Tutorial 5: Probability and Statistics (MAL403/IC105)

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1. If X is a random variable such that $E(X) = 3$ and $E(X^2) = 13$, then determine a lower bound for $P(-2 < X < 8)$.

2. Let the random variable X have the m.g.f. $M(t) = \frac{e^{-2t}}{8} + \frac{e^{-t}}{4} + \frac{e^{2t}}{8} + \frac{e^{3t}}{2}$. Find the distribution function of X and find $P(X^2 = 4)$.

3. Let X be a random variable with m.g.f. $M(t),\ -h < t < h$
   (a) Prove that $P(X \geq a) \leq e^{-at}M(t),\ 0 < t < h$;
   (b) Prove that $P(X \leq a) \leq e^{-at}M(t),\ -h < t < 0$;

4. X be a discrete random variable with probability mass function given as follows

$x:$	-2	-1	0	1	2	3
$p_X(x):$	0.1	k	0.2	$2k$	0.3	$3k$

   (a) Evaluate the value of k
   (b) Obtain $P(X < 2)$ and $P(2 < X < -2)$
   (c) Find the CDF of X
   (d) Find $E(X)$ and $Var(X)$
   (e) Find m.g.f. of X

5. Let be a continuous random variable with pdf

$$f_X(x) = \begin{cases} px & 0 < x \leq 1 \\ p & 1 < x \leq 2 \\ p(3-x) & 2 < x < 3 \\ 0 & \text{Otherwise} \end{cases}$$

   (a) Find the valiue of p.
   (b) Find $E(X)$, $Var(X)$ and m.g.f.

6. For any random variable X having mean μ and finite second moment, show that $E(X-\mu)^2 \leq E(X-c)^2$ for all $c \in \mathbb{R}$.

7. Let X be a continuous random variable with distribution function $F_X(x)$ that is strictly increasing on its support. Let m be the median of (distribution of) X. Show that $E(|X - m|) \leq E(|X - c|),\ \forall\ c \in (-\infty, \infty)$.

8. (a) Let X be a non-negative absolutely continuous random variable and let h be a real-valued function defined on $(0, \infty)$. Define $\psi(x) = \int_0^x h(t)dt \ x \geq 0$, and suppose that $h(x) \geq 0$, for all $x \geq 0$. Show that

$$E(\psi(X)) = \int_0^\infty h(y)P(X > y)dy$$

(b) Let α be a positive real number. Under the assumptions of (a), show that

$$E(X^\alpha) = \alpha \int_0^\infty X^{\alpha-1}P(X > x)dx$$

(c) Let $F(0) = G(0) = 0$ and let $F(t) \geq G(t)$, for all $t > 0$, where F and G are distribution functions of continuous random variables X and Y, respectively. Show that $E(X^k) \leq E(Y^k)$, for all $k > 0$, provided the expectations exist.

9. Let X be an absolutely continuous random variable with p.d.f. $f_X(x)$ that is symmetric about $\mu \in \mathbb{R}$, i.e., $f_X(\mu + x) = f_X(\mu - x)$, for all $x \in \mathbb{R}$. If $E(X)$ finite, then show that $E(X) = \mu$.

10. Let X be a random variable with pdf $f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right]$, $x \in \mathbb{R}$. prove that $X \stackrel{\mathrm{d}}{=} -X$. Hence find $E(X^3)$ and $P(X > 0)$.