Tutorial 1: Probability and Statistics (MAL403/IC105)

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1. Let $\mathscr{F}$ be an algebra. Let $A_1, A_2, \cdots A_n$ belongs to $\mathscr{F}$ (i) Then prove that $\bigcup_{i=1}^{n} A_i \in \mathscr{F}$ and $\bigcap_{i=1}^{n} A_i \in \mathscr{F}$.

2. Let f be a mapping from a set X into a set Y. Let $\mathscr{F}$ be a σ- algebra of subsets of Y. Then prove that $f^{-1}(\mathscr{F})$ is a σ- algebra of subsets of X.

3. Let $\mathscr{A}$ be a σ-algebra of subsets of a set X and let Y be an arbitrary subset of X. Then $\mathscr{D} = \{A \cap Y : A \in \mathscr{A}\}$. Show that $\mathscr{D}$ is a σ-algebra of subsets of Y.

4. Let $\mathscr{F}$ be a collection of subsets of a set Ω with the following properties: $\Omega \in \mathscr{F}$ and if $A, B \in \mathscr{F}$ then $A - B = A \cap B^c \in \mathscr{F}$. Prove that $\mathscr{F}$ is an algebra.

5. Let $\mathscr{F}$ be an algebra of subsets of a set X. Suppose $\mathscr{F}$ has the property that for every increasing sequence $\{A_n\}$ in $\mathscr{F}$, we have $\bigcup_{n=1}^{\infty} A_n \in \mathscr{F}$. Show that $\mathscr{F}$ is a σ-algebra of subsets of the set X.

6. Let Ω be an arbitrary infinite set. We say that a subset A of Ω is co-finite if A^c is a finite set. Let $\mathscr{F}$ consist of all the finite and the co-finite subsets of a set Ω .(a) Show that $\mathscr{F}$ is an algebra of subsets of Ω. (b) Show that $\mathscr{F}$ is a σ-algebra if and only if Ω is a finite set.

7. Define a sequence of sets $\{I_n\}$ where $I_n = \left\{x \in \mathbb{R} : 0 < x < \frac{1}{n}\right\}$. Show that $\bigcap_{n=1}^{\infty} I_n = \phi$.

8. Let $\mathscr{G}$ and $\mathscr{H}$ be to collection of subsets of Ω such that $\mathscr{G} \subseteq \mathscr{H}$. Then prove that $\sigma(\mathscr{G}) \subseteq \sigma(\mathscr{H})$

9. $\mathscr{F}$ be a σ-algebra of subsets of a set Ω. Prove that $\sigma(\mathscr{F}) = \mathscr{F}$. If $\mathscr{C}$ is collection of subsets of Ω then prove that $\sigma(\sigma(\mathscr{C})) = \sigma(\mathscr{C})$.

10. A set function μ on a sigma field $\mathscr{F}$ in Ω satisfies the following conditions

    (i) $\mu(A) \in [0, \infty]$

    (ii) $\mu(\phi) = 0$

    (iii) Let $A_1, A_2, \ldots$ a sequence of disjoint $\mathscr{F}-$ sets then $\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i)$.

    Let μ be a finite that is $\mu(\Omega) < \infty$. Define $P(A) = \frac{\mu(A)}{\mu(\Omega)}$ for $A \in \mathscr{F}$. Prove that P is a probability function on $(\Omega, \mathscr{F})$.

11. Let P_1 and P_2 be two probabilityfunction defined on $(\Omega, \mathscr{F})$. Then prove that $\alpha P_1 + (1-\alpha)P_2$, $\alpha \in [0, 1]$ is a probability function.