① 15% of items produced at a manufacturing facility are defective

Take $p = \frac{15}{100}$

Let X denote the no. of defective items in a lot of 10 items. So $X \sim \text{Bin}\left(10, \frac{15}{100}\right)$

Required probability is

$$P(X>3) = 1 - \sum P(X \le 3)$$

$$= 1 - \sum_{i=0}^{3} \binom{10}{i} (p)^{i} (1-p)^{10-i}$$

② Let X no of train arriving or departing from a railway Station. Then $\lambda = 1/5$

Then $X \sim \mathcal{P}\left(\frac{1}{5}\right)$,

$$\lambda t = \frac{1}{5} \times 60 = 12$$

$$P(X \geq 10) = 1 - P(X < 10) = 1 - \sum_{k=0}^{9} \frac{12^{k} e^{-12}}{k!}$$

$$P(X \leq 4) = \sum_{k=0}^{3} \frac{12^{k} e^{-12}}{k!}.$$

③ Let P_n denote the probability of an n-component system operate effectively

X be the number of components functioning in a n component system.

$P_3 = P(X > 1.5) = \binom{3}{2} p^2 (1-p) + \binom{3}{3} p^3$

$P_5 = P(X > 2.5) = \binom{5}{3} p^3 (1-p)^2 + \binom{5}{4} p^4 (1-p) + p^5$

5 component system is better if

$$P(X > 2.5) > P(X > 1.5)$$

$$10 p^3 (1-p)^2 + 5 p^4 (1-p) + p^5 > 3 p^2 (1-p) + p^3$$

$$\Rightarrow 3(p-1)^2 (2p-1) > 0 \Rightarrow p > 1/2$$

④ The DVD produced by a company are defective with prob $p = 0.01$, independently each other

Let X = no of defective DVD in a pack of 10 DVD

$X \sim Bin(10, p)$. The prob that a pack will be returned is

$p_1 = P(X > 1) = 1 - P(X \le 1) = 1 - P(X=0) - P(X=1)$

$= 1 - (0.01)^0 (0.99)^{10} - 10 (0.01)^1 (0.99)^9$

Y = no of pack will be reduced form 3 packs.

$Y \sim Bin(3, p_1)$.

Required prob. is

$$P(Y \le 1) = P(Y=0) + P(Y=1)$$

(5) Let A be the event that person gets a cold

B denote the event drug is beneficial to him.

X be the number of times an individual contracts the clod in a yeare

$$X|B \sim \mathcal{P}(2), \qquad X|B^c \sim \mathcal{P}(3)$$

P(A person does not get cold | drug is benificial) $= P(A^c|B^c)$

$$= P(X=0|B) = e^{-2}$$

P(A person not get cold | drug is not benificial) $= P(A^c|B^c)$

$$P(X=0|B^c) = e^{-3}$$

$$P(B) = 0.75, \qquad P(B^c) = 0.25$$

P(Drug is benificial to him | the person does not get cold)

$$= P(B|A^c) = \frac{P(A^c|B)\,P(B)}{P(A^c|B)\,P(B) + P(A^c|B^c)\,P(B^c)}$$

(6) Let X denote the length of AP.

$PB = (2a - X)$. $\quad X \sim U(0, 2a)$

$$f_X(x) = \begin{cases} \frac{1}{2a}, & 0 < x < 2a \\ 0, & o/w \end{cases}$$

$$E(AP \cdot PB) = \int_0^{2a} x(2a-x) f_X(x)\,dx = \frac{2a^2}{3}$$

$$|AP - PB| = 2|X - a|$$

$$E|AP - PB| = 2\int_0^{2a} |x-a| f_X(x)\,dx$$

$$= 2\int_0^{a} (a-x) f_X(x)\,dx + 2\int_a^{2a} (x-a) f_X(x)\,dx$$

$$= a.$$

(iii) $\max\{AP, PB\} = \max\{X, 2a - X\} = \begin{cases} 2a - X & \text{if } X < a \\ x & \text{if } X \geq a. \end{cases}$

$$E(\max\{AP, PB\}) = \int_0^{a} (2a - x) f_X(x)\,dx + \int_a^{2a} x f_X(x)\,dx$$

$$= \frac{3}{2}a.$$

(7) Suppose n bombs are dropped and let X denote the number of direct hits. Then $X \sim Bin\left(n, \frac{1}{2}\right)$

We want n s.t.

$$P(X \geq 2) \geq 0.99$$

or $$1 - P(X=0) - P(X=1) \geq 0.99$$

or $$P(X=0) + P(X=1) \leq 0.01$$

or $$\left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^n \leq 0.01$$

or $$2^n \geq 100(n+1) \quad \text{——} \; (*)$$

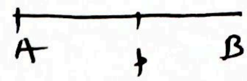
The smallest value of n for which $(*)$ is satisfied is $n=11$.

