Solution Tutorial-2

(1) Soln (a) $\Omega = \{(r,r), (r,g), (r,b), (g,r), (g,g), (g,b), (b,r), (b,g), (b,b)\}$

(b) $\Omega = \{(r,g), (r,b), (g,r), (g,b), (b,r), (b,g)\}$.

(2) Let A be the event that a randomly choosen person is a cigarette smoker and let B be the event that the person is cigar smoker

$P(A) = 0.28$, $P(B) = 0.07$, $P(A \cap B) = 0.05$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.28 + 0.07 - 0.05$
$= 0.30$.

(a) $1 - P(A \cup B) = 0.70$. So 70% smoke neither.

(b) $P(A^c B) = P(B) - P(A \cap B) = 0.07 - 0.05 = 0.02$.

Hence 2% smoke cigars but not cigarettes.

(3) $|\Omega| = 36$.

A be the event that the sum is 7.

$A = \{(3,4), (5,2), (2,5), (6,1), (1,6), (4,3)\}$

$P(A) = 6/36 = 1/6$.

(Q4) Let A denote the event that at least one psychologist chosen.

$A^c \rightarrow$ no psychologist chosen

$$P(A) = 1 - P(A^c) = 1 - \binom{30}{3} \Big/ \binom{54}{3} = 0.8363.$$

(Q5) Let A be the required event.

6 – W
5 – B.

$$P(A) = \frac{\binom{6}{1}\binom{5}{2}}{\binom{11}{3}}$$ is the required prob.

(Q6) A be the required event.

Then $$P(A) = \frac{\binom{13}{5}\binom{39}{8}}{\binom{52}{13}}$$

(Q7) Let E be the event that the committee have 3 men and 2 women.

$$P(A) = \frac{\binom{6}{3}\binom{9}{2}}{\binom{15}{5}} = 0.2398.$$

Q8 There are n people. So there are $(365)^n$ possible choices of birthdays for the set of n peoples.

Let E be the event where we will not able to find two or more people with the same birthday.

This event is equivalent to saying that the n people have n distinct birth-days. So there are

$$365 \times 364 \times 363 \times \cdots \times (365-n) = \binom{365}{n} n!$$

ways that n people can have n distinct birthdays.

$$P(E) = \frac{\binom{365}{n} n!}{(365)^n}.$$

So the required prob. is $= 1 - P(E)$.

$$= 1 - \binom{365}{n} n! / (365)^n.$$

9 Let E denotes the Event That the second die lands on a higher value than the first

$$E = \{ (1,2), \ldots, (1,6), (2,3), (2,4) \ldots, (2,6) \ldots \}$$

$$|E| = 5+4+3+2+1 = 15$$

$$P(E) = \frac{15}{36} = 5/12.$$

(10) E = be the event that two dice land in different no.

F = be the event that at least one 6.

$$P(E) = \frac{30}{36} = 5/6, \quad P(F) = \frac{11}{36}.$$

$$P(F|E) = \frac{P(F \cap E)}{P(E)}, \quad P(F \cap E) = \frac{10}{36}$$

$$= \frac{\cancel{10}^{2}}{\cancel{36}_{6}} \times \frac{\cancel{6}}{\cancel{5}} = 1/3.$$

11) Let $E \rightarrow$ the 1st is 6

$F_i \rightarrow$ the sum $= i$, $i = 7, 8, \cdots, 12$

$$P(E|F_7) = \frac{P(E \cap F_7)}{P(F_7)} = \frac{1/36}{1/6} = 1/6.$$

$\left[\because E \cap F = \{(6,1)\}, \quad F_7 = \{(6,1), (1,6), (5,2), (2,5), (4,3), (3,4)\}. \right]$

$$P(E|F_8) = \frac{P(E \cap F_8)}{P(F_8)} = \frac{1/36}{5/36} = 1/5.$$

$P(E|F_9) = 1/4$, $\quad P(E|F_{10}) = 1/3$, $\quad P(E|F_{11}) = 1/2$

$P(E|F_{12}) = 1.$

12) Let W_A, W_B, W_C be the events that the ball drawn from urn A, B, C respectively white.

A	B	C
2 → W	8 − W	1 − W
4 → R	4 − R	3 → R

$P(W_A) = 2/6$, $\quad P(W_B) = 8/12$, $\quad P(W_C) = 1/4$.

