Tutorial 9: Probability and Statistics (MAL403/IC105)

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1. A boy and a girl plan to meet a coffee shop between 4 pm and 5 pm each of agreeing not to wait more than 20 minutes for the other. They arrived at the coffee shop independently between 4pm to 5 pm. What is the probability that they will meet.

2. The joint pdf of X and Y is given as

$$f_{X,Y}(x,y) = \begin{cases} k(1-x-y), & x > 0, y > 0, x+y < 1 \\ 0, & \text{Otherwise} \end{cases}$$

(a) Find the value of k (b) Find the marginal pdf of X and Y. (c) Find $E(X)$, $E(Y)$, $E(XY)$ and $Cov(X,Y)$ and $\rho_{X,Y}$.

3. A two dimensional discrete random vector (X,Y) having pmf as

$$f_{X,Y}(x,y) = P(X = x, Y = y) = \begin{cases} c(3x+4y), & x = 0,1,2,3, y = 1,2,3,4 \\ 0, & \text{Otherwise} \end{cases}$$

(a) Find the value of c (b) Find the marginal distribution of X and Y. (c) Find $P(X \geq 2|Y \leq 3)$ and $P(Y = 2|X = 3)$.

4. The joint pmf of a discrete random vector (X,Y) given as

X/Y	-1	0	1
0	0	$1/3$	0
1	$1/3$	0	$1/3$

(a) Find the marginal distribution of X and Y. (b) Find $E(X), E(Y), E(XY)$ and $\rho_{X,Y}$. (c) Are X and Y independent?

5. Let (X,Y) with the joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 6xy(2-x-y), & 0 < x < 1, \ 0 < y < 1, \\ 0, & \text{Otherwise} \end{cases}$$

(a) Find the marginal distribution of X and Y (b) Find $E(X), E(Y), E(XY)$ and $\rho_{X,Y}$. (c) Are X and Y independent?

6. Let (X,Y) with the joint pdf

$$f_{X,Y}(x,y) = \begin{cases} \frac{6-x-y}{8}, & 0 < x < 2, \ 2 < y < 4, \\ 0, & \text{Otherwise} \end{cases}$$

(a) Find the marginal distribution of X and Y

(b) $P(X < 1, Y < 3)$, $P(X + Y < 3)$, $P(X < 1|Y = 3)$ and $P(X < 1|Y < 3)$.

7. What is the probability that the sum of two numbers chosen randomly from the interval $(0, 1)$, is grater than 1, while the sum of their squares is less than 1.

8. Let X and Y be to random variable taking values independently in the interval $(-1, 1)$. Find the probability that the roots of the quadratic equation $a^2 + 2aX + Y = 0$ are real.

9. Find the value of $E(X|Y = 1/2)$ and $E(Y|X = 1/2)$ for problem (1).

10. Find $E(X|Y = 1)$ and $E(Y|X = 0)$ for problem (3).

11. Let the random variables X and Y have joint pdf

$$f_{X,Y}(x, y) = \begin{cases} ce^{-(x+y)}, & y > x > 0, \\ 0, & \text{Otherwise} \end{cases}$$

(a) Find the value of c. (b) Find the value of $E(Y|X = 2)$.

12. The conditional probability density function of X given $Y = y(> 0)$ is

$$f_{X|Y}(x|y) = \begin{cases} ye^{-yx}, & x > 0, \\ 0, & \text{Otherwise} \end{cases}$$

and the marginal pdf of Y is

$$f_Y(y) = \begin{cases} \alpha e^{-\alpha y}, & y > 0, \\ 0, & \text{Otherwise} \end{cases}$$

Find the conditional probability density function of Y given $X = x$.