(8) – same as 7 do yourself.

(9) Let $X \to$ time in minutes past 7.00 a.m. that the passenger arrive at the bus stop. Then $X \sim U(0,30)$.

The passenger will have to wait less than 5 min, if he/she arrives between 7:10 & 7:15 or between 7:25 & 7:30 am. (a) Hence required prob $P(10<X<15) + P(25<X<30) = \frac{5}{30} + \frac{5}{30} = \frac{1}{3}$

(b) Similarly the P(wait at least 12 min) $= P(0<X<3) + P(15<X<18)$
$= \frac{1}{5}$.



(10) Let p be the cut point.

Let X be the length of AP.

$X \sim U(0,2)$

Then the required prob is

$$P(\max\{X, 2-X\} \geq 2\min\{X, 2-X\})$$

There are two possibilities. If X is bigger than $(2-X)$

Then $X > 2(2-X) \Rightarrow X > \frac{4}{3}$. —(I)

If X is smaller than $(2-X)$ Then

$(2-X) \geq 2X \Rightarrow X < \frac{2}{3}$ —(II)

Now $\max\{X, 2-X\} \geq 2\min\{X, 2-X\}$ will hold if any one of (I) & (II) holds and $X < \frac{2}{3}$ & $X > \frac{4}{3}$ are mutually disjoint

$$P(\max\{X, 2-X\} \geq 2\min\{X, 2-X\}) = P\left(X < \frac{2}{3}\right) + P\left(\frac{4}{3} < X < 2\right) = \frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\left(\frac{2}{3}\right) = \frac{2}{3}.$$

(11) Let X denote the number candidate qualified for job.

$X \sim Hyp(20, 6, 30)$ $\quad$ $(K = 20,\ n = 6$
$N = 30)$

Required prob.

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$

$$= 1 - \frac{\binom{20}{0}\binom{10}{6}}{\binom{30}{6}} - \frac{\binom{20}{1}\binom{10}{5}}{\binom{30}{6}}$$

(12) (a) Let X be the number of trials needed to open the door. Let S denote success if door opend and f denote failure.

We have $P(S) = \frac{1}{n}$. $P(f) = \frac{n-1}{n}$.

Now X is the number of trial need to open the door

If $X = K$ i.e.. $\underbrace{f\,f\,f \cdots f}_{K-1}\, \underset{\sim}{s}$

$$P(X=K) = \left(\frac{1}{n}\right)\left(\frac{n-1}{n}\right)^{K-1}$$

So $X \sim Geo\left(\frac{1}{n}\right)$, $E(X) = \frac{1}{p} = n$.

(b) If we remove the unsuccessful keys. Then it can take at most n attempts to open the door with the following probabilities. $X \to$ denote # trials

$$P(X=1) = \frac{1}{n}$$

$$P(X=2) = \frac{n-1}{n} \cdot \frac{1}{n-1} = \frac{1}{n}$$

$$P(X=3) = \frac{n-1}{n} \frac{n-2}{n-1} \cdot \frac{1}{n-2} = \frac{1}{n}$$

$$P(X=4) = \frac{n-1}{n} \cdot \frac{n-2}{n-1} \cdot \frac{1}{n-2} = \frac{1}{n}.$$

$$P(X=n) = \frac{n-1}{n} \cdot \frac{n-2}{n-1} \cdot \frac{n-3}{n-2} \cdots \frac{1}{2} \cdot 1 = \frac{1}{n}.$$

i.e. $X \sim U(1, 2, \ldots n)$ is a discrete uniform dist^n

$$E(X) = \frac{n+1}{2}.$$

13 $$P(\text{Ruby will win}) = \frac{\binom{4}{3} + \binom{4}{2}\binom{3}{1}}{\binom{7}{3}} = \frac{22}{35}.$$