Quiz-2: Probability and Statistics (MAL403/IC105)

Indian Institute of Technology Bhilai

Name: Roll: Marks: 10, Time: 1hr

Multiple choice question may have more than one answer. To get mark you have to tick all the correct options.

1. Ramesh and Harish each independently choose real number at random between 0 and 1. Find the probability that the magnitude of the difference of the two number is less than $1/4$=_____7/16.

2. Let (X, Y) have joint bivariate normal distribution with parameters $\mu_1 = 2$, $\mu_2 = 3$, $\sigma_1^2 = 1$, $\sigma_2^2 = 4$ and $\rho = 0.5$. Then
   (a) $E(X|Y = 4) = 9/4$
   (b) $E(X|Y = 4) = \frac{2}{3}$
   (c) $Var(X|Y = 4) = 3/4$
   (d) $P(X < 3|Y = 4) = \frac{4}{9}$

3. Let $X_1, X_2, \ldots, X_{25}$ be independent and identically distributed $N(3, 5)$ random variables. Let $\bar{X} = \frac{1}{25}\sum_{i=1}^{25} X_i$ and $S^2 = \frac{1}{24}\sum_{i=1}^{25}(X_i - \bar{X})^2$. Then which of the following is (are) correct
   (a) $5(\bar{X} - 3)^2 \sim N(0, 1)$
   (b) $5(\bar{X} - 3)^2 \sim \chi_1^2$
   (c) $\frac{5(\bar{X}-3)}{S} \sim t_{24}$
   (d) $\frac{1}{5}\sum_{i=1}^{5}(X_i - 3)^2 \sim \chi_5^2$

4. Let X_1, X_2, X_3 be a random sample from a population with probability mass function

$$p(x) = \begin{cases} 1-\theta, & \text{if } x = 0 \\ \theta, & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases} \qquad \theta > 0$$

   which of the following is (are) unbiased estimator (s) of θ
   (a) $\frac{X_1+X_2+X_3}{3}$
   (b) $\frac{X_1^2+X_2^2+X_3^2}{3}$
   (c) $\frac{X_1^2+X_2^2-X_3^2}{2}$
   (d) $\frac{X_1^2+X_2}{2}$

5. Let (X, Y) be a random vector with joint probability mass function $p(x, y) = \frac{1}{25}(x + y), x = 1, 2;\ y = 0, 1, 2$. Then $P(Y = 1|X = 1) =$ _____1/4

6. Let X_1, X_2, X_3, X_4 be independent random variable with common probability density function
$$f(x) = \begin{cases} 2e^{-2x}, & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$
Define $Y = \min\{X_1, X_2, X_3, X_4\}$. Suppose $E(Y) = \mu_y$ and $Var(Y) = \sigma_y^2$. Then $P(Y > \mu_y + \sigma_y) =$ ____e^{-2}

7. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $U(2\theta - 1, 2\theta + 1)$, $\theta \in \mathbb{R}$. Then which of the following is (are) maximum likelihood estimator(s) of θ
   (a) $\frac{1}{4}(X_{(1)} + X_{(n)})$
   (b) $\frac{1}{6}(2X_{(1)} + X_{(n)} + 1)$
   (c) $\frac{1}{8}(X_{(1)} + 3X_{(n)} - 2)$
   (d) None of the above

8. Let X_1, X_2, X_3 be a random sample. Then find the value of $E\left(\frac{X_2 + X_1}{X_1 + X_2 + X_3}\right) =$ ____$2/3$.

9. Let X and Y be independent random variables such that $X \sim U(0, 2)$ and $Y \sim U(1, 3)$. Then the value of $P(X < Y) =$ ____$7/8$.

10. X and Y two independent random variables with $M_X(t) = \frac{(1+3e^t)^2}{16}$ and $M_Y(t) = \frac{e^t}{2-e^t}$, $t < \ln 2$. Then $P(X + Y = 1) =$ ____$1/32$