$E \rightarrow$ be the event exactly 2 white balls are Selected.

$$= (W_A \cap W_B \cap W_C^c) \cup (W_A \cap W_B^c \cap W_C) \cup (W_A^c \cap W_B \cap W_C)$$

So the required prob. is

$$P(W_B|E) = \frac{P(W_B \cap E)}{P(E)} = \frac{P((W_A \cap W_B \cap W_C^c) \cup (W_A^c \cap W_B \cap W_C))}{P(E)}.$$

$$= \frac{P(W_A \cap W_B \cap W_C^c) + P(W_A^c \cap W_B \cap W_C)}{P(E)}$$

$$= \frac{P(W_A \cap W_B \cap W_C^c) + P(W_A^c \cap W_B \cap W_C)}{P(W_A \cap W_B \cap W_C^c) + P(W_A^c \cap W_B \cap W_C) + P(W_A \cap W_B^c \cap W_C)}$$ [∵ events are mutually disjoint]

$$= \frac{2/6 \cdot \frac{8}{12} \cdot (1-\frac{1}{4}) + (1-\frac{2}{6}) \cdot \frac{8}{12} (\frac{1}{4})}{\frac{2}{6} \cdot \frac{8}{12} (1-\frac{1}{4}) + (1-\frac{2}{6}) \frac{8}{12} \cdot \frac{1}{4} + \frac{2}{8} \cdot (1-\frac{8}{12}) \cdot \frac{1}{4}}$$

(13) Let $S \rightarrow$ be the event the baby survive

$C \rightarrow$ be the event cesarean section

$P(S) = 0.98, \quad P(C) = 0.15$

$P(S|C) = 0.96$. We have to find $P(S|C^c)$

$$P(S) = P(S|C)P(C) + P(S|C^c)P(C^c)$$

$$= 0.96 \times 0.15 + P(S|C^c) \times 0.85$$

$\Rightarrow P(S|C^c) = 0.9835.$

(14) $P(I) = 0.46, \quad P(L) = 0.30, \quad P(C) = 0.24.$

Let $V \rightarrow$ be the event that a person voted.

$P(V|I) = 0.35, \quad P(V|L) = 0.62, \quad P(V|C) = 0.58$

$$P(I|V) = \frac{P(V|I)\,P(I)}{P(V|I)\,P(I) + P(V|L)\,P(L) + P(V|C)\,P(C)}$$

$$= \frac{0.35 \times 0.46}{0.35 \times 0.46 + 0.62 \times 0.3 + 0.58 \times (0.25)}$$

$\approx 0.331.$

(15) Let $E \rightarrow$ be the event that the coin is two-headed

$F \rightarrow$ be " " " " fair.

$G \rightarrow$ " " " biased

One coin is selected at random

$P(E) = P(F) = P(G) = 1/3.$ Let H be the

Let the selected coin is tossed.

event that head occur

$P(H|E) = 1, \quad P(H|F) = \frac{1}{2}, \quad P(H|G) = 0.75$

$$P(E|H) = \frac{P(H|E)\,P(E)}{P(H|E)\,P(E) + P(H|F)\,P(F) + P(H|G)\,P(G)} = \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{1}{2}\cdot\frac{1}{3} + \frac{3}{4}\cdot\frac{1}{3}}$$

(16) Let $E \rightarrow$ be the good Risk

$F \rightarrow$ " " average Risk

$G \rightarrow$ " Bad Risk

$P(E) = 0.2, \quad P(F) = 0.5, \quad P(G) = 0.3$

Let K be the event that a person got an accident

$p(K|E) = 0.05, \quad P(K|F) = 0.15,$

$P(K|G) = 0.30.$

$P(E) = P(K|E)\,P(E) + P(K|F)\,P(F) + P(K|G)\,P(G)$

$= 0.175$

So 17.5 % got an accident.

Let A be a policyholder had no accident in 1997

$$P(E|K^c) = \frac{P(E \cap K^c)}{P(K^c)} = \frac{P(K^c|E)\,P(E)}{P(K^c)}$$

$$= \frac{0.95 \times 0.2}{0.825} = 0.230.$$

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For this case $a, b, c \rightarrow 1, 2, 3, \ldots 6$

$ax^2+bx+c=0$ has real root is $b^2-4ac \geq 0$

$\Rightarrow b^2 \geq 4ac$

A → roots are real

b	b^2	(a, c)	cases
1	1	—	0
2	4	(1,1)	1
3	9	(1,1), (1,2), (2,1)	3
4	16	(1,1), (1,2), (2,1), (2,2) (1,4), (4,1), (1,3), (3,1)	8
5	25	(1,1), (1,2), (2,1), (2,2) (1,4), (4,1), (1,3), (3,1) (1,5), (5,1), (1,6), (6,1) (2,3), (3,2),	14
6	36	14 + (2,4), (4,2) (3,3)	17
			43

$$P(A) = \frac{43}{6^3} = \frac{43}{216}.$$