}{3}$
 - (c) $E(X) = \frac{15}{6}$
 - (d) The m.g.f of X^2 is $\frac{1}{6} + \frac{e^t}{2} + \frac{e^{4t}}{3}$

6. Questions are asked to Girish in a quiz competition one by one until he fails to answer correctly. The probability of his answering correctly a question is p . The probability that he will quit after answering an odd number of questions is 0.9. Then the value of p is _____ $1/9$

7. Let X be a random variable with the probability mass function

$$f_X(x) = P(X = x) = \begin{cases} \frac{1}{10}, & x = 1, 2, \dots, 10 \\ 0, & \text{Otherwise.} \end{cases}$$

Then which of the following statement is (are) correct?

- (a) $E(\max\{X, 5\}) = \frac{13}{2}$
- (b) $E(\max\{X, 5\}) = \frac{13}{12}$
- (c) $P(\max\{X, 5\} = 5) = \frac{1}{2}$
- (d) $P(\max\{X, 5\} = 5) > \frac{1}{2}$

8. Let X be a random variable having moment generation function $M_X(t) = \frac{1}{1728}(10 + 2e^t)^3$, $t \in \mathbb{R}$. Then which of the following statement is (are) correct

- (a) $E(X) = \frac{1}{2}$
- (b) $Var(X) = \frac{5}{12}$
- (c) $P(X > 1) = \frac{2}{27}$
- (d) None of the above

9. Let X be a random variable with support S_X . Then which of the following is (are) correct

- (a) $E(X^2) \leq (E(X))^2$
- (b) $E(\ln X) \leq \ln E(X)$, provided $S_X \subset (0, \infty)$
- (c) $E(e^{-X}) \geq e^{-E(X)}$
- (d) $E\left(\frac{1}{X}\right) \geq \frac{1}{E(X)}$, if $S_X \subset (0, \infty)$

10. Let X be a random variable with m. g. f. $e^{(e^t-1)/2}$, $t \in \mathbb{R}$. Then $E((1+X)!)$ equals _____ $4e^{-1/2}$.