Tutorial 6: Probability and Statistics (MAL403/IC105)

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1. Suppose 15% of items produced at a manufacturing facility are defective. What is the probability that a lot of randomly selected 10 items contains more than 3 defective items?

2. The average number of trains either arriving at or departing from a railway station is one every 5 minutes. What is the probability that at least 10 trains arrive/depart during a selected hour? What is the probability that fewer than 4 such train will take place in an hour?

3. A electronic system consists of n parts each of which function independently with probability p. The entire system will be able to operate effectively, if at least one-half of its components function. For what values of p, a 5-component system more likely to operate effectively than a 3-component system?

4. The DVD produced by a certain company are defective with probability 0.01, independently of each other. The company sells the DVDs in packs of size 10 and offers a money-back guarantee if more than one of the 10 DVDs in the pack is found to be defective. If you buy 3 packs, what is the probability that at most one pack will be returned.

5. The number of times that an individual contracts cold in a given year is a Poisson random variable with parameter $\lambda = 3$. Suppose that a new drug has been just marketed that reduces the parameter λ to 2 for 75% of the population. For the other 25% of the population the drug has no appreciable effect on the cold. If an individual tries the drug for a year and has no cold in that time, how likely is it that the drug is beneficial for him?

6. A point P is chosen at random on a line segment AB of length $2a$. Find the expected values of (i) $AP.PB$ (ii) $|AP - PB|$(iii)$\max\{AP, PB\}$.

7. In a precision bombing attack there is a 50% chance that a bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give at least 99% chance of completely destroying the target?

8. Suppose 5% of chips manufactured at a plant are defective. How many should he/she buy in order that there is more than 99% chance of having at most one defective chip?

9. Buses arrive at a specified stop at 15-minute intervals starting at 7 A.M. That is, they arrive at 7, 7:15, 7:30, 7:45, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30, find the probability that he waits (a) less than 5 minutes for a bus; (b) more than 12 minutes for a bus.

10. If a string of 2 meter is cut into two pieces at a random point along its length, what is the probability that the length of the longer piece is at least twice the length of the shorter piece?

11. There are 30 applicants for a job, out of which only 20 applicants are qualified for the job. Six applicants are selected at random from these 30 applicants. Find the probability that, among the selected candidates, at least two will be qualified for the job. (Use Hyper geometric distribution)

12. A man with n keys wants to open his door and tries the keys at random. Exactly one key will open the door. Let X denote the number of trials required to open the door for the first time. Find $E(X)$ if

    (a) unsuccessful keys are not eliminated from further selections

    (b) unsuccessful keys are eliminated

13. Ruby and Mini tied for the first place in a beauty contest. The winner is to be decided by the majority opinion of a panel of three judges chosen at random from a group of seven judges. If four of these judges favour Ruby and three favour Mini, what is the probability that Ruby will be declared the winner.