1 Let X denote the r.v. daily consumption of oil in city in excess of 30,000 gallons.

$X \sim \text{Gamma}\left(2, \frac{1}{10000}\right)$

$$f_X(x) = \begin{cases} \left(\frac{1}{10000}\right)^2 x e^{-x/10000}, & x > 0 \\ 0 & \text{o/w.} \end{cases}$$

$E(X) = 20000$

The required prob is $P(X > 10000)$

$$= \int_{10000}^{\infty} \frac{x}{(10000)^2} e^{-x/10000}, \quad \text{let } y = \frac{x}{10000}$$

$$= \int_{1}^{\infty} y e^{-y} dy = 2e^{-1}.$$

(2) X denote the the time to failure (in years).

Then a density of X is

$$f_X(x) = \begin{cases} \frac{1}{8} e^{-x/8}, & x > 0 \\ 0 & \text{o/w.} \end{cases}$$

Then the percentage of TV's will fail with warranty period is $= P(X<1) \times 100\% = 0.1175 \times 100\%$

$= 11.75\%$

$E$ ( Profit on sell of one TV ) $= 10000\, P(X>1) -$

$15000\, P(X \leq 1) = \alpha$ (say)

(Find $\alpha$)

$E$ ( Profit on 1000 TV ) $= 1000 \times \alpha$.

3) Given that the lead time of oders of diodes from a certain manufacturer follow a Gamm $(r, \lambda)$ where $\frac{r}{\lambda} = 20$, $\frac{r}{\lambda^2} = 100$

$\Rightarrow r = 4$, $\lambda = 1/5$. Let $X$ denote the lead time $X \sim$ Gamma$(4, 1/5)$

Required prob $P(X \leq 15) = \int_0^{15} \frac{1}{5^4} \frac{x^3 e^{x/5}}{\Gamma(4)} dx.$

4) Let $X$ denote the no. of defects in a 2% area of the total ~~surg~~ surface.

$X \sim \mathcal{P}(\lambda)$, $\lambda = 300 \times 0.02 = 6$.

Required prob

$$P(X \leq 4) = \sum_{x=0}^{4} \frac{e^{-6} 6^x}{x!} \cong 0.285.$$

5) Let $X$ denote the life of a bulb in hours

Given that

$X \sim Exp(\lambda)$, with $E(X) = \frac{1}{\lambda} = 50$

$$\Rightarrow \lambda = \frac{1}{50}.$$

$$f_X(x) = \begin{cases} \frac{1}{50} e^{-x/50}, & x > 0 \\ 0, & o/w. \end{cases}$$

Now $P$(A bulb working after 100 Hrs) $= P(X > 100)$

$= e^{-2}$

Let $Y$ denote the no of bulb working after 100 hrs. Then $Y \sim Bin(10, e^{-2})$, $[p = e^{-2}]$

$$P(Y \geq 2) = 1 - P(Y=0) - P(Y=1)$$
$$= 1 - (1-e^{-2})^{10} - 10(e^{-2})(1-e^{-2})^9$$

[redacted]

6) $X$ denotes the time (in hours) needed to locate and rectify a prob. $X \sim N(10, 9)$

The required prob is $P(X \le 15)$

$$= P\left(\frac{X-10}{3} \le \frac{15-10}{3}\right)$$

$$= P(Z \le 5/3) = \Phi(5/3) = 0.9525$$

7) Prob. of giving opinion in favor is $p = \frac{1}{2}$

so $q = 1/2$, $n = 100$, $np = 50$.

$npq = 25$.

$X \rightarrow$ no of adults in favour of the project

$X \sim Bin(100, \frac{1}{2})$

$$P(X \geq 60) = P\left(\frac{X - np}{\sqrt{npq}} \geq \frac{60 - np}{\sqrt{npq}}\right).$$

$$= P\left(\frac{X-50}{\sqrt{25}} \geq \frac{60-50}{\sqrt{25}}\right) \approx P(Z \geq 2)$$

$$= 1 - \Phi(2) = 1 - 0.9772 = 0.0228.$$

[redacted]

8 X denote the length of dimeter

8

$$X \sim N(3, 0.005^2)$$

Required prob. $P$ (ball bearing is scarpped)

$$= 1 - P(2.99 < X < 3.01)$$

$$= 1 - P(-2 < Z < 2)$$

$$= 2\Phi(-2) \qquad [\because \Phi(2) + \Phi(-2) = 1]$$

$$= 2 \times 0.0228 = 0.0456.$$

9) X be the hight of high jumper will cleare.

$$X \sim N(200, 100)$$

Let a be such that $P(X > C) = 0.95$

$$\Rightarrow P\left(\frac{X-200}{10} > \frac{C-200}{10}\right) = 0.95$$

$$\Rightarrow P\left(Z > \frac{C-200}{10}\right) = 0.95$$

$$\Rightarrow P\left(Z \leq \frac{C-200}{100}\right) = 0.05$$

$$\frac{C-200}{100} = -1.645 \Rightarrow \frac{200-C}{100} = 1.645$$

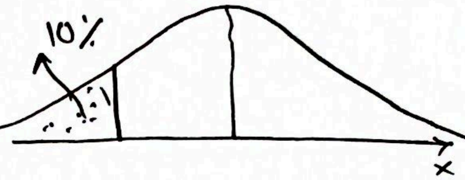
$$\Rightarrow C = 183.55 \text{ cm}.$$

[redacted]

Further d is s.t.

$$P(X>d) = 0.1 \Rightarrow \frac{200-d}{10} = -1.28 \Rightarrow d = 212.80 \text{ cm}.$$

10 X denote the marks. $X \sim N(74, 62.41)$




ⓐ

$$P(X<c) = 0.1$$

$$P\left(Z < \frac{c-74}{\sqrt{62.41}}\right) = 0.1 \Rightarrow \frac{c-74}{\sqrt{62.41}} = -1.28$$

$$\Rightarrow c \approx 64.$$

lowest passing marks = 64.

ⓑ

$$P(X>d) = 0.05$$

$$P\left(Z \le \frac{d-74}{\sqrt{62.41}}\right) = 0.95$$

$$\frac{d-74}{\sqrt{62.41}} = 1.645 \Rightarrow d \approx 86.99.$$

So highest of B is 86.